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VISCOUS DISSIPATION AND RADIATIVE EFFECTS IN THE MAGNETO-MICROPOLAR FLUID WITH PARTIAL SLIP AND CONVECTIVE BOUNDARY CONDITION

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Abstract. In this article, the numerical study of the flow of micro polar fluid above a porous stretching sheet in the presence of viscous dissipation, thermal radiation, flow on an unsteady stretching surface and magnetohydrodynamic with heat transfer through moving fluid is discuses. A proper similarity transformation is utilized to convert the boundary layer equations into the nonlinear and coupled ordinary differential equations. These ODEs are sorted out numerically by applying the shooting mechanism. Graphical representations are also included to explain the effect of evolving parameters against the above-mentioned distributions. Finally, the numerical outcomes are discussed at the end of the article.

1 Introduction

The science which analyzes the movement of a highly conducting fluid within the presence of a magnetic field is termed as the Magnetohydrodynamics (MHD).Analysis of Newtonian and non-Newtonian flows in the presence of magnetic field has an innumerable applications in industries and engineering. Some prominent uses of MHD can be seen in the cooling system with fluid metals, MHD power generators, liquid beads and sprays, accelerators, atomic reactors, preparing of nourishment stuffs, oil industry, microelectronic gadgets, geothermal energy extractions of metals and so on. Ahmad et al.[1] investigated the semi-inverse solutions for non uniform magnetohydrodynamic stream of second grade fluid over stretching surface. Irregular magnetohydrodynamic mixed convection stream of second grade nanofluid induced by stretching plane with thermal radiation was observed by Ramzan and Bilal [2]. Ellahi [3] examined the magnetohydrodynamic flow of non-ideal nano fluid in a tube. Govindaraju et al. [4] studied the entropy modeling of the nano fluid and MHD flow over a nonlinear stretching surface. MHD rotating flow of water based nano fluids

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moving in parallel plates was discussed by Sheikholeslami et al.[5]. Lin et al.[6] examined the heat transfer impacts on MHD flow of pseudo-plastic fluid loaded with antiparticles.

The impact of thermal rays is essential in space technology and high temperature processes. At the point when the temperature variation is very high, the linear thermal radiation causes a noticeable error. To overcome such errors, nonlinear thermal radiation is taken into account. The impact of chemical reaction and thermal radiation on the flow over stretching surface with outer heat source was explained by Krishna et al.[7]. Researchers and scientists have done a series of research work to highlight the importance of thermal radiation [8]-[10].

Mixed convection is one of the transport phenomena which is composed of both natural and forced convection flow. Mixed convection flow appear in many transport processes both naturally and in engineering applications. Industrial and technical processes incorporating the solar central receivers exposed to winds,electronic gadgets cooled by fans and nuclear reactors cooled in case of emergence shutdown. Abo-Eldahab et al. [11] examined the magneto-hydrodynamic free heat transfer flow past a semi-infinite vertical strip with mass exchange and Hall effects. Mixed convection boundary layer is influenced by Hall current and Ohmic heating. Impacts on flow of a micropolar fluid from a circular cone with power-law fluid at stretching surface was studied by Abo-Eldahab et al.[12]. The impact of Hall current and chemical reaction on hydromagnetic flow of a vertical stretching plane with interior heat absorption was presented by Salem and El-Aziz [13].

In this paper, we provide a comprehensive numerical review of [14]. A numerical study of micropolar, partial slip, MHD flow with convective boundary condition is analyzed. The constitutive of the flow model expression are sorted out numerically and the impact of physical parameters concerning the flow model on dimensionless energy, velocity, and microrotation are presented through graphs and tables. A comparison of the achieved numerical results with the published results of Ramzanet al. [14] is also presented.

2 Problem Formulation

Assume that the fluid under discussion is taken as MHD two dimensional incompressible micropolar fluid which passes over porous stretching surface with slip velocity. The governing equations (2.1)-(2.4) are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial y}v=\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma}{\rho}\left(E_0B_0-B_0^2u\right), \tag{2.2}$$

$$\frac{\partial N}{\partial x}u+\frac{\partial N}{\partial y}v=\frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\kappa}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right), \tag{2.3}$$

$$\frac{\partial T}{\partial x}u+\frac{\partial T}{\partial y}v=\frac{\kappa_1}{\rho C_p}\left(\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho c_p}\frac{16\sigma^* {T_\infty}^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\frac{(\mu+\kappa)}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{(uB_0-E_0)^2}{\rho C_p} \tag{2.4}$$

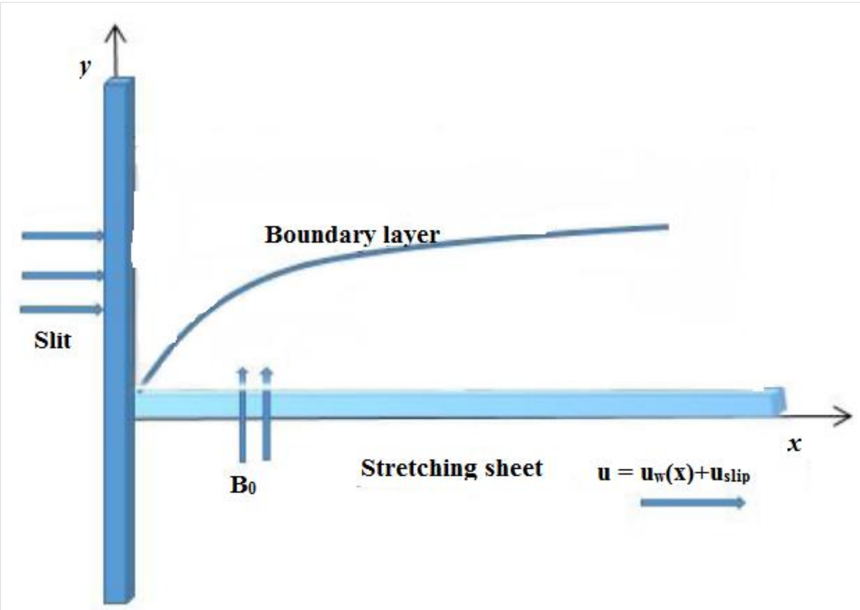

Figure 1: Geometry of the problem

In eqs (2.1)-(2.4),κ^* the mean absorption coefficient, σ^* the Stefen-Boltzman, T the temperature and v the kinematic viscosity, B_0 the applied magnetic field strength, E_0 the applied electric field, $v=\frac{\mu}{\rho}$. The parameters κ and C_p are taken as constants and represents the permeability of porous media, the thermal conductivity and specific heat of the fluid respectively. The associated BCs for the above system of equations are,

$$\left.\begin{array}{l}\left.\begin{array}{l}u=ax+\sigma^*\left[(\mu+\kappa)\frac{\partial u}{\partial y}+\kappa N\right], v=v_w,\\ N=-n\frac{\partial u}{\partial y}, -k\left(\frac{\partial T}{\partial y}\right)=h_f\left[T_f-T\right],\end{array}\right\}\text{ at } y=0\\ \quad u\to 0,\quad N\to 0,\quad T\to T_\infty,\quad \text{as}\quad y\to\infty\end{array}\right\} \tag{2.5}$$

Here N,j,κ_1,α^*,u_w and h_f represent the micro-rotation or angular velocity, micro inertia density, thermal conductivity, slip coefficient, suction/injection velocity,heat transfer coefficient respectively. Further, is a constant and $0 \leq n \leq 10$. Now we convert the system of Eqs. (2.1)-(2.4) following the boundary conditions into a unit less form. For this purpose, we use the following similarity transformation.

$$\eta = \sqrt{\frac{a}{v}}y, N = ax\sqrt{\frac{a}{v}}h(\eta), u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \psi = \sqrt{av}xf(\eta), \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \tag{2.6}$$

Here the parameters K,M,R,E_c,f_w,P_r and α and are the material parameter, Hartmann number, radiation parameter, Eckert number, suction velocity, Prandtlnumber and slip parameter respectively. These quantities are formulated as follows:

$$K = \frac{\kappa}{\mu}, \quad M^2 = \frac{\sigma B_0^2}{\rho a}, \quad f_w = -(av)^{-\frac{1}{2}}v_w, \quad P_r = \frac{\mu C_p}{\kappa}, \quad \alpha = \alpha^* \mu\sqrt{\frac{a}{v}}, \quad E = \frac{E_0}{u_w B_0},$$
$$R = \frac{4\sigma^* T_\infty^3}{\kappa^* \kappa_1}, \quad Ec = \frac{u_w^2}{C_p(T_f - T_\infty)}$$

The dimensionless form of the mathematical model of the present problem is:

$$(1+K)f''' + ff'' + \left(M^2E - (f^{'})^2\right) + Kh^{'} - M^2f^{'} = 0 \tag{2.7}$$

$$(1+\frac{K}{2})h'' + fh' - f'h - K(2h + f'') = 0 \tag{2.8}$$

$$\left(1+\frac{4}{3}R\right)\theta'' + \Pr\left[f\theta' + (1+K)E_c(f'')^2 + EcM^2\left((f')^2 + E^2 - 2Ef^{'}\right)\right] = 0 \tag{2.9}$$

along with BCs:

$$\left.\begin{array}{l}\left.\begin{array}{l} f(\eta) = f_w, f'(\eta) = 1 + \alpha\left(1 + K(1-n)\right)f''(\eta), \\ h(\eta) = -nf''(0), \theta(\eta) = -\gamma(1-\theta(\eta)) \end{array}\right\} as\ \eta = 0 \\ \\ f'(\eta) = 0,\ h(\eta) = 0,\ \theta(\eta) = 0 \text{ as } \eta \to \infty \end{array}\right\} \tag{2.10}$$

The local skin friction and the Nuselt numbers are characterized as

$$C_{fx} = \frac{2\tau_w}{\rho(ax)^2}, Nux = \frac{xq_w}{k\left(T_f - T_\infty\right)} \tag{2.11}$$

where q_w, τ_w are given by

$$\tau_w = \left[(\mu + K)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0}, q_w = -\kappa_1\left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{2.12}$$

The dimensionless form of the above quantities becomes

$$\begin{aligned} &\frac{1}{2}C_f Re_x^{\frac{1}{2}} = [1 + (1-n)K]\, f''(0), \\ &Nu Re_x^{\frac{1}{2}} = -\frac{1}{3}\left[3 + 4R_d\left[(Q_w - 1)\,\theta(0) + 1\right]^3\right]\theta'(0). \end{aligned} \tag{2.13}$$

3 Solution methodology

The analytic solution of the boundary value problem (2.7)-(2.9) cannot be found because these equations are non-linear and coupled. So, we use a numerical technique, i.e.the shooting scheme with fourth order Adams-Moulton method. In order to solve the system of ODEs (2.7)-(2.9), with boundary conditions eqn (2.10),utilizing the shooting method, first of all we have to change these equations into a system of first order differential expressions. Let us use the following notations.

$$f = y_1, \quad f' = y_2, \quad f'' = y_3, \quad h = y_4, \quad h' = y_5, \quad \theta = y_6, \quad \theta' = y_7. \tag{3.1}$$

The system of equation (2.7)-(2.9) following the boundary limits are (2.10) changed into a system of seven first order differential expressions.

$$\left.\begin{aligned} &y_1' = y_2, && y_1(0) = f_w, \\ &y_2' = y_3, && y_2(0) = 1 + \alpha\,[1 + K(1-n)]\,s, \\ &y_3' = \frac{1}{(1+K)}\left[y_2^2 - y_1y_3 - Ky_5 + M^2y_2 - M^2E\right], && y_3(0) = s, \\ &y_4' = y_5, && y_4(0) = -ns, \\ &y_5' = \frac{2}{(2+K)}\left[-y_1y_5 + y_2y_4 + K(2y_4 + y_3)\right], && y_5(0) = t, \\ &y_6' = y_7, && y_6(0) = u, \\ &y_7' = -\left(\frac{3P_r}{3+4R}\right)\left[\begin{array}{c} y_1y_7 - (1+K)E_cy_3^2 \\ -M^2E_c(y_2^2 + E^2 - 2Ey_2) \end{array}\right], && y_7(0) = -\gamma(1-u) \end{aligned}\right\} \tag{3.2}$$

In the above system of equations (3.2), the missing initial conditions and are to be chosen such that

$$y_2(\eta_\infty, s, t, u) = 0, y_4(\eta_\infty, s, t, u) = 0, y_6(\eta_\infty, s, t, u) = 0 \tag{3.3}$$

With these new notation, the Newton's iterative scheme get the following form.

$$\begin{bmatrix} s^{(n+1)} \\ \\ t^{(n+1)} \\ \\ u^{(n+1)} \end{bmatrix} = \begin{bmatrix} s^{(n)} \\ \\ t^{(n)} \\ \\ u^{(n)} \end{bmatrix} - \begin{bmatrix} \frac{\partial y_2}{\partial s} & \frac{\partial y_2}{\partial t} & \frac{\partial y_2}{\partial u} \\ \\ \frac{\partial y_4}{\partial s} & \frac{\partial y_4}{\partial t} & \frac{\partial y_4}{\partial u} \\ \\ \frac{\partial y_6}{\partial s} & \frac{\partial y_6}{\partial t} & \frac{\partial y_6}{\partial u} \end{bmatrix}^{-1}_{\left(s^{(n)}, t^{(n)}, u^{(n)}, \eta_\infty\right)} \begin{bmatrix} y_2^{(n)} \\ \\ y_4^{(n)} \\ \\ y_6^{(n)} \end{bmatrix} \tag{3.4}$$

The convergence criteria are chosen to be successive value agree up to 6 significant digits. The choice of $\eta_{max} = 9$ was more than enough for end condition.

4 Results and discussion

The objective of this section is to analyze the numerical results displayed in the shape of graphs and tables. The computations are carried out for various values of the material parameter K , Hartmann number M , electric parameter E ,radiation parameter R , Eckert number E_c , suction velocity f_w , slip parameter α, and Prandtl number P_r and the impact of these parameters on the velocity,micro rotation and temperature profiles is also discussed in detail.

4.1 Impact of slip parameter and suction parameter on the unitless velocity profile

The effect of slip parameter α on the dimensionless velocity profile $f^{'}(0)$ is presented in Figure [2]. Increasing the values of the slip parameter reduces the velocity field and particular boundary thickness as depicted in Figure[2]. The impact of the section parameter f_w on the dimensionless velocity profile $f^{'}(0)$ is presented in Figure[3]. Velocity profile diminishes and accompanied with boundary layer width increases for gradually growing values of the suction parameter f_w.

4.2 Impact of material parameter on the unitless velocity profile and dimensionless microrotation profile

The impact of the material parameter K on the dimensionless velocity profile $f^{'}(0)$ is presented in Figure[4]. By increasing K, the velocity field reduces in the lower half of the surface whereas it enhances in the upper half. The velocity is going to reduce initially with the mounting values of the material parameter K . However,for $\eta > 1$ there is an elevation in the velocity profile. Figure[5] shows the impact of the material parameter K on the dimensionless micro rotation profile $h(\eta)$. By increasing the values of K the micro rotation decreases.

4.3 Impact of Hartmann number on the unitless velocity profile and dimensionless energy profile.

Figures[6] and [7] depict the impact of Hartman number M on the velocity profile $f^{'}(0)$ and the dimensionless energy profile $\theta(\eta)$. It is shows that the huge values of the magnetic parameter M case a fall down the velocity profile $f^{'}(0)$ and an increase in the dimensionless energy profile $\theta(\eta)$, since the magnetic field introduces a force i.e. the Lorentz force which opposes the stream and the velocity distribution.

4.4 Impact of Biot number and Eckert number on the dimensionless energy profile.

Figure[8] demonstrates the impact of the Biot number γ on the temperature $\theta(\eta)$ .We notice that the enhanced values of Biot number γ cause a higher energy and thicker the thermal boundary layer thickness. Figure[9] displays the influence of Eckert number E_c on the energy profile. Energy profile increases when the Eckert number is increased. Due to friction, the heat energy is kept in owing to accelerating values of Eckert number, which results in the enhancement of the temperature profile.

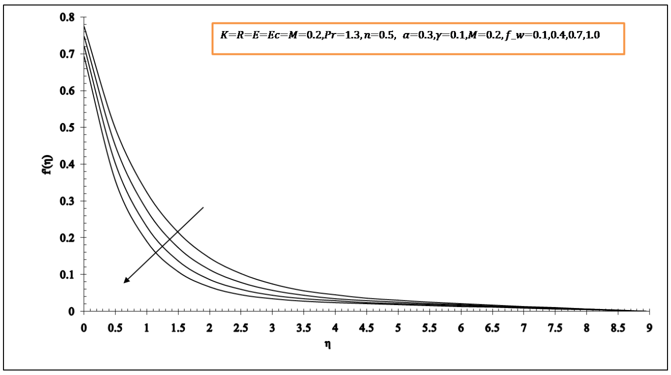

Figure 2: Impact of α on the unitless velocity $f^{'}(\eta)$

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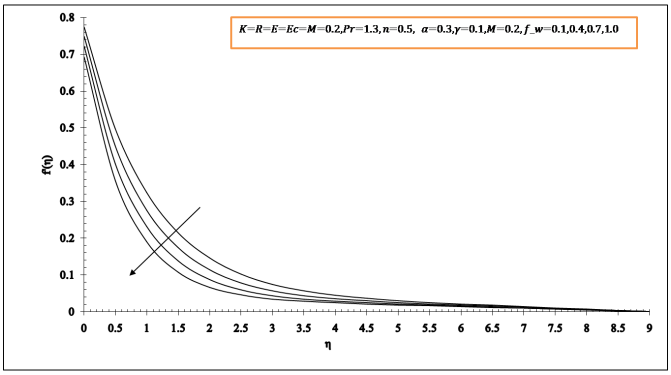

Figure 3: Impact of f_w on the unitless velocity $f^{'}(\eta)$

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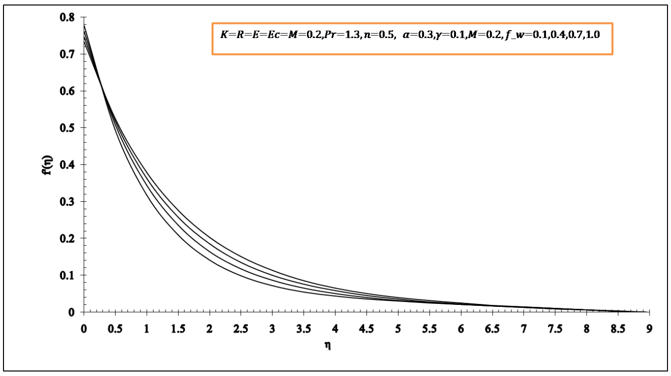

Figure 4: Impact of K on the unitless velocity $f^{'}(\eta)$

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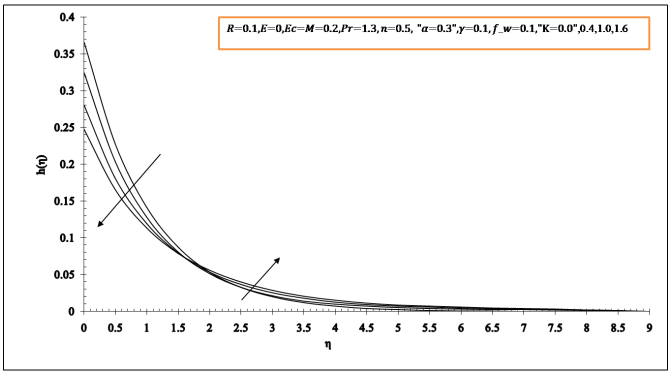

Figure 5: Impact of K on the unitless micro rotation $h(\eta)$

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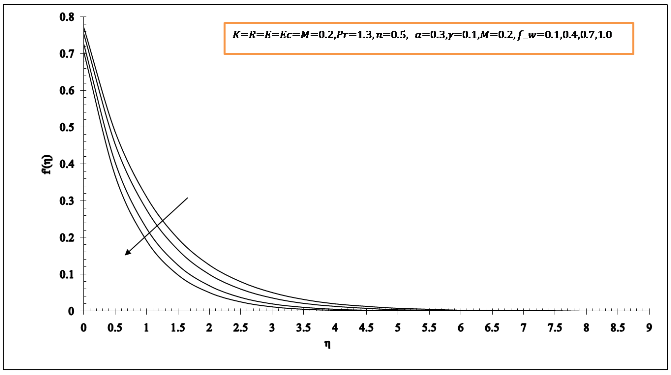

Figure 6: Impact of M on the unitless velocity $f^{'}(\eta)$

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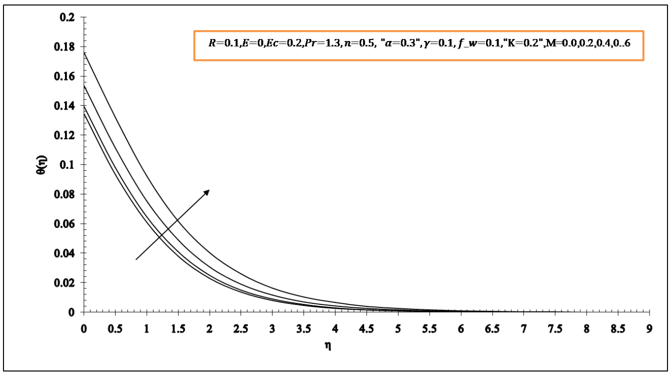

Figure 7: Impact of M on the unitless energy $\theta(\eta)$

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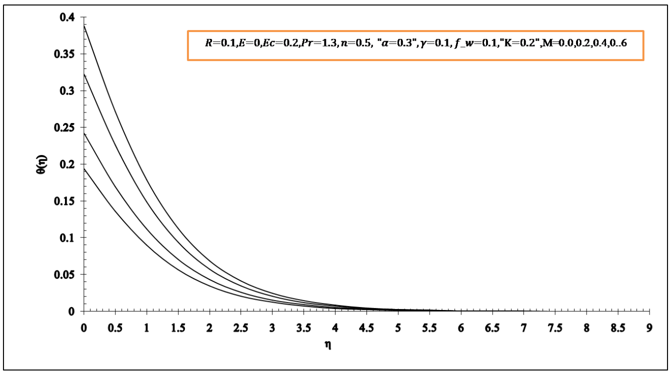

Figure 8: Impact of γ on the unitless energy $\theta(\eta)$

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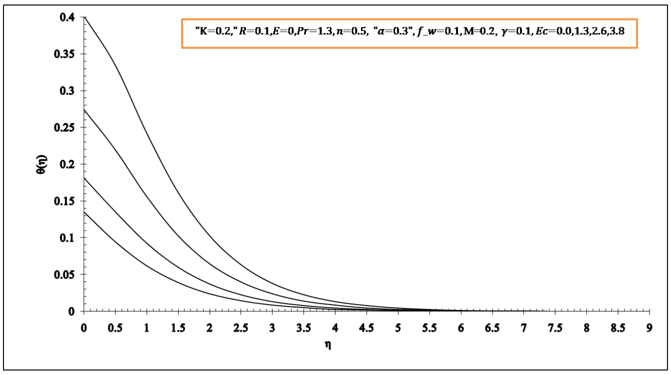

Figure 9: Impact of E_c on the unitless energy $\theta(\eta)$

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4.5 Skin-friction coefficient and Nusselt number.

The Nusselt number Nux is of great interest for engineers. In Table[1], the numerical analysis of Nux for different physical parameters under discussion, is displayed. Here, we use the shooting technique with the Adams – Moulton fourth order mechanism. We compare the results obtained [14] and found in an excellent agreement.It is found that the inflation in the radiation parameter R ,γ,electric parameter E,Eckert number E_c,and Prandtl number P_r results a rise in the Nusselt number.Moreover, the Nusselt number has inverse relation with Hartman number M and the slip parameter α.

Table 1: Numerical results of $NuxRe_x^{-\frac{1}{2}}$ for different values of P_r,K,M,R,E,α,γ and E_c .

Parameters							$\mathbf{NuxRe_x^{-\frac{1}{2}}}$	
M	**R**	**E**	α	γ	$\mathbf{E}_c$	$\mathbf{P}_r$	**Ramzan et al.[14]**	**Present**
0.2	0.1	0.2	0.1	0.1	0.2	1.0	0.08591	0.0852257300
0.3							0.08595	0.0843653500
0.6							0.08490	0.0794487200
0.9							0.08381	0.0716007100
0.2	0.3						0.08540	0.0838840700
	0.6						0.84400	0.0811583800
	0.9						0.08351	0.0786207100
	0.1	0.3					0.08580	0.0854807100
		0.6					0.08605	0.0832169200
		0.9					0.08599	0.0791851400
		0.9	0.3				0.08515	0.0851039500
			0.6				0.08440	0.0639808700
			0.9				0.08339	0.0592653600
			0.1	0.3			0.20212	0.2003953000
				0.6			0.34401	0.3023692000
				0.9			0.36662	0.3641334000
				0.1	0.3		0.08544	0.0848446600
					0.6		0.08455	0.0832361400
					0.9		0.08355	0.0813171300
					0.2	2.0	0.08951	0.0899033800
						3.0	0.09194	0.0914043600
						4.0	0.09340	0.0920416800

5 Conclusion

In this article, the numerical study of the flow of micropolar fluid past a porous stretching sheet in the presence of Joule heating, partial slip and magnetohydrodynamic (MHD), is presented. The properties of the fluid like viscosity and thermal conductivity are taken independent of temperature. The dimensionless velocity,dimensionless temperature and dimensionless micro-rotation are analyzed and presented in the form of graphs and tables. We present the Nusselt number in the tabular form for different values of the distinctive physical parameters. From the above study, we can make the following conclusions.

- The material parameter K diminishes the micro-rotation and distributions of velocity.
- The slip parameter diminishes the micro-rotation and the velocity distributions.
- The energy failed with its boundary layer thickness inflated against the mounting values of the electrical field parameter E.
- The mounting values of the Hartmann number M decrease the velocity distribution and increase the energy field.

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