ON THE UTILITY OF SORNETTE’S CRASH PREDICTION MODEL WITHIN THE ROMANIAN STOCK MARKET

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Abstract:
Stock market crashes have been a constant subject of interest among capital market researchers. Crashes’ behavior has been largely studied, but the problem that remained unsolved until recently, was that of a prediction algorithm. Stock market crashes are complex and global events, rarely taking place on a singular national capital market. They usually occur simultaneously on several if not most capital markets, implying important losses among the investors. Investments made within various stock markets have an extremely important role within the global economy, influencing people’s lives in many ways. Presently, stock market crashes are being studied with great interest, not only because of the necessity of a deep understanding of the phenomenon, but also because of the fact that these crashes belong to the so-called category of “extreme phenomena”. Those are the main reasons that determined scientists to try building mathematical models for crashes prediction. Such a model was built by Professor Didier Sornette, inspired and adapted from an earthquake detection model. Still, the model keeps many characteristics of its predecessor, not being fully adapted to the economic realities and demands, or to the stock market’s characteristics. This paper attempts to test the utility of the model in predicting Bucharest Stock Exchange’s price falls, as well as the possibility of it being successfully used by investors.

Keywords: Sornette’s Model, stock market, speculative bubble, crash prediction

JEL Classification: C58, G14, G15, G17

1. Introduction

Stock market crashes are seen as a very interesting subject among scientists, who consider these extreme events as part of the “complex systems” theory. Within such a complex system, collective and simultaneous large-scale behaviors and actions are likely to appear. These complex structured behaviors appear from traders’ continuous and repeated interactions, which are often impossible to predict. Under these circumstances, the stock market becomes a complicated multilevel system, which depends on the time and size of the interactions occurring between traders.

In terms of spatiality, the global stock market is composed of all the national stock exchanges, which are correlated and interconnected which other, functioning through the interactions and the relations established among traders. In reality, the evolution of the stock market can be highly unsymmetrical. Periods of sustained, continuous growth can alternate with moments of sudden decrease that can occur within a single day. These patterns are due to complex economic, social, political and psychological reasons, difficult to foresee and sometimes difficult to even post-explain. Lately, however, the possibility of prediction of the crash moments of the stock market became possible.

Usually, most scientific papers tend to explain the crash mechanism based on very short time scales. An interesting and very different approach is the one in Professor Didier Sornette’s papers. Professor Sornette considers that the determining cause of the crash is to be found months, or even years before its occurrence, within the escalating levels of market participants’ cooperation or multiplying interactions between traders, which often triggers a quick ascension of the market prices. This phenomenon previously described is called “speculative bubble” by the capital market scientists. Form this point of view, a crash is mainly triggered by an endogenous cause, within a trading environment characterized and determined by a systemic instability, which, often, develops into an augmented growth of the stock prices, without any economic or financial foundation. The rest of the causes contributing to the crash are regarded as secondary. Subsequently, the problem remains to explain how such large-scale behavioral patterns develop from various interactions among traders, starting from small time frames and scales and evolving onto big time frames and scales.

Specialists consider that the actual moment of the crash presents numerous differences in respect to a normal trading day. In other words, a proper definition of the crash is necessary, in order to be able to determine if a crash is only a great price fluctuation or it is indeed an anomaly. The method preferred by specialists is to calculate and analyze the daily returns (in fact, the daily modifications of the main indexes). The analysis made by Professor Didier Sornette will be resumed below. The following figure presents the distributions of the daily returns of the two most important

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American Stock Exchange Indexes. These two indexes are Dow Jones Industrial Average and Nasdaq. The analyzed period is 2nd of January 1990 – 29th of September 2000. It can be easily noticed the exponential nature of the graph. The normal trading days, having average daily returns are extremely well approximated by the distribution graph. Extremely large returns present an obvious tendency of deviation from the distribution graph.

Fig. 1. The distribution of the daily returns for Dow Jones Industrial Average and Nasdaq Indexes.
(Source: Sornette, 2002)

The convexity of Nasdaq Index, which leads to a right tailed asymmetrical distribution, conducts to the conclusion of the presence of an important drawdown risk. Professor Sornette calculates, under the assumption of the independency of returns, the probability of an occurrence of the return variation equal to 10 standard deviations, which is $p = 0.000045$. This value corresponds to one event at 22026 days, or one such event in 88 years. The probability of occurrence of the crash of 19th of October 1987, known as “The Black Monday”, (with an attached drawdown of 22.62%) corresponds to the possibility of having a similar phenomenon once in 5.2 million years. This fact confers the crash phenomenon the attributes of an “outlier”. So, under these circumstances, Professor Sornette proved that a 10% variation of the daily returns cannot be included in the category of “outliers”.

So, the crash is defined by the specialists as a major drawdown of the financial market. A negative variation of the stock prices is included in the “crashes” category if the drawdown consists of at least 20% of the main stock index value. The main conclusion drawn from here is that a crash cannot occur every day. Taking into consideration this conclusion, the risk rate associated with the possibility of occurrence of a crash, crucially depends of whether a crash had or not appeared in the previous period. The more one crash occurred in the previous period, the smaller the probability of occurrence of another crash in the current days.

2. A Model of Crash Prediction

The hypothesis approached by the econo-physicists states that the first causes that determine a financial crash appear with the occurrence of strong correlations within the stock market. The reason for this fact is that the correlations mentioned above determine large-scale collective behaviors of the traders within the market, radically affecting the market’ allocational efficiency. During a certain time period, the phenomenon described above has as a result the stock market collapse. Sornette’s team of researchers stated that this collapse is, in fact, a critical point. He supposed that the stock market crash could be determined by the imitative behavior of the traders. This imitative behavior is the one that determines the explosion of the speculative bubble. If the traders’ imitation tendency grows sufficiently as to reach a critical point, most of the traders will place simultaneously the same market order, causing this way the stock market’s crash. The progressive growth of the imitation level conducts to the conclusion that the crash is not a natural consequence of the existence of a speculative bubble. The probability of a crash occurrence is given by the probability that the crash would occur in the next moment, taking into consideration that it didn’t occur in the current moment. Since the crash doesn’t necessarily follow the speculative bubble, it is logical to say that the investors would rather stay on a market where the speculative bubble phenomenon is present, because the assumed risk (knowing that a crash could occur anytime) is compensated by the return growth rate. This phenomenon brings to the investors higher and higher profits. Another reason for the investors to stay on the market is their hope that the bubble would not end with a crash, but it will have a slow ending.
Sornette noticed a resemblance between the log-periodic functions and the evolution of prices before the crash, especially the evolution of stock markets indexes. That was the reason for him to try to use for the financial crashes prediction, models derived from the mathematical models used to detect and predict earthquakes. The resemblance between the graphs of evolution of the ion concentrations in ground-water before the earthquake and the evolution of stock prices before the crash is illustrated in the figures presented below.

Fig. 2 The evolution of the Cl ion concentration in the groundwater before Kobe earthquake, Japan, 1995  
(Source: Sornette, 2002)

Fig. 3 The evolution of the Mg ion concentration in the groundwater before Kobe earthquake, Japan, 1995  
(Source: Sornette, 2002)

Fig. 4 The 23.6% drawdown of the Dow Jones Industrial Average in October 1929  
(Source: Sornette, 2002)

Fig. 5 The drawdown of the NASDAQ Index in October 1987  
(Source: Sornette, 2002)

In the last two graphs presented above, illustrating possible endings of the speculative bubble, one can notice log-periodic oscillations. This type of graphs made Sornette think that the mathematical and physical model used to predict earthquakes (based on the log-periodic oscillations of the ions in the groundwater) could be adapted to the very particular characteristics of the stock market in order to predict crashes (based, this time, on the log-periodic oscillations of the stock’s close-price).

Inside the stock market, there are mechanisms and behaviors of the investors which can lead to identical actions. Such a mechanism is the imitative behavior. It has been scientifically proven (Shiller, 2000) the existence of large-scale collective behaviors derived from imitation, inside the stock market.

All stock market investors are part of a system within which they enter into various relationships with their family, friends, colleagues, acquaintances, etc., all of whom can become sources of information and influence for each other. Suppose we have investor X, acting on the stock market. In the vicinity of investor X, all those with whom he comes into contact are considered to be included. Other sources of influence are newspapers, web sites, television, etc. Thus, if the investor is directly connected to k "neighbors", then there are two forces of influence acting upon him:  
a) the opinions of those k people he comes into contact with, as well as the influence of the media;  
b) his personal opinions, which may be acquired or generated by him.

Under the concept of "group behavior", it is assumed that investors tend to imitate the "neighbors" rather than contradict them. Thus it can be said that force a) will lead to group behaviors (it will create a so-called "order" between the behaviors of stock market investors, meaning that, at some point, everyone will want to do the same thing), and force b) will oppose the "ordering", making investors aim towards individual behaviors. The competition between these two states, "order" and "disorder" is expected to lead to the emergence of critical phenomena.
In other words, we can note with \( i \) each of the stock market investors, \( i = 1, \ldots, I \), while \( N(i) \) stands for the set of those who have direct contact with \( i \). If we randomly choose investor \( X \), then \( N(X) \) will be the number of those entering in direct contact with \( X \) and with whom \( X \) can exchange information, so that have a direct influence upon him. For simplicity, we will assume that each investor resembling \( X \) can be found in one of several possible states. In the simplest version of this assumption, we have only two possible states as follows: \( s_X = -1 \) and \( s_X = +1 \). These two states can be interpreted as "investor \( X \) places a buying order" and "investor \( X \) places a selling order" or "\( X \)'s portfolio return is on an upward trend" and "\( X \)'s portfolio return is on a downward trend" or even "investor \( X \) acts following an optimistic scenario" and "investor \( X \) acts following a pessimistic scenario", and so on.

If investor \( X \) has considered only the information on the actions \( s_j(t-1) \) undertaken yesterday (at the time \( t-1 \)) by the set \( N(X) \) (\( X \)'s "neighbors"), this means that at the time \( t-1 \), \( X \) tried to maximize his profit by undertaking action \( s_X(t-1) \), action determined by the sign of the sum of actions \( s_j(t-1) \) undertaken by the set of his "neighbors", \( N(X) \). In other words, investor \( X \)'s optimal decision, based on his "neighbors"' decisions (which he sees as a market "barometer") represents the imitation of group \( N(X) \)'s behavior.

This optimal decision can change when the investor decides to follow his own intuition, rather than accepting his group’s influences. Such a behavior can be modeled using an independent stochastic component of the "neighbors" decisions or of any other investor’s decisions. The general reason for which \( X \) will consider that \( N(X) \) group’s decisions are the optimal ones resides in the fact that prices will tend to evolve in the direction of the actions of the majority, due to the supply – demand ratio.

Let’s consider a network of \( N \) investors who mutually exchange information. This network will describe the relations between any two investors in the world (direct or indirect). Investors buy or sell an asset at price \( p(t) \), which evolves as a function of time. Time is considered to be a discrete variable, measured in units of time \( \Delta t \). In the simplest version of the model, each investor can either buy or sell a single unit of a single asset. This is quantified by the state \( s_i = +1 \) when buying and by the state \( s_i = -1 \) when selling. Each investor may trade at time \( t-1 \) and at the price \( p(t-1) \), based on the information held by him until the time \( t-1 \), inclusively. The price variation is considered to be proportional to the amount \( \sum_{i=1}^{N} s_i(t-1) \) (the sum of all market investors’ states). If this amount is zero, there is an equal number of sellers and buyers and the price does not change because of the state of equilibrium between supply and demand. Conversely, if this amount is positive, there are more buy orders than sell orders for the asset and its price will increase according to the new supply – demand ratio.

Within the stock market there are many influences acting on the daily price variations. They can be quantified using a stochastic component added to the price change. Taken separately, this term describes a regular "random-walk" process. The supply – demand ratio and the imitation effect lead to a more organized (meaning easier to understand and predict) market behavior.

At the time \( t-1 \) corresponding to price \( p(t-1) \), the investor defines his strategy \( s_i(t-1) \), which he will apply from time \( t-1 \) to time \( t \). From applying the chosen strategy, he will have either profit or loss, based on the difference \( \{p(t)-p(t-1)\} \). To define the optimal strategy, the investor must establish the expected profit \( P_E \) and then, depending on his information, to choose the strategy \( s_i(t-1) \) that maximizes his profits. Since the price evolution will take place according to the general opinion, given by \( \sum_{i=1}^{N} s_i(t-1) \), the best possible decision would be to buy if this amount is positive and to sell if this amount is negative. To be able to follow the described mechanism, each investor \( i \) needs to know the probabilities \( P_+ \) and \( P_- \), corresponding to those cases in which each of the other investors would buy or would sell. If he knows these probabilities, he will be able to form an opinion on the future price changes. Within a market where the number of those wishing to sell is equal to the number of those wishing to buy, these probabilities will satisfy the relationship:

\[
P_+ = P_- = 1/2.
\]

Based on the previously stated rule, that the price change is proportional to the sum of all market investors’ states \( \sum_{i=1}^{N} s_i(t-1) \), investor’s \( i \)'s best guess about the future price trend would be to calculate the sum of his "neighbors" states, hoping that it will provide a fairly faithful approximation of all investors’ actions. Then, the strategy that maximizes his profits will be the one given by the sign of the sum of his "neighbors" actions:
The function $\text{sign}(x)$ is defined as follows:

$$\text{sign}(x): \mathbb{R} \rightarrow \{-1,0,1\} \text{ with } \text{sign}(x) = \begin{cases} -1, & x \in (-\infty,0) \\ 0, & x = 0 \\ 1, & x \in (0,\infty) \end{cases},$$

where:

- $s_j$ = every stock market investors’ state;
- $K$ is a positive constant, expressing the relationship between the price evolution and the total number of buy or sell orders. $K$ is inversely proportional to the market size: the larger the market, the smaller the relative impact of a certain imbalance between buy or sell orders, at a smaller price change.
- $\varepsilon_i$ is a "noise" and $N(i)$ is the number of "neighbors" to whom the investor has significant contact.

In common terms, the rule described by (1) argues that the best investment decision for an investor is to use his "neighbors" known strategies, being aware of some certain risk given by the possibility that most of his "neighbors" to provide him with an incorrect forecast regarding the overall market evolution.

The imitative behavior discussed and described in the relationship above belongs to a general class of stochastic models, developed in order to describe interacting elements, particles, agents, in various contexts, particularly in the fields of physics and biology. The tendency towards imitation is governed by coefficient $K$. The tendency towards individual behaviors is governed by "the noise" $\varepsilon_i$. This leads to the conclusion that the percentage represented by $K$’s value towards $\varepsilon_i$’s value, gives the result of the confrontation between "order" and "disorder", finally providing information on market prices.

When the mass behaviors begin to appear, and the effect of imitation is more and more powerful, the system becomes more sensitive to disturbances that are significant, although they are becoming smaller and smaller. Eventually, the system becomes unstable: a very small external perturbation can cause a large reaction from investors, dramatically affecting their decisions. This way, large and sudden imbalances between supply and demand can be reached, this being the first step towards a crash.

A crash is not a certain event but it is characterized by a risk rate $h(t)$, which is actually the probability per unit of time that the crash to happen next, considering that it has not already occurred. This fact influences the general evolution of prices. The risk rate of a crash occurrence encompasses uncertainties that concern the investors, such as the date of the next crash. In terms of stock prices, the crash appears when the state of "order" settles (due to the imitation effect everyone wants to do the same thing: all investors place sell orders), and within a normal state there will be "disorder" (buyers and sellers disagree with each other and not everyone places sell orders).

The easiest way to describe a process of imitation is to assume that risk rate evolves according to the following relationship:

$$\frac{dh}{dt} = Ch^{\delta}, \text{ cu } \delta > 1,$$

where:

- $\frac{dh}{dt}$ = risk rate derivative with respect to time;
- $C$ is a positive constant. Researchers try to find a unified form of expression through which to quantify the various investor behaviors caused by their interactions. In this regard, $h(t)$ expresses the collective result of investors’ interactions.

The term $h^{\delta}$ means that the risk rate will increase or decrease depending on the interactions between investors. The exponent $\delta > 1$ quantifies the number of $\delta - 1$ interactions to which an investor is subjected to. By integrating the above relationship yields:

$$h(t) = \frac{B}{(t_c - t)^\alpha}, \text{ where } \alpha = \frac{1}{\delta - 1},$$

where:

- $B$ is a positive constant;
- $t_c$ is the critical time.

The critical time $t_c$ is determined by the initial conditions, from the selected time origin. For economic reasons, $\alpha$ should be between zero and one. This condition leads to the conclusion that $2 < \delta < +\infty$: an investor enters
into interaction with more than another investor. \( t_c \) is not the crash, but the end of the speculative bubble. It is also most probably the crash time also, as the risk rate is at its maximum rate at this time. However, given its probabilistic nature, the crash may also occur some other time, with a probability that changes over time with the change of the risk rate. There is a probability

\[ 1 - \int_0^t h(t) dt > 0 \]

that the speculative bubble end "smoothly" and not with crash. This residual probability is crucial for the model correctness, as otherwise all market participants should anticipate the crash time and would exit the market before its occurrence.

Moreover, the \( \alpha \) exponent can be a complex number. The solution for the risk rate is given by:

\[ h(t) = B_0 (t_c - t)^\alpha + B_1 (t_c - t)^\alpha \cos[\omega \ln(t_c - t) - \psi] \]

where: \( \psi \) and \( \omega \) are real constants.

The risk rate "explodes" in the vicinity of the critical point. In addition, it describes log-periodic oscillations. In order to build a relation between the risk rate and the price trend, the following relation is found:

\[ dp = k[p(t) - p_1]h(t) dt \]

where: \( p_1 \) is the reference value.

The researchers adapted the earthquake prediction model, finding that the price evolution before the crash is given by:

\[ p(t) = p_c - \frac{k}{z} \left[ B_0 (t_c - t)^\alpha + B_1 (t_c - t)^\alpha \cos[\omega \ln(t_c - t) - \varphi] \right] \]

\[ z = 1 - \alpha \in (0,1) \]

\( p_c \) is the price at the critical time (or the crash time). The key characteristic is that the log-periodic oscillations occur in the stock’s price before the critical time.

The key feature is that the oscillations occur within the asset price before the critical time. This means that local maxima of the function are separated by intervals tending to zero, evolving in geometric progression, so that the ratio of this progression is \( \lambda = e^{\omega} \). Such oscillations are more graphically visible than a simple power law.

The logical structure of this model states that the evolution of the stock market is influenced by the risk of occurrence of a crash. Moreover, the imitation effect and the large-scale collective behaviors determine the level of the crash risk. Still, the model does not describe the way the large-scale collective behaviors appear, it only assumes their presence inside the market.

3. Testing the Effectiveness of the Model on the Romanian Stock Market

The effectiveness of the model has been tested within the Romanian Stock Market, using the evolution of BET Index, the main Index of Bucharest Stock Exchange (BSE). The model’s parameters were determined taking into account BET’s 11.22% fall on the 28.03.2005. BET’s specific parameters, as indicated by the model, are presented below:

\( k = 2.29295745067; \ z = 0.901514019429; \ B_0 = 0.00141871175529228; \ B_1 = 0; \ \omega = 5.00005479008999; \ \varphi = 0. \)

Based on these parameters, it was further attempted to forecast the next major 12.29% decrease, on the 01.07.2009. The results were not satisfactory. The model fails to capture the critical time associated with the day of 01.07.2009. It, also, doesn’t succeed to indicate the following significant breakdown, the one of 10.96% on the 25.05.2010.

One of the possible explanations for the model’s failure when used within the Romanian market would be that the BET Index evolution does not comply with the required assumptions of the model. Only in the case of the 28.03.2005 price fall, one can say that there was a clear strong previous growth trend. However, this trend wasn’t present in the other price fall cases. Before the fall of 2005 one can notice a strong upward trend, which has started several years before. Within this strong upward trend, the formation of a speculative bubble is obvious. This bubble’s formation is triggered by the exponential growth of all Bucharest Stock Exchange stocks prices, a phenomenon clearly reflected by BET’s evolution. Similar speculative bubbles are not to be found within the other price fall cases, as shown by the chart below (which presents the entire BET Index evolution since the first day it was calculated by BSE):
Since the main premise of the model is not met, the results provided are not satisfactory. Going forward, another significant problem in applying the model within the Romanian market is the amplitude of the price falls. In a previous paper (Pătru, 2006), the effectiveness of the model has been proved, by applying it on the American Index Dow Jones Industrial Average (DJIA). In that case, the model gave excellent results, accurately predicting the exact day of the 1997 crash, using parameters determined within the 1987 crash. But the DJIA 1987 and 1997 falls were, in fact, real crashes. The decrease of the DJIA value in 1987 was of about 23%, a crash in the true sense of the word, an extreme phenomenon as required by the model’s assumptions. Unlike the DJIA case, in the BET case, the price decrease is not to be considered a real stock market crash, but rather a correction, as the magnitude of the fall is only of 12%. Thus, the parameters found within the 2005 crash, that will be used to further predict the 2009 crash, are not necessarily specific to a genuine crash moment and therefore they fail to catch the 2009 price fall. On the other hand, BSE has a volatility limitation rule, which suspends from trading any stock which has an intraday variation of +/- 15%. The rule is meant to protect investors from major sudden losses induced by large unexpected price variations. Due to this rule, a real crash, an over 20% fall in a single day isn’t even possible on BSE. Thus, the biggest BET fall ever registered remains the 01.07.2009 12.29% fall, not really a crash.

Through a detailed study of the evolution of BET index over time, one can notice that although there are many important decreases, they do not occur suddenly, in a single trading session, but over several consecutive days. So, significant falls exist, but they do not appear in a single day, thus not qualifying as crashes as defined by Sornette’s model.

4. Conclusions

First of all, we must mention and analyze few of the limitations of the model, residing in its restricting hypothesis. The model makes the assumption that every placed order will be executed, a deeply incorrect assumption, as the execution of traders’ orders mainly depends on the supply – demand balance and on the price. Also, the model doesn’t take into consideration the presence of slippage during the trading process, nor the trading commissions and taxes. Another critic may refer to the fact that the investors don’t place orders for equal amounts of shares, as one of the model’s hypothesis states.

Another economically unrealistic hypothesis of the model states that the actions of a trader are influenced by his “neighbors” actions, understanding by “neighbors” – relatives, friends, whose actions and decisions are supposed to be known by the trader. In reality, an economically rational trader’s decision is based on the price, supply and demand, his expectations and predictions as is it practically illegal to look for, try to obtain and use information about other traders’ actions. A main role in the functioning of the stock market is that of the budgetary constraint, also, not taken into consideration by the model. Even if the imitation of neighbors could be accepted, the group of neighbors a trader will try to imitate is nothing else but a statistical sample, about which we do not have the certainty that it has been chosen according to any scientific statistical principles, being otherwise not representative. So, the conclusions drawn for this small sample cannot be extended to the entire population the sample has been chosen from.

Leaving aside all the subjective hypothesis of this model, we must take into consideration the fact that the results may also depend on the changing characteristics of each market. Such an issue is the one regarding the evolution of the volume of the transactions, as well the evolution of their value. Bucharest Stock Exchange indicates a number of
16.934.865.957 transactions in 2005, with a total value of 7.809.734.452 RON. As a consequence of the global economic crisis, in 2009 BSE reports a number of 14.431.359.301 transactions, having a total value of 5.092.691.411 RON. For the year 2010, even a smaller number of transactions (13.339.282.639) are reported, summarizing a total value of 5.600.619.918 RON. It is proven that the market became less liquid, and this can be another possible explanation for the fact that the parameters don’t fit anymore and the model doesn’t succeed in correctly indicating the critical times in 2009 and 2010.

All these identified critiques are the reason for which the model fails to predict the critical time of the crash in many cases. Such an example is the one analyzed in this paper, the one referring to the Romanian Stock Market. However, there are several cases, such as the case of the Dow Jones Industrial Average (Pătru, 2006), where the model succeeds to correctly indicate the critical time. As a final conclusion, we can say that the model may be a useful tool if applied on markets which satisfy the model’s hypothesis (from a regulations’ point of view) and show a high atomicity degree. Is also is to be seen as an interesting research and improving objective for economists.

5. Bibliography


