LINIAR ANALYSE VERSUS NONLINIAR ANALYSE FOR SEWAGE AND PIPING SYSTEM

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Abstract: Numerical methods known in our days in order to address the calculus for a building structure involves the nonlinear calculus. Non linear calculus is basically a repetition of the linear calculus until a certain sectional equilibrium is acquired. Nevertheless a calculus that involves the stiffness matrix is also an approach often used in such a problem. As for the sewage and piping system is needed a calculus within the limits of linear limits. More often the need for a better approximation of phenomena that take place in the piping system is needed due to the fact that reliable and sustainable piping and sewage system are a great demand. More over I would say that piping is a crucial factor in some cases like fire or seismic event. For this purpose the present paper will consider some of the main important techniques for express the sectional and global equilibrium and stability. This paper will make a comparative analyze for the most important techniques with comparative results. The results are relevant in order to address the problem for further estimation and in order to address new challenges like ways and methods to compute errors of execution, seismic movements and aging/degradation or malfunctions of the piping system.

Key words: stiffness matrix; sewage and piping system; numerical method; the age factor; sewage and piping system; comparative study.

1. INTRODUCTION

The paper deals with the non-linear techniques used in structural analyses for reinforced concrete structures as well as metal/other material structures. The techniques will show the numerical comparative analyze and will focus on the matrix stiffness method due to possible advantages that this method can bring. More over the study will express the Finite element method and techniques to address a non linear analyze by using linear analyze that is most convenient and implies the less amount of calculus. The volume of calculus needed for a non linear analyze will be an important issue treated by the present due to precision needed.

For further estimation of the sectional efforts that take place into the solids analyzed I will express the mathematical formulation for the stiffness matrix with 1 and 3 degrees of freedom. Also an analyze of compounding the final stiffness matrix is expressed in the present paper for the stiffness matrix with 6 degrees of freedom. In the end an algorithm will be propose capable of implementing this techniques that leads to good results comparative with laboratory tests. Please note that such an algorithm was tested for structural elements only as for sewage and piping elements it is only at an early stage.
2. MAIN IDEA

There are two major ways to estimate the cross sectional effort for a piping line/connection. This sectional effort, as expressed by Zienkiewicz [10] leads to the estimation of the minimal material/with and physical properties necessary in order to create a sustainable piping/sewage line. Most of the piping available on the market is certified through laboratory tests conducted by INCCD (National Institute of Certified Testing and Accreditations) in Romania and elsewhere by similar institutions. The main problem remains the viability, the time testing of the products; which cannot be tested on usually laboratory tests. Therefore some of the leading industry players offer warranties long periods of time. Even though the warranty is due, in fact the warranty cannot cover collateral damages caused by the degradation of other goods and values caused by a malfunction of a piping/sewage system. Often times the collateral damage is more significant in financial terms. That is why the study is a financial demand and the behavior of piping system is crucial in order to estimate reliable economical forecasts.

Next I will mention the Finite Element Method and the Matrix Method as main important ways to do the static and dynamic predictions, as expressed by A. Borosnyoi [2]. Because the topic is vast I will focus on Matrix Method. For an easy understanding of the matter I will express the Direct Method, which is characterized as Matrix Method, for a solid with 2 DOF (Degrees of Freedom), as expressed by Faur [4], as seen in (fig. no. 1).

\[ \begin{align*}
F_1 &= k_{11} = -k_{21} = \frac{ES}{L} \\
F_2 &= k_{22} = -\frac{ES}{L}
\end{align*} \]  

(1)

(2)

The low of static expressed by Hooke sais that the forces has to be in an equilibrium as (3):

\[ F_1 + F_2 = 0 \]  

(3)

Another rewriting of (1), (2) and (3) is expressed in (4) by a matrix.

\[ \begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} = 
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix} \]  

(4)

As expressed above, in a similar manner it is considered a movement in point A as \( u_1 = 0 \) and in point B \( u_2 = 1 \) as seen in (fig. no. 2).
The forces in point A and in point B become:

\[ F_2 = K_{22} = K_{11} = \frac{ES}{L} \]  \hspace{1cm} (5)

and:

\[ F_1 = K_{21} = K_{12} = -\frac{ES}{L} \]  \hspace{1cm} (6)

As a conclusion a final stiffness matrix for a solid with 1 degree of freedom is created and expressed in (7). Please note that a similar result is obtain by using Indirect Method.

\[ [K_i] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \frac{ES}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]  \hspace{1cm} (7)

As for a solid with 6 degrees of freedom as in (fig. no. 3) where the forces/efforts are \( N_x, S_y, S_z, M_z, M_y, M_x \) and along each force/effort there is a degree of freedom, means there is a possibility for the point/node to move as in (fig. no. 4) and expressed as three linear movements along axes X,Y and Z (linear movements are: u, v, w) and three rotational movements around axes X,Y and Z (angular movements are: \( \phi_x, \phi_y, \phi_z \)), as expressed by Hutchinson [6].

\[ \begin{align*}
T_{y1} & = \frac{N_1}{L} \\
M_{y1} & = \frac{M_x}{L} \\
M_{x1} & = \frac{M_z}{L}
\end{align*} \]  \hspace{1cm} (8)
Figure 4- Schematic figure for a solid with 6 degree of freedom-DOF-movements

Similar with procedure applied for solid with one degree of freedom, as expressed by Chiorean [3], a matrix 12x12 as in (10) would be the stiffness matrix and the movement vector \( U_i \), and force vector \( F_i \) as in (8) would create the final mathematical formulation as in (9).

\[
\{U_i\} = \begin{bmatrix}
    u_t \\
    v_t \\
    w_t \\
    \phi_{x1} \\
    \phi_{y1} \\
    \phi_{z1} \\
    u_2 \\
    v_2 \\
    w_2 \\
    \phi_{x2} \\
    \phi_{y2} \\
    \phi_{z2}
\end{bmatrix}
\]

\[
\{F_i\} = \begin{bmatrix}
    N_t \\
    T_{y1} \\
    T_{z1} \\
    M_{x1} \\
    M_{y1} \\
    M_{z1} \\
    N_2 \\
    T_{y2} \\
    T_{z2} \\
    M_{x2} \\
    M_{y2} \\
    M_{z2}
\end{bmatrix}
\]  \hspace{1cm} (8)

\[
\{F_i\} = [K_i]\{U_i\}
\]  \hspace{1cm} (9)
And the final mathematical formulation for stiffness matrix for a solid with 6 degree of freedom is expressed in (10). The main notice, according to Nam-Il Kim [8], that is to be made is that a good sense of understanding math principles would underline the fact that efforts are interdependent with some exceptions. The exceptions, [1] are among shear forces $T_{y,z}$ and moments $M_{y,z}$. A very odd observation leading to practical issues is an example of a solid that is solicited by a rotational effort along the axis $X$ leads to conclusion that there are not created efforts of elongation along axes $X$, with other words effort along axes $X$. This note is at least strange. Many researchers ignored that connection due to the fact that in structural static analyze of a building structure that effect is under 15 % and mainly due to the fact that efforts of rotation along axes $X$ is rare in construction industry. Nevertheless for piping the effort interconnection is stronger due to circular shape and empty core cross section.

If a system of axes considered as coincident with forces and movements as expressed in (fig. no. 3) and (fig. no. 4), than the stiffness matrix is as expressed in (10). Please note that in a nonlinear analyze that is to be made the system of axes is mobile as the solid moves. Therefore it is necessary to express the connection among two different systems of axes: the local system represented by the solid and the global system represented by initial position of the solid. If the angles among axes are $\alpha_i, \beta_i, \gamma_i$, with $i=1,2,3$ ca as in (fig. no. 5) a mathematical formulation can be made according to Herno [5].

\[
\{K\} = \begin{bmatrix}
\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_y}{L^3} & 0 & 0 & 0 & -\frac{6EL_y}{L^2} & 0 & -\frac{12EI_y}{L^3} & 0 & 0 & 0 & \frac{6EL_y}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{GI_y}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_y}{L} & 0 & 0 \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} \\
\end{bmatrix}
\] (10)
Figure.5 - Schematic figure for a solid with a different system of axis than the coincident system

The mathematical formulation is found in (11).

\[ \{K_i\} = \{K_i\}T_j \]  \hspace{1cm} (11)

With:

\[
\begin{bmatrix}
\{T_j\} & \{0\} & \{0\} & \{0\} \\
\{0\} & \{T_j\} & \{0\} & \{0\} \\
\{0\} & \{0\} & \{T_j\} & \{0\} \\
\{0\} & \{0\} & \{0\} & \{T_j\}
\end{bmatrix}
\]

and

\[
\{0\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]  \hspace{1cm} (14)

With notations:

\[ a_i = \cos \alpha_i, \quad b_i = \cos \beta_i, \quad c_i = \cos \gamma_i \]  \hspace{1cm} (15)

With i=1,2,3 and:

\[ \alpha_{1-\text{angle}} (x,x'), \quad \beta_{1-\text{angle}} (y,y'), \quad \gamma_{1-\text{angle}} (z,z') \]

\[ \alpha_{2-\text{angle}} (x,y'), \quad \beta_{2-\text{angle}} (y,y'), \quad \gamma_{2-\text{angle}} (z,y') \]

\[ \alpha_{3-\text{angle}} (x,z'), \quad \beta_{3-\text{angle}} (y,z'), \quad \gamma_{3-\text{angle}} (z,z') \]  \hspace{1cm} (16)

The global stiffness matrix is composed by combining stiffness matrix for every component of the solid eith respect to the common point/nodes and common effort/deformations. For the connections among deformations and efforts is easy to underline that connection in the (fig. no. 6) for a solid element with 3 degrees of freedom.
Please note that a global stiffness matrix from each component stiffness matrix is done according to (fig. no. 7).

The final remark is that phenomena that take place at the level of the constituent element as seen in (fig. no. 7) is repeating at the level of global solid. With other words the connection among efforts is still missing sometimes. Therefore the valuable remark that interconnection among rotational effort and other efforts is of crucial importance in piping and sewage system especially when it comes to used systems.
Please note that piping and sewage system has a special demand in making connections by using torsional forces and movements as a screw is doing. The study above shows that such a demand is not clearly fundamental understood and therefore cannot be implemented with a real success. However there is a small amount of knowledge conducted by some pioneers that tries to make a connection among efforts as in (fig. no. 8) showing that effort outside the surrounding is eligible to fail, according to Liu [7].

Figure 7- Graphic estimation of the interconnection among strain/stress relationship

3. CONCLUSIONS

When it comes to new piping and sewage system the sustainability is insured by nothing else but laboratory tests and limited knowledge obtained by simple observation. Nevertheless there is no calculus or theoretical knowledge on how the piping system behaves in time on points like U turns, or where the material is partially melted due to installation process.

The most sensitive part of a sewage/piping system is where the connections are. The knowledge does not take into account long period of time testing of the connections tested in laboratory, according to Pfrang [9]. This would lead to significant delay of a certain material or process to be certified before being allowed on the market in our country or elsewhere. Therefore an effort of researchers is needed in order to create valuable knowledge about this calculus for piping and sewage system. This knowledge would lead to know when is appropriate or not to exchange the piping sewage system and so huge financial opportunities and demand is created. Nevertheless the education system is benefitting out of this proposed research as well as general well fair as well. More over a legislative proposal can be created as well. However linear analyses can be enough with the condition of making a connection among efforts, especially rotational and axial.

References
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