THE ENTROPY OF THE SUSTAINABLE DEVELOPMENT OF RURAL TOURISM ORGANIZATIONS IN THE CONTEXT OF COLLABORATIVE ECONOMY

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Abstract
In this paper, the author suggests a mathematical model for the entropy of the sustainable development of rural tourism organizations. The author uses the entropy formula defined by C. Shannon for each component of the sustainable development: the economic component, the sociocultural component, and the environmental component, applying the maximum of the entropies of these components. The model has the advantage that it can be easily understood and applied also by individuals who do not have a very strong mathematical background. The constants within this model can be chosen by the management of the organization according to the organization’s specificity and objectives, and according to the managers’ professional experience. The model suggested within the paper was applied for the entropy of the sustainable development of five rural tourism organizations from Sibiu County, Romania. The result can be compared with the result for the coming years, 2018, 2019, etc. or with the result calculated for other rural tourism organizations from other regions.

Keywords: entropy, sustainable development, rural organizations, tourism, collaborative economy

Jel Classification: C 63

1. Introduction
Over the years, the mathematical model has proved to be a tool for the scientific research of the world, of immediate analysis of reality, through which specialists have been able to evaluate the consistency of various ideas and scientific theories, in order to find explanations, new concepts and relations between phenomena, processes, and activities from the socio-human field.

At the present time, modeling and simulation as methods of scientific and practical approach of the surrounding world have become an objective necessity, as stringent need to introduce rigor and abstraction in analyzing the essence and laws of the phenomena from the real world.

In the context of increasing sustainable development and quality assurance of systems, processes, activities, products, and services, the human society can perform only by imposing and complying with certain conditions that target: “the identification and understanding of quality characteristics and attributes specific to each field, their management in the complex and interactive relation with people’s requirements, needs, and expectations”.

Applying the principle of sustainable socioeconomic development, and not only in this field, can be done only through a harmonious and dynamic engagement with the changes from the field of quality and management of sustainable development that take place in the evolutionary sense in the world of the future. Ultimately, the problematic of sustainable development is very vast, containing all aspects of everyday life and of people’s perspective.

It is the task of the management of the organization to approach and establish concrete ways of designing and implementing a system of sustainable development, which is subordinated to the objectives, products, processes, and practices specific to each organization.

Due to the complexity of the situations, of measuring the diversity of the need for sustainable development and the need to satisfy consumer needs, researchers and practitioners have to do thorough research, approached with scientific rigor in assuring sustainable development in all fields, including the rural one.
Now and in the future, in order to operatively and efficiently achieve the ever increasing objectives of sustainable development, in order to obtain remarkable results, the attention is focused on the mathematical modeling of processes from the internal and external environment of each organization.

The trends in the modeling of sustainable development have focused on using weighted averages in the formulas of some indicators of sustainable development [2], aggregated indicators [3], functional models [14], multicriterial decision models and game theory models [10].

Obviously, taking into consideration the complexity of processes from the real world, as well as the mathematical limitations known to date, no matter the mathematical model used or created, the model will have an idealized character and will refer only to a part of the properties of the sustainable development system [5].

**Why do we need to continuously adapt the model of sustainable development of rural tourism to the new requirements brought by the changes?**

The answer to this question was already given by various specialists from the field of mathematics and sustainable development. A newer opinion [6], expressed in the specialty literature, better contours the subject of modeling and mathematical knowledge by analogy with the abstract subject of any knowledge, which is: *establishing mathematical identity* (quantitatively, structurally, evolutionary procedural).

Change is present in the entire universe; it is, in fact, a characteristic of permanent transformations. In human life, change refers to qualitative accumulations and jumps in the field of social organization, economic growth, environment protection, of developing work and communication technologies, etc.

From this perspective, modeling as a quantitative tool of forecast and evaluation of human results, individual and organizational, represents major imperatives of continuously adapting to the needs and benefits of the people from the present and future generations.

In the end, the results of adapting to the changes from the organizational and surrounding environment are nothing else but a systemic understanding of reality, a premise of continuous and sustainable development of the people, in order to accumulate new experiences, information, knowledge, for improving knowledge in predicting the natural and social environment and, lastly, to maximize profits of any kind and to minimize the loss of material and spiritual goods [5].

In the field of sustainable development of rural tourism, the focus should be on the top fields of change, on communication [8] and, last but not least, on creating new knowledge and its dissemination.

Modeling the sustainable development of rural tourism is compatible with the ever increasing needs of the people, but also in accordance with the trends in the development of the human society.

The present paper wishes to bring a quantitative and qualitative contribution to the field of the sustainable development of rural tourism.

### 2. Literature review

The economists of the 20th century, among which are Georgescu – Roegen [9], have contributed to the development and promotion of the concept of economic and social sustainable (viable) development. This concept was adopted at the highest political level, at the ECOSOC (United Nations Economic and Social Council) Conference in Rio de Janeiro, in June 1992, as the aim of international cooperation and national plans of action [13].

The sustainable development of rural tourism organizations or of the rural areas was analyzed in numerous recent specialty works, with tools pertaining to management, economy, quantification, etc.
An entropic approach to capitalizing human resources from the perspective of sustainable development and collaborative economy was presented by Bran et al in the book published in the year 2013 [4]. In 2014, Ioan et al analyzed the strong and weak points of developing tourism in the rural areas of Romania [12]; Stanciu and Hapenciuc [16] conducted a statistic research regarding tourism in the area of Bucovina, Romania; Hapenciuc et al [11] conducted a case study to highlight the touristic potential of the Sucevița area of Romania; Tigu and Tulcea [17], Zorzoliu and Iatagan [18] highlighted details regarding rural tourism advertising in Romania.

Baum [1] highlighted elements pertaining to the potential of rural areas from Poland by using statistical data and the SWOT analysis. Øian [19] highlighted two specific activities of the rural areas from Norway, hunting and fishing; Patterson et al [15] discussed aspects pertaining to the ecological and environmental component of the rural tourism of Val di Merse, Italy; etc.

3. Mathematical modeling of the sustainable development of rural tourism organizations in the context of collaborative economy

In this section, the author created an entropic model, the mathematical model of the sustainable development of rural tourism organizations in the context of collaborative economy. The aim of this model is to quantify the uncertainty regarding the impact of the collaborative economy on the sustainable development of rural tourism organizations.

In fig. 1, the author presents the underlying descriptive model of the mathematical model.

Variables from fig. 1 are numeric values from the range [0,1] associated to the factors from tables 1-4:

| $x_1^{ec}$ | number of tourists |
| $x_2^{ec}$ | average length of stay |
| $x_3^{ec}$ | the satisfaction of local tourists in the region (0-10) |
| $x_4^{ec}$ | the satisfaction of foreign tourists in the region (0-10) |
| $x_5^{ec}$ | the tourists’ opinion on the relationship between the accommodation price and the services offered (0-10) |
| $x_6^{ec}$ | assessment of the quality of the personnel from the tourism sector (0-10) |
| $x_7^{ec}$ | three, four, and five-star hotels per capita |
| $x_8^{ec}$ | restaurant per capita |
| $x_9^{ec}$ | tourist information centers (per capita) |
| $x_{10}^{ec}$ | regional tourism websites/internet sites allowing tourists to "visualize" holiday [7] |
| $x_{11}^{ec}$ | the ratio between seasonal and off-season tourists |
| $x_{12}^{ec}$ | with or without airport access |
| $x_{13}^{ec}$ | rail access has or doesn’t have a rail link |
| $x_{14}^{ec}$ | assessment of the viewpoint of locals on the impact of tourism on the enrichment of regional crafts (0-10) |

where:

$x_1^{ec}$, ..., $x_{16}^{ec}$ represent mathematical notations of the economic factors selected after bibliographic research, which give subjective satisfaction to people from a society at time “$t$”, in the hypothesis of their statistical independence.
Figure 1. General elements of a mathematical model for the sustainable development of rural tourism
Table 2. Sociocultural factors

| \( x^t_{1,\text{sociocultural}} \) | organizing culture festivals to preserve and introduce customs |
| \( x^t_{2,\text{sociocultural}} \) | number of cultural exhibitions (per capita) |
| \( x^t_{3,\text{sociocultural}} \) | percentage of young population in the region |
| \( x^t_{4,\text{sociocultural}} \) | percentage of population aged >60 years in the region |
| \( x^t_{5,\text{sociocultural}} \) | number of people per unit area |
| \( x^t_{6,\text{sociocultural}} \) | natural rate of population growth |
| \( x^t_{7,\text{sociocultural}} \) | income per capita |

where:
\( x^t_{1,\text{sociocultural}}, \ldots, x^t_{7,\text{sociocultural}} \) represent mathematical notations of the sociocultural factors selected after bibliographic research, which give subjective satisfaction to the people from a society at time ‘\( t \)’, in the hypothesis of their statistical independence.

Table 3. Environmental factors

| \( x^t_{1,\text{environmental}} \) | percentage of local population that has access to clean and healthy water |
| \( x^t_{2,\text{environmental}} \) | the volume of waste produced by the tourism sector |
| \( x^t_{3,\text{environmental}} \) | tourist assessment of environmental health (0-10) |
| \( x^t_{4,\text{environmental}} \) | daily energy consumption (per capita) |
| \( x^t_{5,\text{environmental}} \) | water consumption in the tourism sector |
| \( x^t_{6,\text{environmental}} \) | daily water consumption (per capita) |
| \( x^t_{7,\text{environmental}} \) | construction density in the unit area |
| \( x^t_{8,\text{environmental}} \) | number of natural attractions in the region |
| \( x^t_{9,\text{environmental}} \) | percentage of agricultural lands from the total surface of the region |
| \( x^t_{10,\text{environmental}} \) | number of tourists in the area of the regional unit |
| \( x^t_{11,\text{environmental}} \) | diversity of plant species (per unit area) |

where:
\( x^t_{1,\text{environmental}}, \ldots, x^t_{11,\text{environmental}} \) represent mathematical notations of the environmental factors selected after bibliographic research, which give subjective satisfaction to the people from a society at time ‘\( t \)’, in the hypothesis of their statistical independence.

Table 4. Factors related to the safety of the tourists and of the region the rural tourism organizations are part of

| \( x^t_{1,\text{safety}} \) | tourist satisfaction regarding the safety of the region |
| \( x^t_{2,\text{safety}} \) | safety equipment from the region (ambulance, road emergency) per capita |
| \( x^t_{3,\text{safety}} \) | assessment of military cooperation and of the local or governmental agencies to apply laws that ensure the safety of the tourists (0-10) |

where:
\( x^t_{1,\text{safety}}, x^t_{2,\text{safety}}, x^t_{3,\text{safety}} \) represent mathematical notations of the safety factors selected as a result of bibliographic research, which give subjective satisfaction to the people from a society at time ‘\( t \)’, in the hypothesis of their statistical independence.

In order to develop the mathematical model of sustainable development of rural tourism organizations in the context of a collaborative economy, the author defined an entropy of sustainable development, based on the entropy formula established by Claude Shannon.

Let the following probabilities be:
\[ p^t_{k,\text{economic}} = P(x^t_{k,\text{economic}} > a_k), \quad k = 1,14; \quad p^t_{i,\text{sociocultural}} = P(x^t_{i,\text{sociocultural}} > b_i), \quad i = 1,7. \]
Let the following be the probability field of the economic factors at time $t$:

$$\text{Fe}_{\text{economic}}^t:\begin{pmatrix} x^t_{1,\text{economic}} & x^t_{2,\text{economic}} & \ldots & x^t_{14,\text{economic}} \\ p^t_{1,\text{economic}} & p^t_{2,\text{economic}} & \ldots & p^t_{14,\text{economic}} \end{pmatrix},$$

$$p^t_{1,\text{economic}} + p^t_{2,\text{economic}} + \ldots + p^t_{14,\text{economic}} = 1.$$  \hspace{1cm} (1)

Let the following be the probability field of the sociocultural factors at time $t$:

$$\text{Fs}_{\text{sociocultural}}^t:\begin{pmatrix} x^t_{1,\text{sociocultural}} & \ldots & x^t_{7,\text{sociocultural}} \\ p^t_{1,\text{sociocultural}} & \ldots & p^t_{7,\text{sociocultural}} \end{pmatrix},$$

$$p^t_{1,\text{sociocultural}} + \ldots + p^t_{7,\text{sociocultural}} = 1.$$  \hspace{1cm} (2)

Let the following be the probability field of the environmental factors at time $t$:

$$\text{Fe}_{\text{environmental}}^t:\begin{pmatrix} x^t_{1,\text{environmental}} & x^t_{2,\text{environmental}} & \ldots & x^t_{11,\text{environmental}} \\ p^t_{1,\text{environmental}} & p^t_{2,\text{environmental}} & \ldots & p^t_{11,\text{environmental}} \end{pmatrix},$$

$$p^t_{1,\text{environmental}} + p^t_{2,\text{environmental}} + \ldots + p^t_{11,\text{environmental}} = 1.$$  \hspace{1cm} (3)

Let the following be the probability field of the safety factors at time $t$:

$$\text{Fs}_{\text{afety}}^t:\begin{pmatrix} x^t_{1,\text{afety}} & x^t_{2,\text{afety}} & x^t_{3,\text{afety}} \\ p^t_{1,\text{afety}} & p^t_{2,\text{afety}} & p^t_{3,\text{afety}} \end{pmatrix},$$

$$p^t_{1,\text{afety}} + p^t_{2,\text{afety}} + p^t_{3,\text{afety}} = 1.$$  \hspace{1cm} (4)

Definition. The author defines the mathematical model of the entropy of sustainable development of rural tourism organizations in the context of a collaborative economy at time $t$ as being the maximum entropy of the economic, sociocultural and environmental factors, with the formula:

$$H(DD^t) = \max \left\{ \begin{array}{l}
- \sum_{k=1}^{14} p^t_{k,\text{economic}} \ln p^t_{k,\text{economic}}, \\
- \sum_{i=1}^{7} p^t_{i,\text{sociocultural}} \ln p^t_{i,\text{sociocultural}}, \\
- \sum_{j=1}^{11} p^t_{j,\text{environmental}} \ln p^t_{j,\text{environmental}} 
\end{array} \right\}.$$  \hspace{1cm} (5)

In order to correctly define $H(DD^t)$, we will agree that

- $p^t_{k,\text{economic}} \ln p^t_{k,\text{economic}}$ is equal to zero for $p^t_{k,\text{economic}} = 0$;
- $p^t_{i,\text{sociocultural}} \ln p^t_{i,\text{sociocultural}}$ is equal to zero for $p^t_{i,\text{sociocultural}} = 0$;
- $p^t_{j,\text{environmental}} \ln p^t_{j,\text{environmental}}$ is equal to zero for $p^t_{j,\text{environmental}} = 0$;
- $p^t_{j,\text{afety}} \ln p^t_{j,\text{afety}}$ is equal to zero for $p^t_{j,\text{afety}} = 0$;

which means that the function $-x \ln x$ is extended by continuity in $x = 0$.

By defining the concept of entropy of sustainable development of rural tourism organizations in the context of a collaborative economy at time $t$, we can quantify the uncertainty regarding the sustainable development of rural tourism organizations at time $t$.

**Observation 1.** $H(DD^t) \geq 0$ for any probabilities $p^t_{k,\text{economic}}, k = 1,14$; $p^t_{i,\text{sociocultural}}, i = 1,7$; $p^t_{j,\text{environmental}}, j = 1,11$, with $p^t_{1,\text{economic}} + p^t_{2,\text{economic}} + \ldots + p^t_{14,\text{economic}} = 1$.
\[ 1, p^t_{1, sociocultural} + \cdots + p^t_{7, sociocultural} = 1, p^t_{1, environmental} + p^t_{2, environmental} + \cdots + p^t_{11, environmental} = 1. \]

**Demonstration**: Based on the agreement that \( p^t_{k, economic} \ln p^t_{k, economic} \) is equal to zero for \( p^t_{k, economic} = 0; p^t_{i, sociocultural} \ln p^t_{i, sociocultural} \) is equal to zero for \( p^t_{i, sociocultural} = 0 \); \( -p^t_{j, environmental} \ln p^t_{j, environmental} \) is equal to zero for \( p^t_{j, environmental} = 0 \), and the fact that the probabilities are quantities less than or equal to 1, thus their natural logarithms are negative, the observation is correct.

**Interpretation**: The probabilistic experiment regarding the sustainable development of rural tourism organizations has a certain quantity of uncertainty, of indeterminacy, at any time \( t \). The minimum value of uncertainty is zero.

**Observation 2**.
If \( p^t_{k_0, economic} = 1 \) and \( p^t_{k, economic} = 0 \) for all \( k = 1,14, k \neq k_0; p^t_{i, sociocultural} = 1 \) and \( p^t_{i, sociocultural} = 0 \) for all \( i = 1,7, i \neq i_0; p^t_{j, environmental} = 1 \) and \( p^t_{j, environmental} = 0 \) for all \( j = 1,11, j \neq j_0 \), then \( H(DD^t) = 0 \).

**Demonstration**: Taking into consideration the agreement for the cases of probabilities equal to zero and the fact that the natural logarithm of 1 is zero, the observation is correct.

**Interpretation**: An experiment regarding the sustainable development of rural tourism organizations in the context of collaborative economy in which all results have a probability equal to zero and contain no indeterminacy, except for one result from each economic, sociocultural, and environmental field has a probability equal to 1, is a deterministic experiment.

**Observation 3**. For any probabilities \( p^t_{k, economic}, k = 1,14; p^t_{i, sociocultural}, i = 1,7; p^t_{j, environmental}, j = 1,11 \), with \( p^t_{i, economic} + p^t_{2, economic} + \cdots + p^t_{16, economic} = 1, p^t_{1, sociocultural} + \cdots + p^t_{7, sociocultural} = 1, p^t_{1, environmental} + p^t_{2, environmental} + \cdots + p^t_{11, environmental} = 1 \), we have \( H(DD^t) \leq ln14 \).

**Demonstration**: We use Jensen’s inequality as inequality for real function, which is stated as follows:

Let \( f: [a, b] \rightarrow R \) be a real function. This function is concave if, for any \( x_1, x_2 \in [a, b] \) and any real constant \( \lambda \in [0,1] \), we have: \( f(\lambda y_1 + (1 - \lambda) y_2) \geq \lambda f(y_1) + (1 - \lambda) f(y_2) \).

We analyze the continuous and concave function \( f: [a, b] \rightarrow R \). \( y_1, ... , y_n \) from the range \( [a, b] \) and the real numbers \( \lambda_1, ... , \lambda_n \geq 0 \) with \( \Sigma_{k=1}^n \lambda_k = 1 \). Jensen’s inequality establishes the following relation:

\[ f(\Sigma_{k=1}^n \lambda_k y_k) \geq \Sigma_{k=1}^n \lambda_k f(y_k). \]  \( (6) \)

We will apply this inequality to:

- the concave and continuous function \( f(\gamma) = -\gamma \ln \gamma \),
- \( [a, b] = [0,1] \),
- \( n = 16 \),
- \( (y_1, \ldots, y_{16}) = (p^t_{1, economic}, \ldots, p^t_{16, economic}) \), \( p^t_{k, economic} \in [0,1] \) \( k = 1, \ldots, 14 \),
- \( (\lambda_1, \ldots, \lambda_{14}) = \left( \frac{1}{14}, \ldots, \frac{1}{14} \right) \), \( \Sigma_{k=1}^{14} \lambda_k = \Sigma_{k=1}^{14} 1 = 14 \).

We have:

\[ -(\Sigma_{k=1}^{14} \frac{1}{14} p^t_{k, economic}) \ln(\Sigma_{k=1}^{14} \frac{1}{14} p^t_{k, economic}) \geq -\Sigma_{k=1}^{14} \frac{1}{14} p^t_{k, economic} \ln p^t_{k, economic}. \]

This means that:

\[ -\frac{1}{14} (\Sigma_{k=1}^{14} p^t_{k, economic})(\ln \frac{1}{14} \Sigma_{k=1}^{14} p^t_{k, economic}) \geq -\frac{1}{14} \Sigma_{k=1}^{14} p^t_{k, economic} \ln p^t_{k, economic} \]

As \( \Sigma_{k=1}^{14} p^t_{k, economic} = 1 \), it implies that:
We apply inequality (6) to:

- the concave and continuous function \( f(\gamma) = -\gamma \ln \gamma \),
- \([a, b] = [0,1] \),
- \( n = 7 \),
- \((\gamma_1, \ldots, \gamma_7) = \left( p_{1,sociocultural}^t, \ldots, p_{7,sociocultural}^t \right), \quad p_{i,sociocultural}^t \in [0,1] \) \( i = 1, \ldots, 7 \),
- \((\lambda_1, \ldots, \lambda_7) = \left( \frac{1}{7}, \ldots, \frac{1}{7} \right), \quad \Sigma_{i=1}^7 \lambda_i = \Sigma_{i=1}^7 \frac{1}{7} = \frac{7}{7} = 1.\)

We have:

\[-(\Sigma_{i=1}^7 \frac{1}{7} p_{i,sociocultural}^t) \ln(\Sigma_{i=1}^7 \frac{1}{7} p_{i,sociocultural}^t) \geq -\sum_{i=1}^7 p_{i,sociocultural}^t \ln p_{i,sociocultural}^t.\]

We have:

\[-(\sum_{k=1}^7 p_{k,sociocultural}^t) \left( \frac{1}{7} \sum_{k=1}^7 p_{k,sociocultural}^t \right) \geq -\frac{1}{7} \sum_{k=1}^7 p_{k,sociocultural}^t \ln p_{k,sociocultural}^t.\]

As \(\sum_{k=1}^7 p_{k,sociocultural}^t = 1\), it implies that:

\[-\sum_{k=1}^7 p_{k,sociocultural}^t \ln p_{k,sociocultural}^t \leq -\frac{1}{7} \ln 7.\]

Then we apply Jensen’s inequality to:

- the concave and continuous function \( f(\gamma) = -\gamma \ln \gamma \),
- \([a, b] = [0,1] \),
- \( n = 11 \),
- \((\gamma_1, \ldots, \gamma_{11}) = \left( p_{1,environmental}^t, \ldots, p_{11,environmental}^t \right), \quad p_{j,environmental}^t \in [0,1] \) \( j = 1, \ldots, 11 \),
- \((\lambda_1, \ldots, \lambda_{11}) = \left( \frac{1}{11}, \ldots, \frac{1}{11} \right), \quad \Sigma_{j=1}^{11} \lambda_j = \Sigma_{j=1}^{11} \frac{1}{11} = 11 \frac{1}{11} = 1.\)

We have:

\[-(\Sigma_{j=1}^{11} \frac{1}{11} p_{j,environmental}^t) \ln(\Sigma_{j=1}^{11} \frac{1}{11} p_{j,environmental}^t) \geq -\sum_{j=1}^{11} p_{j,environmental}^t \ln p_{j,environmental}^t.\]

This means that:

\[-\frac{1}{11} \left( \sum_{j=1}^{11} p_{j,environmental}^t \right) \left( \ln \sum_{j=1}^{11} p_{j,environmental}^t \right) \geq -\frac{1}{11} \sum_{j=1}^{11} p_{j,environmental}^t \ln p_{j,environmental}^t.\]

As \(\Sigma_{j=1}^{11} p_{j,environmental}^t = 1\), it implies that:

\[-\sum_{j=1}^{11} p_{j,environmental}^t \ln p_{j,environmental}^t \leq -\frac{1}{11} \ln 11.\]

Thus, we have \(H(DD^t) \leq \max \{\ln 14, \ln 7, \ln 11\} = \ln 14\).

The interpretation of this property is the following: the maximum quantity of uncertainty or indeterminacy is contained within the probability fields that have equally probable elementary events.

The author proposes the mathematical model of sustainable development of rural tourism organizations with the formula system (7) - (17) as being composed of the formula for the entropy of sustainable development of rural tourism in the context of a collaborative economy at time expressed in years, to which the author added a restriction related to the Shannon entropy of the safety factors:
4. Assessment of the model

The assessment of the model can be done by successively applying it at different times $t$. The model has the advantage that it can be easily understood and applied also by individuals who do not have a strong mathematical background. The constants from the proposed model can be chosen by the management of the organization according to their specifics and objectives and according to the professional experience of the manager.

A case study in which we can highlight how to apply the model suggested in this paper and composed of formulas (7) - (17) is one in which the necessary data was obtained by applying a survey in the year 2017 in five rural tourism organizations in Sibiu County, Romania.

We consider the percentage values of the favorable responses from the survey, each divided by their sum, to be values of the probabilities from the mathematical model, as presented in table 5:

Table 5. Values of the probabilities

<table>
<thead>
<tr>
<th>Probability</th>
<th>Natural logarithm of the probability</th>
<th>The first two columns multiplied</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{1, economic}^t$ = 0.03</td>
<td>$\ln 0.03 = -3.50$</td>
<td>-0.1050</td>
</tr>
<tr>
<td>$p_{2, economic}^t$ = 0.02</td>
<td>$\ln 0.02 = -3.91$</td>
<td>-0.0782</td>
</tr>
<tr>
<td>$p_{3, economic}^t$ = 0.03</td>
<td>$\ln 0.03 = -3.50$</td>
<td>-0.1050</td>
</tr>
<tr>
<td>$p_{4, economic}^t$ = 0.07</td>
<td>$\ln 0.07 = -2.65$</td>
<td>-0.1855</td>
</tr>
<tr>
<td>$p_{5, economic}^t$ = 0.05</td>
<td>$\ln 0.05 = -2.99$</td>
<td>-0.1495</td>
</tr>
<tr>
<td>$p_{6, economic}^t$ = 0.11</td>
<td>$\ln 0.11 = -2.20$</td>
<td>-0.2420</td>
</tr>
<tr>
<td>$p_{7, economic}^t$ = 0.18</td>
<td>$\ln 0.18 = -1.71$</td>
<td>-0.3078</td>
</tr>
<tr>
<td>$p_{8, economic}^t$ = 0.07</td>
<td>$\ln 0.07 = -2.65$</td>
<td>-0.1855</td>
</tr>
<tr>
<td>$p_{9, economic}^t$ = 0.05</td>
<td>$\ln 0.05 = -2.99$</td>
<td>-0.1495</td>
</tr>
<tr>
<td>$p_{10, economic}^t$ = 0.10</td>
<td>$\ln 0.10 = -2.30$</td>
<td>-0.2300</td>
</tr>
<tr>
<td>$p_{11, economic}^t$ = 0.07</td>
<td>$\ln 0.07 = -2.65$</td>
<td>-0.1855</td>
</tr>
<tr>
<td>$p_{12, economic}^t$ = 0.09</td>
<td>$\ln 0.09 = -2.40$</td>
<td>-0.2160</td>
</tr>
<tr>
<td>$p_{13, economic}^t$ = 0.05</td>
<td>$\ln 0.05 = -2.99$</td>
<td>-0.1495</td>
</tr>
<tr>
<td>$p_{14, economic}^t$ = 0.09</td>
<td>$\ln 0.09 = -2.40$</td>
<td>-0.2160</td>
</tr>
<tr>
<td>$p_{1, sociocultural}^t$ = 0.10</td>
<td>$\ln 0.10 = -2.30$</td>
<td>-0.2300</td>
</tr>
</tbody>
</table>

$$H (DD^t) = \max \left\{ -\sum_{k=1}^{14} p_{k, economic}^t \ln p_{k, economic}^t, \right.$$  
$$-\sum_{i=1}^{7} p_{i, sociocultural}^t \ln p_{i, sociocultural}^t, \right.$$  
$$-\sum_{j=1}^{11} p_{j, environmental}^t \ln p_{j, environmental}^t \right\}, \quad (7)$$

$$-\sum_{s=1}^{3} p_{s, safety}^t \ln p_{s, safety}^t \leq \alpha, \quad (8)$$

$$p_{k, economic}^t = P (x_{k, economic}^t > a_k), k = 1, 14, \quad (9)$$
$$p_{i, sociocultural}^t = P (x_{i, sociocultural}^t > b_i), i = 1, 7, \quad (10)$$
$$p_{j, environmental}^t = P (x_{j, environmental}^t > c_j), j = 1, 11, \quad (11)$$
$$p_{s, safety}^t = P (x_{s, safety}^t > d_s), s = 1, 3, \quad (12)$$
$$P_{1, economic}^t + P_{2, economic}^t + \cdots + P_{14, economic}^t = 1, \quad (13)$$
$$P_{1, sociocultural}^t + \cdots + P_{7, sociocultural}^t = 1, \quad (14)$$
$$P_{1, environmental}^t + P_{2, environmental}^t + \cdots + P_{11, environmental}^t = 1, \quad (15)$$
$$P_{1, safety}^t + P_{2, safety}^t + P_{3, safety}^t = 1, \quad (16)$$
$$a_k, b_i, c_j, d_s, \alpha \geq 0. \quad (17)$$
In these case, \( p_{1,safety}^t = 0.33; \ p_{2,safety}^t = 0.33; \ p_{3,safety}^t = 0.34 \) and \( \alpha = 2 \).

Thus, according to formula (7), \( H(DD_t) = max\{2.5050, 1.8591, 2.3696\} = 2.5050 \). This result can be compared to the results for the upcoming years, 2018, 2019, etc. or with the results obtained for other rural tourism organizations from other regions.

5. Conclusions

The mathematical model of the entropy of sustainable development of rural tourism organizations in the context of collaborative economy at time \( t \) can highlight uncertainty related to the strong or weak points of rural tourism organizations from Romania or from another country, in order to achieve their sustainable development, depending on different optimistic or pessimistic scenarios.

Rural tourism has created its own emerging market, characterized by factors with specific action and heterogeneous elements. On this slightly atypical market, the quality of touristic products and services tends to become an element of competitive differentiation. Constantly applying superior quality services allows to obtain a certain level of notoriety, both for the providers of touristic services, and for the advertised destination.

Continuously monitoring and improving the quality of rural and touristic products should be based on the link between the tourist satisfaction, perception, and expectations. Naturally, the tourist wishes to receive more options, in order to discover the suitable tourism offer that satisfies the most diverse preferences. The tourist does not wish to make any concessions regarding the quality of the touristic products received. Dissatisfactions could be related to the poor performance of the personnel training activities, to the lack of equipment and facilities compared to other states in the world, to the poor condition of the infrastructure, the low safety level, etc.

The sustainable development of rural tourism organizations could be optimized after a market research in all the regions of each country, by developing and applying regional tourism strategies based on principles of collaborative economy, after establishing cross-sectorial partnerships in every region and after increasing the level of interest and initiative of some rural populations to develop their creativity and innovation regarding rural tourism.
6. References

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