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RIEMANN SOLITONS ON GENERALIZED WEAKLY ω-SYMMETRIC α-COSYMPLECTIC MANIFOLDS

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Abstract. Generalized quasi-conformal curvature tensor (ω-tensor) has the flavour of conformal, conharmonic, concircular, projective, m-projective, $\mathcal{W}_1$-curvature, $\mathcal{W}_2$-curvature and $\mathcal{W}_4$-curvature tensors. In the present paper we have investigated the nature of Riemann solitons in α-cosymplectic manifold in the light of generalized weakly ω-symmetric structure.

1 Introduction

In 2016, Hirică and Udrişte [4] introduced and studied the notion of Riemann solitons. A smooth manifold M with Riemannian metric g is called a Riemann soliton if g satisfies

$$2R + \lambda(g \wedge g) + (g \wedge \pounds_X g) = 0, \tag{1.1}$$

where X is a potential vector field, $\pounds_X$ denotes the Lie-derivative and λ is a constant and for $(0, 2)$-tensors μ and ν, the Kulkarni-Nomizu product $(\mu \wedge \nu)$ is given by

$$\begin{aligned} & (\mu \wedge \nu)(Y, U, V, Z) \\ = \; & \mu(Y, V)\nu(U, Z) + \mu(U, Z)\nu(Y, V) \\ & -\mu(Y, Z)\nu(U, V) - \mu(U, V)\nu(Y, Z). \end{aligned} \tag{1.2}$$

A Riemann soliton is called expanding, steady and shrinking when $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$ respectively. The Riemann soliton are also studied in [2], [5–8] and also references there in.

Recently, the authors in [9] have introduced and studied generalized quasi-conformal curvature tensor (which will be denoted by ω-tensor) in the frame of

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$N(k,\mu)$-manifold. For n-dimensional manifold, ω-tensor is defined as follows:

$$\begin{aligned} & \omega(Y,U)V \\ = \ & R(Y,U)V + a_1[S(U,V)Y - S(Y,V)U] + b_1[g(U,V)LY - g(Y,V)LU] \\ & -\frac{c_1 r}{n}\left(\frac{1}{n-1} + a_1 + b_1\right)[g(U,V)Y - g(Y,V)U], \end{aligned} \tag{1.3}$$

for all Y, U & $V \in \chi(M)$, the set of all smooth vector fields on the manifold M, where scalar triples (a_1, b_1, c_1) are real constants. It is to be noted that ω-tensor has the flavour of conformal curvature tensor C [13] if $(a_1,b_1,c_1) \equiv (-\frac{-1}{n-2}, -\frac{-1}{n-2}, 1)$, Riemann curvature tensor R if $(a_1,b_1,c_1) \equiv (0,0,0)$, concircular curvature tensor E [15] if $(a_1,b_1,c_1) \equiv (0,-0,1)$, conharmonic curvature tensor K [14] if $(a_1,b_1,c_1) \equiv (-\frac{-1}{n-2}, -\frac{-1}{n-2}, 0)$, projective curvature tensor P [15] if $(a_1,b_1,c_1) \equiv (-\frac{-1}{n-1}, 0, 0)$, m-projective curvature tensor $\mathcal{M}$ [19] if $(a_1,b_1,c_1) \equiv (-\frac{-1}{2n-2}, -\frac{-1}{2n-2}, 0)$, the $\mathcal{W}_1$-curvature tensor [20] if, $(a_1,b_1,c_1) \equiv (\frac{1}{(n-1)}, 0, 0)$, the $\mathcal{W}_2$-curvature tensor [19] if $(a_1,b_1,c_1) \equiv (0, -\frac{1}{(n-1)}, 0)$, the $\mathcal{W}_4$-curvature tensor [21] if $(a_1,b_1,c_1) \equiv (0,0,\frac{n}{r})$, where ([2], [11, 12])

$$\begin{aligned} C(U,V) &= R(U,V) \\ &\quad -\frac{1}{n-2}\left[(U \wedge_g QV) + (QU \wedge_g V) + \frac{r}{(n-1)}(U \wedge_g V)\right], \\ E(U,V) &= R(U,V) - \frac{r}{n(n-1)}(U \wedge_g V), \\ P(U,V) &= R(U,V) - \frac{1}{n-1}(U \wedge_S V), \\ K(U,V) &= R(U,V) - \frac{1}{n-2}[(U \wedge_g QV) + (QU \wedge_g V)], \\ \mathcal{M}(U,V) &= R(U,V) - \frac{1}{2(n-1)}[(U \wedge_g QV) + (QU \wedge_g V)], \\ \mathcal{W}_1(U,V) &= R(U,V) - \frac{1}{(n-1)}(U \wedge_S V), \\ \mathcal{W}_2(U,V) &= R(U,V) - \frac{1}{(n-1)}[(QU \wedge_g V) + (U \wedge_g QV) - (U \wedge_S V)], \\ \mathcal{W}_4(U,V)W &= R(U,V)W - \frac{1}{(n-1)}[g(U,W)QV - g(U,V)QW], \end{aligned}$$

and

$$(U \wedge_B V)W = B(V,W)U - B(U,W)V.$$

In [10] author studied GRW-spacetime and certain type of energy momentum tensor. A Riemannian manifold of dimension n is said to be a generalized weakly symmetric [1], if it admits the following equation

$$\begin{aligned} &(\nabla_X \bar{R})(Y,U,V,Z) \\ = \; & F(X)\bar{R}(Y,U,V,Z) + G(Y)\bar{R}(X,U,V,Z) + G(U)\bar{R}(Y,X,V,Z) \\ & +H(V)\bar{R}(Y,U,X,Z) + H(Z)\bar{R}(Y,U,V,X) \\ & +\Upsilon(X)\bar{G}(Y,U,V,Z) + \beta(Y)\bar{G}(X,U,V,Z) + \beta(U)\ \bar{G}(Y,X,V,Z) \\ & +\gamma(V)\bar{G}(Y,U,X,Z) + \ \gamma(Z)\bar{G}(Y,U,V,X), \end{aligned} \tag{1.4}$$

where

$$\bar{G}(Y,U,V,\ Z) = g(U,V)g(Y,Z) - g(Y,\ V)g(U,Z), \tag{1.5}$$

for all vector fields X, Y and U and the 1-forms $F(X) = g(X,\pi_1)$, $G(X) = g(X,\pi_2)$, $H(X) = g(X,\varrho)$, $\Upsilon(X) = g(X,\delta_1)$, $\beta(X) = g(X,\delta_2)$ and $\gamma(X) = g(X,\sigma)$.

On the analogy of the above definition, an n-dimensional Riemannian (or semi-Riemannian) manifold is said to be generalized weakly ω-symmetric, if the ω-tensor admits the following:

$$\begin{aligned} &(\nabla_X \omega)(Y,U,V,Z) \\ = \; & F(X)\omega(Y,U,V,Z) + G(Y)\omega(X,U,V,Z) + G(U)\omega(Y,X,V,Z) \\ & +H(V)\omega(Y,U,X,Z) + H(Z)\omega(Y,U,V,X) \\ & +\Upsilon(X)\bar{G}(Y,U,V,Z) + \beta(Y)\bar{G}(X,U,V,Z) + \beta(U)\ \bar{G}(Y,X,V,Z) \\ & +\gamma(V)\ \bar{G}(Y,U,X,Z) + \ \gamma(Z)\bar{G}(Y,U,V,X). \end{aligned} \tag{1.6}$$

The paper is arranged as follows: Section-2 is concerned with some basic results of α-cosymplectic manifolds. In section-3, we have investigated generalized weakly ω-symmetric α-cosymplectic manifold whose metric is Riemann soliton. It is shown that in any case of generalized weakly symmetric, generalized weakly concircularly symmetric, generalized weakly conformally symmetric, generalized weakly conharmonically symmetric, generalized weakly projectively symmetric and generalized weakly m-projectively symmetric, an α-cosymplectic manifold is an η-Einstein manifold as well as Ricci symmetric. We also found curvature restrictions for which Riemann soliton of each of generalized weakly concircularly symmetric, generalized weakly conformally symmetric, generalized weakly conharmonically symmetric, generalized weakly projectively symmetric, generalized weakly m-projectively symmetric, generalized weakly $\mathcal{W}_1$-symmetric, generalized weakly $\mathcal{W}_2$-symmetric and generalized weakly $\mathcal{W}_4$-symmetric α-cosymplectic manifold is expanding, steady and shrinking.

2 α-cosymplectic manifolds and structures

It is well known that [3], an *almost contact structure* (ϕ, ξ, η) on an $n = (2m+1)$-dimensional Riemannian manifold satisfies the following:

$$\phi^2 = -I + \eta \otimes \xi, \tag{2.1}$$

$$\eta(\xi) = 1, \tag{2.2}$$

$$\phi\xi = 0,\ \ \eta \circ \phi = 0,\ \ \text{rank } \phi = \frac{n-1}{2}. \tag{2.3}$$

Moreover, if g is a Riemannian metric on M^n satisfying

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.4}$$

$$g(X, \xi) = \eta(X), \tag{2.5}$$

$$g(\phi X, Y) = -g(X, \phi Y), \tag{2.6}$$

for any vector field X, Y on M^n, then the manifold M^n [3] is said to admit an *almost contact metric structure* (ϕ, ξ, η, g). For an almost contact metric structure (ϕ, ξ, η, g) on an $n = (2m+1)$-dimensional manifold M^n, its Kähler form $\psi \in \Omega^2(M)$ is given by $\psi(X, Y) = g(\phi X, Y)$. The almost contact metric structure (ϕ, ξ, η, g) is cosymplectic if $d\eta = d\psi = 0$; i.e., η and ψ are closed, such that $\eta \wedge \psi^m$ is a volume form.

In an α-cosymplectic manifold, the following relations hold ([16–18], [22]):

$$(\nabla_X \phi)Y = \alpha[g(\phi X, Y)\xi - \eta(Y)\phi X], \tag{2.7}$$

$$\nabla_X \xi = \alpha[X - \eta(X)\xi], \tag{2.8}$$

$$\eta(R(X, Y)Z) = \alpha^2[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)], \tag{2.9}$$

$$R(X, Y)\xi = \alpha^2[\eta(X)Y - \eta(Y)X], \tag{2.10}$$

$$R(X, \xi)\xi = \alpha^2\phi^2 X, \tag{2.11}$$

$$R(X, \xi)\xi - \phi R(\phi X, \xi)\xi = 2\alpha^2\phi^2 X, \tag{2.12}$$

$$R(\xi, X)Y = \alpha^2[\eta(Y)X - g(X, Y)\xi], \tag{2.13}$$

$$S(X, \xi) = -(n-1)\alpha^2\eta(X), \tag{2.14}$$

$$(\nabla_X S)(Y, \xi) = -\alpha S(X, Y) - (n-1)\alpha^3\eta(X)\eta(Y), \tag{2.15}$$

for any vector field X, Y, Z on M^n.

Definition 1. *An α-cosymplectic manifold M^n is said to be an η-Einstein manifold if the Ricci curvature tensor S is of the form*

$$S = ag + b\eta \otimes \eta,$$

where a and b are smooth functions on M^n and η is an 1-form.

In particular, if $b = 0$, then M^n is said to be an *Einstein manifold.* Now, contracting over U and V we get from (1.3)

$$\sum_{i=1}^{n} \omega(Y, e_i, e_i, Z) = \kappa_1 S(Y, Z) + \kappa_2 r g(Y, Z) \tag{2.16}$$

and

$$\sum_{i=1}^{n} (\nabla_X \omega)(Y, e_i, e_i, Z) = \kappa_1 (\nabla_X S)(Y, Z) + \kappa_2 dr(X) g(Y, Z), \tag{2.17}$$

where

$$\begin{aligned} \kappa_1 &= [1 - a_1 + (n-1)b_1], \\ \kappa_2 &= (n-1)\left[a_1 - \frac{c_1}{n}\left(\frac{1}{n-1} + a_1 + b_1\right)\right], \end{aligned}$$

$\{e_i : 1 \leq i \leq n\}$ is an orthonormal frame at any point of the manifold. We also get from (1.5)

$$\sum_{i=1}^{n} \bar{G}(Y, e_i, e_i, Z) = (n-1) g(Y, Z). \tag{2.18}$$

3 Generalized weakly ω-symmetric α-cosymplectic manifold when the metric is Riemann soliton

An n-dimensional α-cosymplectic manifold is said to be generalized weakly ω-symmetric, if the ω-tensor satisfies the curvature condition (1.6). Now contracting over U and V, and then using (2.16), (2.17) and (2.18) in (1.6), we obtain

$$\begin{aligned} &\kappa_1 (\nabla_X S)(Y, Z) + \kappa_2 dr(X) g(Y, Z) \\ = \; &F(X)[\kappa_1 S(Y, Z) + \kappa_2 r g(Y, Z)] + G(Y)[\kappa_1 S(X, Z) + \kappa_2 r g(X, Z)] \\ &+ H(Z)[\kappa_1 S(X, Y) + \kappa_2 r g(X, Y)] + G(\omega(X, Y,)Z) + H(\omega(X, Z,)Y) \\ &+ (n-1)\Upsilon(X) g(Y, Z) + (n-1)\beta(Y) g(X, Z) + (n-1)\gamma(Z) g(X, Y) \\ &+ \beta(\bar{G}(X, Y,)Z) + \gamma(\bar{G}(X, Z,)Y). \end{aligned} \tag{3.1}$$

Setting $Z = \xi$ in (3.1) and then using (2.5), (2.14) and (2.15), we have

$$\begin{aligned} & \kappa_1[-\alpha S(X,Y) - (n-1)\alpha^3\eta(X)\eta(Y)] + \kappa_2 dr(X)\eta(Y) \\ = \; & F(X)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r]\eta(Y) + G(Y)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r]\eta(X) \\ & +H(\xi)[\kappa_1 S(X,Y) + \kappa_2 r g(X,Y)] + G(\omega(X,Y,)\xi) + H(\omega(X,\xi,)Y) \\ & +(n-1)\Upsilon(X)\eta(Y) + (n-1)\beta(Y)\eta(X) + \; (n-1)\gamma(\xi)g(X,Y) \\ & +\beta(\bar{G}(X,Y,)\xi) \; + \gamma(\bar{G}(X,\xi,)Y). \end{aligned} \tag{3.2}$$

Again, putting $X = \xi$ in (3.2) and then using (2.2), (2.14), we get

$$\begin{aligned} & \kappa_2 dr(\xi)\eta(Y) \\ = \; & F(\xi)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r]\eta(Y) + G(Y)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r] \\ & +H(\xi)[-(n-1)\alpha^2\eta(Y)\kappa_1 + \kappa_2 r\eta(Y)] + G(\omega(\xi,Y,)\xi) + H(\omega(\xi,\xi,)Y) \\ & +(n-1)\Upsilon(\xi)\eta(Y) + (n-1)\beta(Y) + \; (n-1)\gamma(\xi)\eta(Y) \\ & +\beta(\bar{G}(\xi,Y,)\xi) \; + \gamma(\bar{G}(\xi,\xi,)Y). \end{aligned} \tag{3.3}$$

Again, setting $Y = \xi$ in (3.2) and in view of (2.2), (2.14), we have

$$\begin{aligned} & \kappa_2 dr(X) \\ = \; & F(X)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r] + G(\xi)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r]\eta(X) \\ & +H(\xi)[-(n-1)\alpha^2\eta(X)\kappa_1 + \kappa_2 r\eta(X)] + G(\omega(X,\xi,)\xi) + H(\omega(X,\xi,)\xi) \\ & +(n-1)\Upsilon(X) + (n-1)\beta(\xi)\eta(X) + \; (n-1)\gamma(\xi)\eta(X) \\ & +\beta(\bar{G}(X,\xi,)\xi) \; + \gamma(\bar{G}(X,\xi,)\xi). \end{aligned} \tag{3.4}$$

Now, plugging $X = Y = \xi$ in (3.2) and by virtue of (2.2), (2.14), we obtain

$$\begin{aligned} & \kappa_2 dr(\xi) \\ = \; & F(\xi)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r] + G(\xi)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r] \\ & +H(\xi)[-(n-1)\alpha^2\kappa_1 + \kappa_2 r] + G(\omega(\xi,\xi,)\xi) + H(\omega(\xi,\xi,)\xi) \\ & +(n-1)\Upsilon(\xi) + (n-1)\beta(\xi) + \; (n-1)\gamma(\xi) \\ & +\beta(\bar{G}(\xi,\xi,)\xi) \; + \gamma(\bar{G}(\xi,\xi,)\xi). \end{aligned} \tag{3.5}$$

Making use of (3.3), (3.4) and (3.5) in (3.2), we get

$$\begin{aligned} & [-\kappa_1\alpha - \kappa_1 H(\xi) + a_1 H(\xi)]S(X,Y) \\ = \; & [\{\kappa_2 r + \alpha^2 + b_1(n-1)\alpha^2 + a_1 r - \frac{\kappa_2 r}{(n-1)}\}H(\xi) + (n-2)\gamma(\xi)]g(X,Y) \\ & -[\{\kappa_2 r + \alpha^2 + b_1(n-1)\alpha^2 + a_1 r - \frac{\kappa_2 r}{(n-1)}\}H(\xi) + (n-2)\gamma(\xi) \\ & +(n-1)\alpha^2\{-\kappa_1\alpha - \kappa_1 H(\xi) + a_1 H(\xi)\}]\eta(X)\eta(Y). \end{aligned} \tag{3.6}$$

Since η is closed and ∇ is a metric connection, taking covariant derivative on the both sides of (3.6) we obtain by straight forward calculation

$$(\nabla_Z S)(X, Y) = 0. \tag{3.7}$$

This leads to the following:

Theorem 2. *Let (M^n, g) be an α-cosymplectic manifold. Then for each of the following:*

(i) generalized weakly symmetric structures,
(ii) generalized weakly concircularly symmetric structures,
(iii) generalized weakly conformally symmetric structures,
(iv) generalized weakly conharmonically symmetric structures,
(v) generalized weakly projectively symmetric structures,
(vi) generalized weakly m-projectively symmetric structures,
(vii) generalized weakly $\mathcal{W}_1$-symmetric structures,
(vii) generalized weakly $\mathcal{W}_2$-symmetric structures,
(ix) generalized weakly $\mathcal{W}_4$-symmetric structures,

the manifold M^n is an η-Einstein manifold as well as Ricci symmetric provided $(a_1 - \kappa_1)H(\xi) \neq \kappa_1\alpha$.

Taking ξ as the potential vector field and using (1.2) in (1.1) and then contracting over U and V, we obtain

$$S(Y, Z) = [\lambda(n-1) + \alpha(2n-3)]g(Y, Z) - \alpha(n-2)\eta(Y)\eta(Z). \tag{3.8}$$

From (3.8) and (3.6), we have the following:

Theorem 3. *The Riemann soliton on generalized weakly ω-symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $[\{\alpha(2n-3)\kappa_1 + \frac{(n-2)r}{(n-1)}\kappa_2 + \alpha^2 + b_1(n-1)\alpha^2 + a_1 r - a_1\alpha(2n-3)\}H(\xi) + (n-2)\gamma(\xi)] + \alpha^2(2n-3)\kappa_1 >=< 0$ *provided* $(a_1 - \kappa_1)H(\xi) \neq \kappa_1\alpha$.

Corollary 4. *The Riemann soliton on generalized weakly conformally symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $\left[\left\{-\frac{(n-1)r}{n} - \frac{r+\alpha^2}{(n-2)} + \frac{\alpha(2n-3)}{(n-2)}\right\}H(\xi) + (n-2)\gamma(\xi)\right] >=< 0$ *provided* $H(\xi) \neq 0$.

Corollary 5. *The Riemann soliton on generalized weakly conharmonically symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $[\{-\alpha^2 - (n-1)r + \alpha(2n-3)\}H(\xi) + (n-2)^2\gamma(\xi)] >=< 0$ *provided* $H(\xi) \neq 0$.

Corollary 6. *The Riemann soliton generalized on weakly concircularly symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $[\{\alpha(2n-3) - \frac{r}{n} + \alpha^2\}H(\xi) + (n-2)\gamma(\xi)] + \alpha^2(2n-3) >=< 0$ *provided* $-H(\xi) \neq \alpha$.

Corollary 7. *The Riemann soliton on generalized weakly projectively symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $\left[\left\{\frac{\alpha(2n-3)(n+1)}{(n-1)} - r + \alpha^2\right\} H(\xi) + (n-2)\gamma(\xi)\right] + \alpha^2(2n-3)\frac{n}{(n-1)} >=< 0$ *provided* $-(n+1)H(\xi) \neq n\alpha$.

Corollary 8. *The Riemann soliton on generalized weakly m-projectively symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $\left[\left\{\frac{\alpha(2n-3)(n+1)}{2(n-1)} - \frac{r}{2} + \frac{\alpha^2}{2}\right\} H(\xi) + (n-2)\gamma(\xi)\right] + \alpha^2(2n-3)\frac{n}{2(n-1)} >=< 0$ *provided* $-(n+1)H(\xi) \neq n\alpha$.

Corollary 9. *The Riemann soliton on generalized weakly $\mathcal{W}_1$-symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $\left[\left\{\alpha(2n-3)\frac{(n-3)}{(n-1)} + \alpha^2 + r\right\} H(\xi) + (n-2)\gamma(\xi)\right] + \alpha^2(2n-3)\frac{(n-2)}{(n-1)} >=< 0$ *provided* $-(n-1)H(\xi) \neq (n-2)\alpha$.

Corollary 10. *There does not exist an α-cosymplectic manifold with generalized weakly $\mathcal{W}_2$-symmetric structure.*

Corollary 11. *The Riemann soliton on generalized weakly $\mathcal{W}_3$-symmetric α-cosymplectic manifold is expanding, steady or shrinking according as* $\left[\left\{\alpha(2n-3) - \frac{(n-2)}{(n-1)} + \alpha^2\right\} H(\xi) + (n-2)\gamma(\xi)\right] + \alpha^2(2n-3) >=< 0$ *provided* $-H(\xi) \neq \alpha$.

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