AUTOREGRESSIVE EVOLUTIONS FOR MACROECONOMIC INDICATORS DO CONFIRM CHAOS THEORIES IN UNITED STATES

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Abstract

Ten years ago, international crisis (beginning 2008) has become a fundamental element for national economies. In addition to economic burden, the crisis has put governments in an instability financial situation also, by both domestic and overseas transactions. National Governments main purpose is to provide sustainable development by attempting to prospect and plot economic evolutions in order to reduce financial influences overall. The core question is then: Are these countries able to anticipate these evolutions? This paper aims to determine how Chaos Theories analyses for Real Gross Domestic Product (GDP) and Consumption per Capita influence the trend of these indicators, but not in the classic macroeconomic dependency direction. The purpose of this article is to provide a core framework from the Chaos Theory perspective in order to create the context for Auto Regressive and Mobile Average (ARMA Models) alternatives methodology in practice, alternatives, and forecasts. The estimations based on econometric and cybernetic models by using ARMA and Hurst Coefficient highlight the fact that Real GDP and Consumption per capita have similar evolutions, but are not directly correlated in the classic theory because here, similarity and persistency are main factors for general evolutions of these macroeconomic indicators.

Keywords: ARMA Modelling, Chaos Theory, Fractal Theory, Time Series

JEL Classification: C1: Econometric and Statistical Methods and Methodology: General, **E17**: Forecasting and Simulation: Models and Applications, **O47:** Measurement of Economic Growth; Aggregate Productivity; Cross-Country Output Convergence

1. Introduction

Government institutions have increased constantly their requirements towards analyses and prognoses such as forecasts or economic trends built on econometric modeling (in terms of time series forecasts) in order to find better financial and economic estimators that can prevent severe impacts over national economies. Such an example can be found in the United States during 1947 – 2017, where macroeconomic indicators such as Real GDP per Capita and Real Consumption per Capita present similar evolutions in the long run, but with different variations at different moments of time. Applying Chaos Theory in order to find a time-series fractal dimension is the beginning of time-series analysis procedure possibilities such as ARMA modeling methodology (Box and Jenkings, 1970). Usually, most time series are not stationary, therefore standard methodologies cannot be applied on primary data in most of the econometric analyses.

This paper analyses if Real GDP and Real Consumption per Capita have similar fractal dimensions (which would make possible time-series forecasts for their respective estimations if lower than 1,5), and whether their long-run evolutions are persistent in Chaos Theory terms. Findings based on Hurst coefficient values showed that both indicators are persistent, aspect which explains why Real GDP and Real Consumption per Capita evolve slightly different, resulting in different generalized formulas for estimated models for the above-mentioned indicators.

Granger and Newbold (1974) and Nelson and Kang (1981) explain how a regression model is not established only based on independent variables, but also with the help of timeline, of the explanatory power of the time-series relationship, and of the autocorrelation of the estimated residuals which provide accuracy in the proposed model. Statistics on econometric models (when using time-series analyses) can be applied only on stationary data, adding another perspective to time series, namely the non-stationary nature of data (if present), and the importance of standardized data.

A standard test in determining first-order auto-correlation (Dickey and Fuller, 1979), but this procedure is a weak hypothesis compared to several alternatives for autocorrelation and mobile average models, such as BDS test which tries to find patterns of evolution in different spatial dimensions created by using original data, but rearranged as vectors with two to six dimensions.

In the time series studies has been a tendency to determine whether United States Real GDP per Capita is deterministic or stochastic. In general, many researchers studied variations around a deterministic trend line (Nelson and Plosser, 1982) which concluded that between 1907 – 1970 aggregate outputs acted like non-stationary stochastic processes, similar to a random walk. A deterministic trend was suggested by Perron (1989) which declined the stochastic hypothesis by inserting a permanent shock upon GDP between analyzed years (1907 – 1970), with disruptive shocks used for short-term periods of the same period. The only significant breakage took place in 1929, but real shocks continuously generated changes on the United States macroeconomic outputs. The unit-root autoregressive evolution has its origins into first-order auto-correlation time series, thus implying first order differencing in order to obtain stationary series.

Fractals discovery (Mandelbrot, 1982)) regarding self-similarities in non-integer dimensions made possible the econometric modeling through chaos and time series analysis. . In the beginning, fractal dimension was applied to finance time series, but later (Michelacci and Zaffaroni, 2000) explained that the presence of unit roots and long-term exponential trends in the growth process related to GDP per capita for 16 OECD countries were not contradictory with the econometric theory regarding time series.

As a combination between ARMA modeling procedure and fractal waves for business cycle (Dudukovic, 2010) compared US and BRIC countries GDP evolutions in order to analyze if fractal wave model (based on Elliot Wave) explained the volatility of the shocks better than traditional regression models.

To the author's knowledge, the researchers focused mainly on either finding time-series procedures (such as ARMA modeling) in order to obtain good estimators for future trends, or determining fractal dimensions for various macroeconomic indicators up to this date. Among these studies cannot be found a research that determines fractal dimensions for both Real GDP and Consumption per Capita in period 1947 – 2017 by using Hurst coefficient, and that uses fractal values in order to apply and validate time-series modelling by ARMA procedure.

This research shows that Real GDP per Capita is less persistent than Real Consumption per Capita due to a higher fractal dimension (1,14614 compared to 1,139744), despite the fact that these macroeconomic indicators are positively-correlated in the traditional macroeconomic theory. In terms of ARMA (Auto Regressive and Mobile Average) procedure, the author chose for Real GDP per Capita the model ARMA (1, 1) due to better validation criteria for both Akaike and Schwarz values (compared to ARMA (1, 2)), while for Real Consumption per Capita the model chosen was ARMA (2, 0) due to the fact that Akaike and Schwarz values were not lower for only one of the selected models, but due to the values in the regression models, closer to 0 for ARMA (2, 0) than for ARMA (3 0).

The structure of the paper is the following: *Section 2* presents a short literature review regarding chaos theory and time series analyses; *Section 3* explains the chaos (or fractal) theory by using BDS statistics and methodology of ARMA procedure built by Box and Jenkings as traditional time-series methodology; *Section 4* shows how both fractal theory and ARMA work in practice by suggesting various alternatives for both Real GDP and Consumption per Capita which are validated by different statistics and tests; and *Section 5* concludes with the fact that fractal value is a good estimator for similar macroeconomic series, showing in the same time possible differences in terms of time series modeling (such as ARMA methodology).

2. Literature Review

Macroeconomic indicators such as GDP or Consumption do not have a linear trend in general because their evolution depends on social factors mainly. Mathematic and economic researchers tried to find patterns in time series in order to predict non-linear stochastic trends. In order to discover similarities in timely evolutions, long-term concept is used often, and resonates with persistency in terms of Chaos Theory. Lorenz (1963) showed that initial conditions can determine long-term evolutions, therefore using this tool in order to predict the weather.

This theory analyses dynamic systems, both through differential (or continuous time) or difference (discrete time) equations (Li and Yorke, 1975), (Procaccia and Saphiro, 1987), while for economic series with non-linear evolutions the chaos theory uses fractals (Mandelbrot, 1982). Fractals are self-similar in their non-integer dimensions, and their structure is determined by rotation and displacement, or by bounded deformation.

Poincaré (1899) highlighted the fact that, for dynamic systems, when studying phase space for cross-section data, the difference between discrete and continuous time does not exist, by underlining invertibility and differentiability as fundaments for motion in the phase space description. Based on his idea, (Aoki,

1987) built econometric models based on the usage of a multivariate state space representation of time series.

Ruelle and Takens (1971) inserted in the chaos theory literature the concept of strange attractor in order to verify the dynamic convergence to a certain form or number. Time-delayed reconstruction (data redesign in terms of multi-spatial dimensions) was lately introduced (Roux et al., 1983) in order to check long-term convergence. This information rearrangement about a time series provides useful insights when trying to find the fractal dimension of a real form (the minimum dimension of the underlying process), establishing the long-term behavior.

A certain trend (or pattern) based on observations of the same indicator at different moments of time was built by Box and Jenkings through their Auto Regressive and Mobile Average (ARMA) methodology, designed on lag differences for a specific data set, developed in late years (Granger and Joyeux, 1980), (Hosking, 1981) while trying different properties of fractionally differenced models.

How accurate or long the prediction problem is depends on whether we take into account an infinite past history and whether the parameters of the processes generating the time series are assumed to be known (or not). Even if the number of observations can be high for a given indicator (Grether and Nerlove, 1970), included annually records (due to problems of trend extraction and seasonal adjustment).

The essential question when analyzing time series indicators from fractal theory perspectives is then if timely theories can be considered the base for long-term persistency in chaos theory terms. Fractal values highlight correlated variables similarities, while ARMA models show similarity (and/or) auto-similarity characteristics of the same time series. In this case, time-series modeling seeks to find the correlation (AR(p)) and autocorrelation (MA(q)) among the items of the same time series. Depending on the fitness of the model and its degree of accuracy are chosen the degrees for p and q in the AR(p) and MA(q) time series modeling, based on either penalty terms (Akaike, 1970, 1974), (Schwarz, 1978)) or on the maximum likelihood estimation of ARMA models (Granger and Newbold, 1974), (Anderson ,1977), (Ansley, 1979), and (Harvey, 1981).

3. Methodology

3.1. Hurst Coefficient and Koch curve methods for long-term memory

Since Mandelbrot discovery about fractals (with the self-similarity and non-integer aspects), chaos and economic theories have defined their purposes with respect to timely evolutions. A time series is based on economic indicators with no linear trend because non-integer numbers gives its dimension, fact that leads to fractals usage in terms of chaos theory. For time series, fractal dimensions have values in the interval (1, 2) in the Euclidian space due to their real structure (axes time and value), with the following properties: fine structure, too abnormal for an

Euclidian descriptive writing, auto-similarity, higher fractal dimension than the topologic one (1), and recursively rules definition.

Future evolutions can be estimated depending on the fractal value from a persistency point of view. Among the methods used in finding fractal dimension and space representation can be found the Hurst coefficient (1951) that determines the extent of segmentation in terms of fractal theory for dynamic evolutions (based on axes time and value) or Koch curve technique for graphic design (Koch, 1904).

Obtaining Hurst coefficient by applying BDS statistics is made by redistributing data in various spatial dimensions (with space values starting at dimension two - R^2 up to six- R^6, the maximum allowed), in order to validate if there are persistency patterns (or long-term memory possibilities). Data formed is based on new vectors with insignificant distance between close values, where r – radius (taking values in the interval (0.015; 0.1), with r ϵ {0.015, 0.016, 0.017, ..., 0.1}), due to the numerator ($C(m, r) \neq 0$) having the formula:

$$C_{r_k,m} = r^D \ (\neq 0) \qquad (1)$$

Where

m – 2, 3, …, 6 (according to the 6 spatial dimensions)

$C_{r_k,m}$ - the integral correlation coefficient

$D = \frac{\log(C_{r_k,m})}{\log(r_k)}$ - fractal dimension, given the formula for every fractal coefficient equations (2):

$$\log\left(C_{r_k,m}\right) = a * \log(r_{k)} + b \qquad (2)$$

Resulting in final fractal value for each dimension of the corresponding dimension (3):

$$\max(\hat{a}) \qquad (3)$$

With Hurst coefficient value formula (4):

$$H = 2 - \ \hat{a} \qquad (4)$$

Fractal dimension determined by applying Hurst coefficient method in time series analyses discriminates between values lower or greater than 1,5 (out of 2). For those time series with a value lower than 1,5, a proper time series analysis can be used, while for the opposite case the evolution is too fragmented, making impossible time series' forecast.

Moreover, for a graphic representation of time series in terms of fractal theory, can be applied the smaller-area square coating technique based on Koch curve. The main property of this curve is that the interior area of Koch curve remains lower than the area of the circle reaching its original triangle peaks, resulting into an infinite length that surrounds a finite area, causing a paradox where the indefinite figure never crosses any of its previous points. Fractal dimension (based on the smaller-area square coating technique) has the following formula (5):

$$D = \frac{\log n}{\log(\frac{l}{s})} \qquad (5)$$

Where

D – fractal (or non-integer dimension)

$n = \frac{1}{s^D}$ - takes different integer values

l – represents the level of fractal "seed" replicated (level of replication)

$s = \frac{1}{n}$

3.2. Autoregressive and Mobile Average (ARMA) procedure for estimation and econometric persistency

Box and Jenkings introduced a new theory related to time series evolution and forecast by applying a simple auto regressive (AR) and mobile average (MA) methodology which takes into account previous values and errors of the same time series. The general formula when applying ARMA(p,q), where p is the autoregressive coefficient and q the mobile average coefficient, is depicted at (6):

$$Y = \mu + \varphi_1 * Y_{t-1} + \varphi_2 * Y_{t-2} + \cdots + \varphi_p * Y_{t-p} + e_t + \theta_1 * e_{t-1} + \theta_2 * e_{t-2} + \cdots + \theta_q * e_{t-q} \qquad (6)$$

Where

μ = the displacement parameter, model's constant

$\varphi_1, \varphi_2, \ldots, \varphi_p$ – parameters for previous years of the autoregressive component, given p

$Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$ – observations of the same time series at different moments of time, give p

$\theta_1, \theta_2, \ldots, \theta_q$ – parameters for previous years of the mobile average component, given q

$e_{t-1}, e_{t-2}, \ldots, e_{t-q}$ – errors of the same time series at different moments of time, given q

e_t – the random variable (or white noise component)

$AR(p) = \mu + \varphi_1 * Y_{t-1} + \varphi_2 * Y_{t-2} + \cdots + \varphi_p * Y_{t-p}$- Autoregressive component

$MA(q) = e_t + \theta_1 * e_{t-1} + \theta_2 * e_{t-2} + \cdots + \theta_q * e_{t-q}$- Mobile Average component

In order to apply the ARMA (p,q) methodology, Box and Jenking suggested several phases, in the following order: *time series transformation* which leads to stationarity in average, variance and covariance; *time series modeling* given different p and q degrees; *parameters estimators* for ARMA(p,q); *performance accuracy and fitness* by using various methods such as residuals diagnosis, white noise (or tests for errors' independence validation (as, for example BDS statistics, (Durbin and Watson, 1951), etc.)); and *best model choice* techniques by using different strategies (either best coefficient criteria or coefficient values in the newly-estimated regression models).

3.3. Applicability

3.3.1. Stationary processes

The indicators analyzed in this paper are Real GDP per Capita and Real Consumption per Capita. Normally, these indicators are positively correlated in terms of macroeconomic theory. Although their evolutions are based on increasing trends, their persistency differs in terms of terms of chaos theory. Figure 1 shows a graphic representation of both Real GDP per Capita and Real Consumption per Capita in period 1947-2017, with data extracted from the website http://www.data360.org/ds_list.aspx, by calculating real averaged values reported to the number of people.

Figure 1. Real GDP and Real Consumption per Capita evolutions in U.S. in period 1947-2017, based on Nicolae (2010, p. 38)

Data are standardized (by series transformation processes) in order to apply Box & Jenkings methodology since both indicators have an increasing trend (see figures 2 and 3 with standardized representations for both indicators). This procedure is used because original data can result in low-accuracy for both estimation and fitness tests of the above-mentioned indicators.

Figure 2. Standardized evolution for Real GDP per capita in U.S. in period 1947-2017, based on Nicolae (2010, p. 25, a)

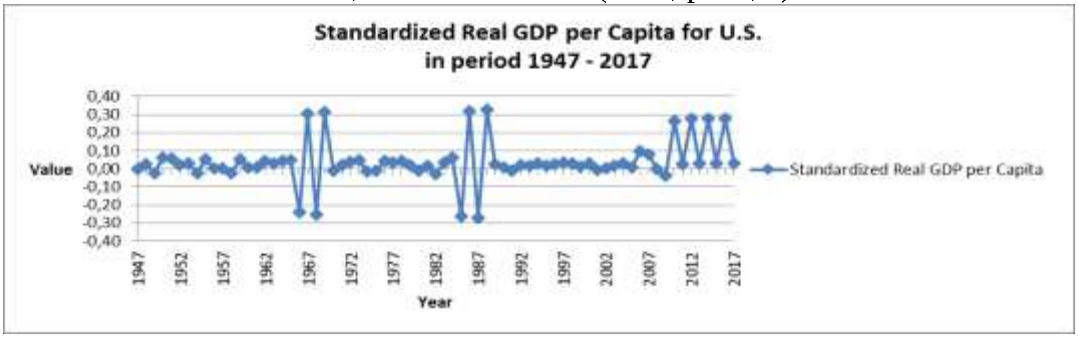
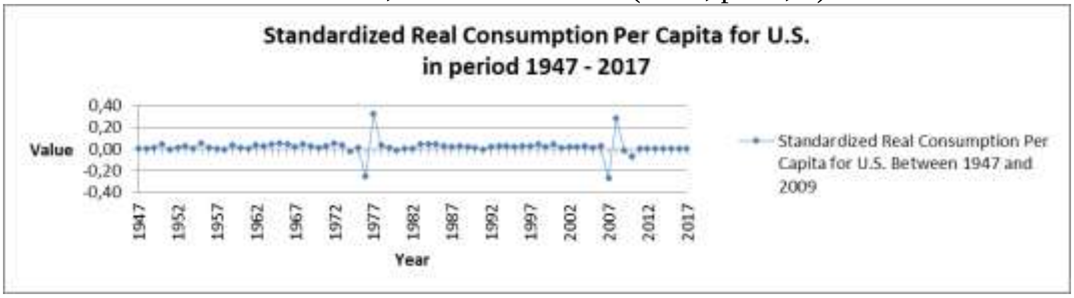


Figure 3. Standardized evolution for Real Consumption per capita in U.S. in period 1947-2017, based on Nicolae (2010, p. 25, b)



Both time series are complex and present different evolutions, fact that leads to using significant statistical test such as Dickey-Fuller (D-F) or Durbin Watson (DW) since original data are correlated, and need stationarity in order to make possible ARMA procedure, allowing at the same time best-fitted form of data processed. At this point, BDS statistics test (Brock et al., 2010)) can be used also for checking data stationarity (in linear dependency terms). From a chaos theory point of view, BDS statistics verifies if newly formed data sets are auto-correlated by redistributing primary data in vectors of 2 up to 6 spatial dimension.

At this point, in terms of chaos theory, ARMA modeling can be possible only if fractal dimension (based on Hurst coefficient) is less than 1,5, otherwise the time series is not persistent.

3.3.2. Estimation

The estimation procedure for a time series should generate as many feasible solutions as possible. Without loss of generality, autoregressive and mobile average coefficients show a time series persistency over time (past values dependency). At this step of the ARMA modeling procedure, absolute values for each coefficient of the newly formed regression models needs to be less than 1 (for an invertible process), therefore rejecting Durbin-Watson statistics that would imply autocorrelation in the stationary process. Moreover, the variables taken into account in the ARMA estimation equations should present important information and a degree of trust of more than 10% in order to validate the alternatives found.

3.3.3. Validity

The decision of choosing best-fitted ARMA(p.q) models is based normally on Akaike and Schwarz criteria in terms of lowest variance of the estimated models but, due to macroeconomic indicators different evolutions generally, the choices can also be based on other criteria such as coefficient values in the regression models closer to 0 (which show a lower autocorrelation degree). In our case, for data analyzed (Real GDP and Consumption per Capita), the indicators have fluctuations at different moments of time, making necessary the introduction of a dummy variable in each time series for those years with abnormal evolutions (0 – where evolutions are normal, and 1 – for shocks in normal trends). At this point, dummy variable introduction is tested before final decision of the chosen estimated ARMA (p,q) model.

3.3.4. Empirical results

The above-described methodology when using ARMA analysis was applied on US macroeconomic indicators Real GDP and Consumption per Capita. Original data for both indicators were standardized using real data per capita logarithm and first order degree procedures (see figures 2 and 3). Furthermore, the sensitiveness of the analyzed indicators was tested based on Durbin-Watson (or D-W), Dickey-Fuller (or D-F) and BDS statistical tests for data reliability control.

4. Chaos Theory and ARMA Modeling in practice for Real GDP and Consumption per Capita

4.1. Fractal Theory

In order to verify if a time series analysis can be used for both Real GDP and Consumption per Capita, BDS statistics is applied in order to extract Hurst coefficient, and Koch curve in order is represented to show fractal evolution and estimation.

4.1.1. Hurst coefficient and long-term memory patterns

According to figure 1, Real Consumption per Capita has a better persistency than Real GDP per Capita due to less fluctuations (two compared to three) in period 1947 - 2017, occurring at different moments of time, involving a higher Hurst coefficient and, implicitly, a higher long-term memory. Applying BDS statistics in order to check this hypothesis, data are rearranged in different spatial dimensions, with small distance between radius values (as mentioned at section 3.1), obtaining the following fractal dimensions for Real GDP and Consumption per Capita:

4.1.2. Fractal dimension for Real GDP per Capita

For m=2, $\log C_{(r_k,2)} = 1{,}14614 * \log(r_k) + 7{,}756161$ (7)

For m=3, $\log C_{(r_k,3)} = 0{,}96502 * \log(r_k) + 7{,}016657$ (8)

For m=4, $\log C_{(r_k,4)} = 0{,}920014 * \log(r_k) + 6{,}700974$ (9)

For m=5, $\log C_{(r_k,5)} = 0{,}610942 * \log(r_k) + 5{,}728875$ (10)

For m=6, $\log C_{(r_k,6)} = 0{,}705303 * \log(r_k) + 5{,}836099$ (11)

Fractal dimension is $\max(\hat{a})$ (see (3)), and is chosen among the values obtained for m= 2,3, …, 6 respectively: 1,14614; 0,96502; 0,920014; 0,610942; 0,705303, resulting into fractal final value 1,14614 which is lower than 1,5 (corresponding to m=2), where Hurst coefficient is larger than 0,5 (based on (4) - see (12)):

$$H = 2 - 1{,}14614 = 0{,}85386 > 0{,}5 \qquad (12)$$

4.1.3. Fractal dimension for Real Consumption per Capita

For m=2, $\log C_{(r_k,2)} = 0{,}940715 * \log(r_k) + 7{,}183026$ (13)

For m=3, $\log C_{(r_k,3)} = 1{,}038590 * \log(r_k) + 7{,}591285$ (14)

For m=4, $\log C_{(r_k,4)} = 1{,}139744 * \log(r_k) + 8{,}014761$ (15)

For Real Consumption per Capita, when m > 4, some of the estimated coefficients are 0, resulting into the impossibility of obtaining relevant values for $\log C_{(r_k,m)}$. In this case, the fractal value is also $\max(\hat{a})$ (see (3)), having a more restrictive selection of maximal values, with m= 2, 3, and 4, respectively: 0,940715; 1,038590; 1,139744, resulting into fractal final value 1,139744 (value lower than 1,5) corresponding to m=4, with Hurst coefficient larger than 0,5 (based on (4) - see (16)):

$$H = 2 - 1{,}139744 = 0{,}860256 > 0{,}5 \qquad (16)$$

Fractal dimensions for both indicators have values lower than 1,5 (Real GDP per Capita fractal value 1,14614, and Real Consumption per Capita fractal value 1,139744), results which make possible the application of time series procedure ARMA (p,q). While comparing long-term persistence for Real GDP and Consumption per Capita, Hurst coefficient values are taken into account in order to verify the macroeconomic indicator with the highest long-term memory. Based on Hurst coefficient values, Real Consumption per Capita has a better persistency (0,860256 > 0,85386) confirming the graphic representation with only two fluctuations for Real Consumption per Capita, compared with three fluctuations for Real GDP per Capita. Moreover, analyzed macroeconomic indicators are correlated in macroeconomic theory terms, with fluctuations happening at various moments of time (see figure 1).

4.1.2. Koch curve and fractal dimension representation

In order to obtain fractal representations for both Real GDP and Consumption per Capita, the author used the wavelet function in the program Matlab **coif3**, function with the propriety of "denoising" algorithms in terms of segmentation and compression, with time series observations analyzed as either uni-dimensional signals, various types of images, or classification (see figures 4.a and 4.b). Based on the graphs, it can be noted that Koch curve for Real GDP per Capita has more variations for the same period than Real Consumption per Capita.

Figure 4.a. Koch curve for Real GDP per capita in U.S. in period 1947-2017
Source: Nicolae (2010, p. 39, a)

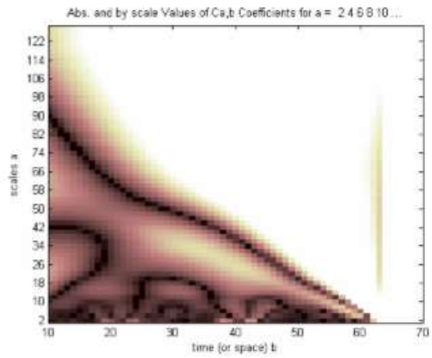
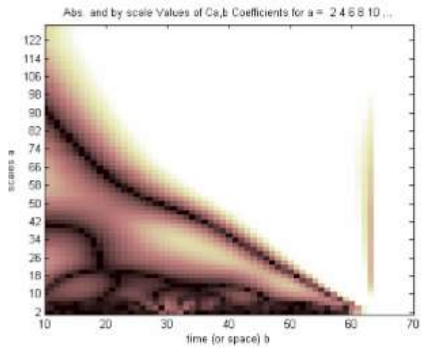


Figure 4. b. Koch curve for Real Consumption per capita in U.S. in period 1947-2017
Source: Nicolae (2010, p. 39, b)



4.2. ARMA models for Real GDP and Consumption per Capita

The simulation and forecast procedures for these indicators was obtained by using the econometric program **Eviews**, which allows a series of statistical tests and regression analyses for different series such as time series or cross-sectional. Eviews had as inputs standardized data for better fitness of the time series corresponding to Real GDP and Consumption per Capita, taking into account standardization (needed for ARMA modeling), length (or lag) of past data (applying the methodology described at *section 3* for stationary data (logarithm and first degree differences in the case analyzed, by using specification tests such as D-W, D-F or BDS statistics in order to verify data standardization, and estimation (by applying ARMA methodology found at *section 3*). Different ARMA(p,q) options for both Real GDP and Consumption per Capita are depicted in the following sub-section:

4.2.1. Real GDP per capita

In this case, the following ARMA(p,q) estimation equations were simulated for values of p = 1,2, and 3, and p = 1,2, and 3, eliminating the models with p>3 and q>3, and p=3 together with q=1, 2 or 3 due to the fact that these modeling alternatives were inaccurate due to their high probabilities of failure for estimated coefficients (17) – (25):

ARMA (1,0): $Y_t = -0{,}743894 * y_{t-1} + 0{,}022074$ (17)

ARMA (1,1): $Y_t = -0{,}649416 * y_{t-1} - 0{,}217246 * e_{t-1} + 0{,}022240$ (18)

ARMA (0,1): $Y_t = -0{,}997306 * e_{t-1} + 0{,}022115$ (19)

ARMA (2,0): $Y_t = -0{,}872063 * y_{t-1} - 0{,}171125 * y_{t-2} + 0{,}022283$ (20)

ARMA (2,1): $Y_t = -0{,}130998 * y_{t-1} + 0{,}387111 * y_{t-2} - 1{,}196539 * e_{t-1} + 0{,}022170$ (21)

ARMA (0,2): $Y_t = -1{,}027751 * e_{t-1} + 0{,}030213 * e_{t-2} + 0{,}022129$ (22)

ARMA (1,2): $Y_t = -0{,}648895 * y_{t-1} - 0{,}217185 * e_{t-1} + 0{,}001719 * e_{t-2} + 0{,}022239$ (23)

ARMA (2,2): $Y_t = -0{,}217215 * y_{t-1} + 0{,}412280 * y_{t-2} - 1{,}077548 * e_{t-1} - 0{,}143513 * e_{t-2} + 0{,}022231$ (24)

ARMA (3,0): $Y_t = -0{,}859202 * y_{t-1} - 0{,}093646 * y_{t-2} + 0{,}093155 * y_{t-3} + 0{,}022675$ (25)

Among these models, ARMA(1,1) and ARMA(1,2) fulfil model's hypotheses (D-W statistics close to 2, low probabilities for regression coefficients and invertible roots, with coefficients in the interval (-1,1)).

4.2.2. Real Consumption per capita

For this indicator, following ARMA(p,q) estimation equations were simulated ((26) - (34)), with values for p = 1,2, and 3, and p = 1,2, and 3, eliminating the models with p>3 and q>3, and p=3 together with q=1, 2 or 3 due to same considerations as for Real GDP per Capita (inaccurate options due to high probabilities of failure for estimated coefficients:

ARMA (1,0): $Y_t = -0{,}48318 * y_{t-1} + 0{,}021519$ (26)

ARMA (1,1): $Y_t = -0{,}164285 * y_{t-1} - 0{,}969370 * e_{t-1} + 0{,}022977$ (27)

ARMA (0,1): $Y_t = -0{,}969754 * e_{t-1} + 0{,}022929$ (28)

ARMA (2,0): $Y_t = -0{,}613419 * y_{t-1} - 0{,}283268 * y_{t-2} + 0{,}021181$ (29)

ARMA (2,1): $Y_t = -0{,}25519 * y_{t-1} + 0{,}232884 * y_{t-2} - 1{,}173845 * e_{t-1}$ (30)

ARMA (0,2): $Y_t = -0{,}830768 * e_{t-1} - 0{,}145453 * e_{t-2} + 0{,}0022951$ (31)

ARMA (1,2): $Y_t = 0{,}149629 * y_{t-1} - 0{,}959947 * e_{t-1} - 0{,}012967 * e_{t-2} + 0{,}022979$ (32)

ARMA (2,2): $Y_t = -0{,}238383 * y_{t-1} + 0{,}198892 * y_{t-2} - 0{,}606828 * e_{t-1} - 0{,}606828 * e_{t-2} + 0{,}023063$ (33)

ARMA (3,0): $Y_t = -0{,}662110 * y_{t-1} - 0{,}399561 * y_{t-2} - 0{,}305817 * y_{t-3} + 0{,}021368$ (34)

Here, ARMA(2,0) and ARMA(3,0) fulfil model's hypotheses as stated for Real GDP per Capita (see section **4.2.1**) in order to be statistically correct.

4.3. Choosing optimal alternatives for Real GDP and Consumption per Capita

Each of the analyzed indicators in this paper has two alternatives that can be used to forecast Real GDP and Consumption per Capita. If for Real GDP per Capita the alternatives are ARMA(1,1) and ARMA(1,2), for Real Consumption per Capita the options are ARMA(2,0) and ARMA(3,0). The decision for best ARMA model is first verified by Akaike and Schwarz criteria values. In the case of different models' choices, an additional criteria is used, by comparing coefficient values in the regression models (closer to 0).

For Real GDP per Capita the following values have been obtained: Akaike value for ARMA(1,1) was -2,382088 and Schwarz value – 2,27916, while for ARMA(1,2) Akaike criteria had the value -2,349831 and Schwarz criteria -2,212596. Since both criteria have a lower value for ARMA(1,1), chosen model for Real GDP per Capita is then ARMA(1,1).

In the case of Real Consumption per Capita, Akaike and Schwarz criteria generated different results for time series models ARMA(2,0) and ARMA(3,0). Here, for ARMA(2,0) the Akaike value was -2,597850 and Schwarz value -2,494037, while for ARMA(3,0) the Akaike value was -2,611432 and Schwarz value -2,471809, not making possible the final decision by using these criteria since chosen alternative was different for each of the analyzed criteria above-mentioned. Therefore, the choice of selecting optimal model was made based on the coefficient values in the regression model, closer to 0 for ARMA(2,0) than for ARMA(3,0) (see equations (29) and (34)).

5. Conclusions

Macroeconomics are an important factor for countries' wealth and stability. One of the challenges that national governments face are related to unpredictable economic evolutions that might lead to economic crises or financial instability.

In this paper are studied two macroeconomic indicators in the United States (Real GDP and Consumption per Capita) in period 1947 – 2017 (a couple of years after last economic crisis) in order to see how fragmented are both trends in chaos theory terms, to determine whether they are long-term persistent, and to check if time series methodologies can be applied. Results show that both time series are persistent, having three (and respective two) major fluctuations, corresponding to registered economic crises, and that their evolutions can be estimated by applying time series procedure such as ARMA modeling developed by Box and Jenkings.

In terms of Chaos Theory, the author used both Hurst coefficient and Koch curve in order to determine fractal dimension and fractal graphic representation. Results based on BDS statistics in determining fractal values show values below 1,5 for both Real GDP per Capital (fractal value 1,14614) and Real Consumption per Capita (fractal value 1,139744), making possible the estimation process based on ARMA design. Moreover, the Hurst coefficient for Real Consumption per Capita (0,860256) is larger than the coefficient value for Real GDP per Capita (0,85386), aspect reflected in these indicators' evolutions, where Real Consumption per Capita has two macroeconomic fluctuations, while Real GDP per Capita has three such deviations, all these deviations happening at different moments of time. Nevertheless, Koch curve reveals the evolutions of Real GDP per Capita and Real Consumption per Capita separately in order to see their similarities in chaos theory terms. Their evolutions has been simulated in the program **Matlab**, by using the wavelet function **coif3.**

The joined analysis for Real GDP per Capita and Real Consumption per Capita highlights another estimation procedure than the classic macroeconomic dependency because it shows how these indicators can be predicted separately, by applying time series.

The author's contribution to the research domain is given by the parallel study between fractal dimension (Chaos Theory) and ARMA(p,q) procedure (Time Series methodology) through creating a general framework that shows economic evolutions persistency, and how empirical results can be used in order to forecast various time series such as Real GDP per Capita and Real Consumption per Capita.

Data simulation and testing was made in the **Eviews** program, with original data as inputs, by applying logarithms and first degree order degree procedures in order to obtain standardized data, in addition to adding a dummy variable, attached to fluctuation years (value 0 for normal evolutions, and 1 for fluctuating years).

ARMA(p,q) procedure defined by Box and Jenkings is applied on standardized data, validated based on D-W, D-F and BDS statistics tests (in order to check auto-correlation errors). Results show that chosen models for the above-mentioned indicators do not have the same p (autoregressive) and q (mobile average) components, resulting into different ARMA estimation alternatives.

If for Real GDP per Capita, optimal time series model is ARMA(1,1) because both validation criteria used in this paper (Akaike and Schwarz) are lower for ARMA (1,1) than for ARMA (1,2) (with Akaike value -2,382088 and Schwarz value – 2,27916 compared to Akaike value -2,349831 and Schwarz value -2,212596), in the Real Consumption per Capita case, Akaike and Schwarz criteria result into different model choices (ARMA (3,0) chosen based on Akaike value of -

2.597850, larger than Akaike value -2.611432, and ARMA(2,0) chosen based on Schwarz value of -2,494037, lower than Schwarz value -2.471809). Here, the coefficient closeness to 0 in the estimated equations gives the optimal model, which is in this case ARMA(2,0).

Fractal dimension in terms of long term persistency, and used as a base for time series evolutions simulated with different methods can be applied on a large amount of economic indicators, not only in the US, but also in other countries that need to find those time series forecasting methodologies based on econometric modeling in order to find closer-to-reality models.

An extension of this paper can be the analysis of fractal dimension intervals as potential predictors for chosen ARMA models, as well as the study of future evolutions of the above-mentioned macroeconomic indicators.

Note: A preliminary draft was presented as a working paper at The 17th International Conference on Informatics in Economy (IE 2018) in May 2018, in Iasi (Romania).

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