MULTIPLICATION OPERATION ON INTUITIONISTIC FUZZY NUMBERS

Iuliana Carmen Bărbăcioru, Lecturer PhD, *"Constantin Brâncuși" University from Târgu Jiu, ROMANIA*

ABSTRACT: *In this article I will present how to demonstrate, other than using the extension principle, that the product of two trapezoidal intuitionistic fuzzy numbers (TrIFN) can be approximated by a TrIFS. Here only one type of intuitionistic fuzzy sets, namely trapezoidal intuitionistic fuzzy number is presented.*

KEY WORDS: *Intuitionistic fuzzy numbers, (α,β)-cut and core of a intuitionistic fuzzy sets, extension principle*

1. INTRODUCTION

A generalized fuzzy number [8] $a=\langle(a,b,c,d);w_a\rangle$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by:

$$\mu_a(x)=\begin{cases}0,\ x<a\\ (x-a)w_a/(b-d),\ a\le x<b\\ w_a,\quad b\le x<c\\ (d-x)w_a/(d-c),\ c<x\le d\\ 0,\ d\le x\end{cases}$$

Where w_a represent the maximum membership degree of a such that they satisfy the conditions: $0\le w_a\le 1$.

Atanassov [1] introduces for the first time the concept of intuitionistic fuzzy sets (IFS), an IFS *A* in *X* being

$$A=\{(x,\mu_A(x),\nu_A(x))|x\in X\}$$

Where $\mu_A : X\to[0,1]$ is degree of membership and $\nu_A : X\to[0,1]$ is degree of non-membership, with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$, $\forall x\in X$.

The concept of the trapezoidal intuitionistic fuzzy number

$$a=\langle(\underline{a},a_1,a_2,\overline{a});w_a,u_a\rangle \quad (1)$$

(TrIFS) is a generalization of the trapezoidal fuzzy number. A TrIFS is a special intuitionistic fuzzy set on the real number set **R**, whose membership and non-membership functions are defined as follows:

$$\mu_a(x)=\begin{cases}0,\quad x<\underline{a}\\ (x-\underline{a})w_a/(a_1-\underline{a}),\ \underline{a}\le x<a_1\\ w_a,\quad a_1\le x<a_2\\ (\overline{a}-x)w_a/(\overline{a}-a_2),\ a_2<x\le\overline{a}\\ 0,\quad x>\overline{a}\end{cases} \quad (2)$$

$$\nu_a(x)=\begin{cases}1,\quad x<\underline{a}\\ (a_1-x+u_a(x-\underline{a}))/(a_1-\underline{a}),\quad \underline{a}\le x<a_1\\ u_a,\quad a_1\le x\le a_2\\ (x-a_2+u_a(\overline{a}-x))/(\overline{a}-a_2),\ a_2\le x\le\overline{a}\\ 1,\quad x>\overline{a}\end{cases}$$

(3)

Where w_a and u_a represent the maximum membership degree and the minimum non-membership degree of a such that they satisfy the conditions: $0 \le w_a \le 1$, $0 \le u_a \le 1$, and $0 \le w_a + u_a \le 1$. $[a_1; a_2]$, $\underline{a}$ and $\bar{a}$ are called the mean interval and the lower and upper limits of the trapezoidal intuitionistic fuzzy number a , respectively.

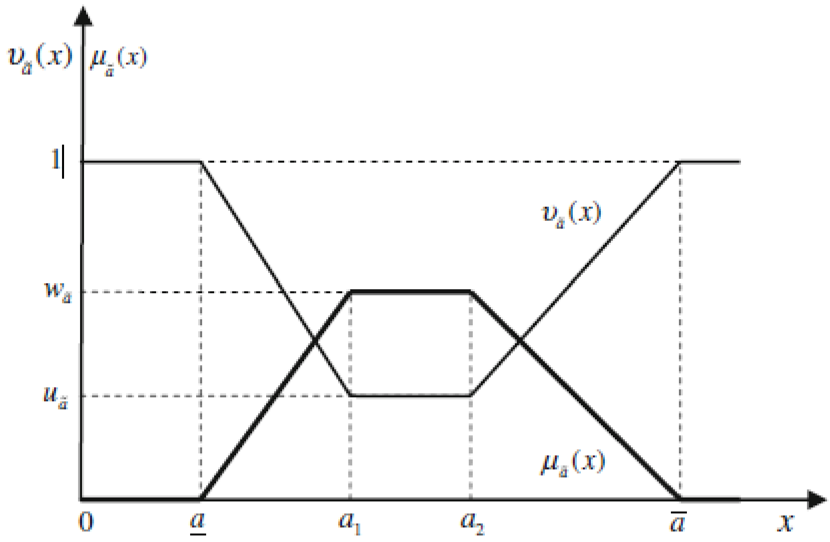

Figure 1. [5] A trapezoidal intuitionistic fuzzy number $a = \langle (\underline{a}, a_1, a_2, \bar{a}); w_a, u_a \rangle$

In [3] We introduced the notion of (α,β) -cut as the IFS A, α,β∈ [0.1], as

$$[A]^{(\alpha,\beta)} = \begin{cases} \left\{ x \in X \middle| \begin{array}{l} \mu_A(x) \ge \alpha, \nu_A(x) \le \beta, \\ 0 \le \mu_A(x) + \nu_A(x) \le 1 \end{array} \right\} \\ \text{if} \quad 0 < \alpha, \beta \le 1 \\ \hline \left\{ x \in X \middle| \begin{array}{l} \mu_A(x) > 0, \nu_A(x) < 0, \\ 0 \le \mu_A(x) + \nu_A(x) \le 1 \end{array} \right\} \\ \text{if} \quad \alpha = 0, \beta = 0 \end{cases}$$

(4)

1.1. Cartesian Product of Intuitionistic Fuzzy Sets [5]

To consider $\tilde{A}_1, \ldots, \tilde{A}_n$ be IFSs in $X_1,...,X_n$ with the corresponding membership functions $\mu_{A_1}(x), \ldots, \mu_{A_n}(x)$ and non-membership function $\nu_{A_1}(x), \ldots, \nu_{A_n}(x)$ respectively.

The Cartesian product of the IFSs $\tilde{A}_1, \ldots, \tilde{A}_n$ is defined as IFS in $X_1,...,X_n$ whose membership functions and non-membership functions are given by:

$$\mu_{A_1 \times \ldots \times A_n}(x_1, \ldots, x_n) = \min\left\{ \mu_{A_1}(x_1), \ldots, \mu_{A_n}(x_n) \right\}$$

and

$$\nu_{A_1 \times \ldots \times A_n}(x_1, \ldots, x_n) = \max\left\{ \nu_{A_1}(x_1), \ldots, \nu_{A_n}(x_n) \right\}.$$

1.2. Extension Principle in Cartesian Space [5]

We consider the function $f : X_1 \times \ldots \times X_n \to Y$ $f(x_1, \ldots, x_n) = y$. The extension principle allows us to define the IFS $\tilde{B}$ in Y induced by the IFS $\tilde{A}_1 \times \ldots \times \tilde{A}_n$ in $X_1 \times \ldots \times X_n$ through f as follows: $B = \left\{ \left(y, \mu_B(y), \nu_B(y) \right) \middle| y = f(x) \right\}$ whose membership and non-membership functions are defined as follows:

$$\mu_B(y) = \begin{cases} \sup_{y=f(x)} \mu_B(x) & , f^{-1}(y) \ne \emptyset \\ 0 & , f^{-1}(y) = \emptyset \end{cases}$$

(5)

$$\nu_B(y) = \begin{cases} \inf_{y=f(x)} \nu_B(x) & , f^{-1}(y) \ne \emptyset \\ 0 & , f^{-1}(y) = \emptyset \end{cases}$$

(6)

where $f^{-1}(y)$ is the inverse image of y .

2. Multiplication of Two TrIFN by (α, β)-cut Method

2.1. Arithmetic Operations of Intuitionistic Fuzzy Numbers based on Extension Principle [5]

[5] It looked as if $\tilde{A}_1$and $\tilde{A}_2$ are IFN then arithmetic operations (*) of IFN are determined by the formula:

$$(\tilde{A}_1 * \tilde{A}_2)(y) = \left\{\left(y, \sup_{y=x_1*x_2}\left[\min\left(\mu_{\tilde{A}_1}(x_1),\mu_{\tilde{A}_2}(x_2)\right)\right], \inf_{y=x_1*x_2}\left[\max\left(\nu_{\tilde{A}_1}(x_1),\nu_{\tilde{A}_2}(x_2)\right)\right]\right) \Big| x_1,x_2,y\in \mathbb{R}\right\}$$

The membership function and non-membership function as follows:

$$\mu_{A_1*A_n}(y) = \sup_{y=x_1*x_2}\left[\min\left(\mu_{\tilde{A}_1}(x_1),\mu_{\tilde{A}_2}(x_2)\right)\right]$$

$$\nu_{A_1*A_n}(y) = \inf_{y=x_1*x_2}\left[\max\left(\nu_{\tilde{A}_1}(x_1),\nu_{\tilde{A}_2}(x_2)\right)\right]$$

2.2. Arithmetic Operations of Intuitionistic Fuzzy Numbers based on (α, β)-cut Method [5]

As we have seen in the relationship (4), if $A=\left\langle\left(\underline{a},a_1,a_2,\overline{a}\right);w_A,u_A\right\rangle$ is a TrIFN then (α,β) -cut is given for instance by

$[A]^{(\alpha,\beta)} =$

$\left\{\left[\underline{a}+\alpha w_A,\overline{a}-\alpha w_A\right];\left[a_1-\beta u_A,a_2+\beta u_A\right]\right\}$,

$0\le\alpha+\beta\le1, \alpha,\beta\in[0,1]$.

Property 2.1 If $A=\left\langle\left(\underline{a},a_1,a_2,\overline{a}\right);w_A,u_A\right\rangle$ and $B=\left\langle\left(\underline{b},b_1,b_2,\overline{b}\right);w_B,u_B\right\rangle$ are two TrIFN's, then $A\times B=\left\langle\left(\underline{ab},a_1b_1,a_2b_2,\overline{ab}\right);w_Aw_B,u_Au_B\right\rangle$ is approximated TrIFN.

Proof: We are transforming $z=x\times y$.

[5] proposes to demonstrate this property using (α,β) -cut method.

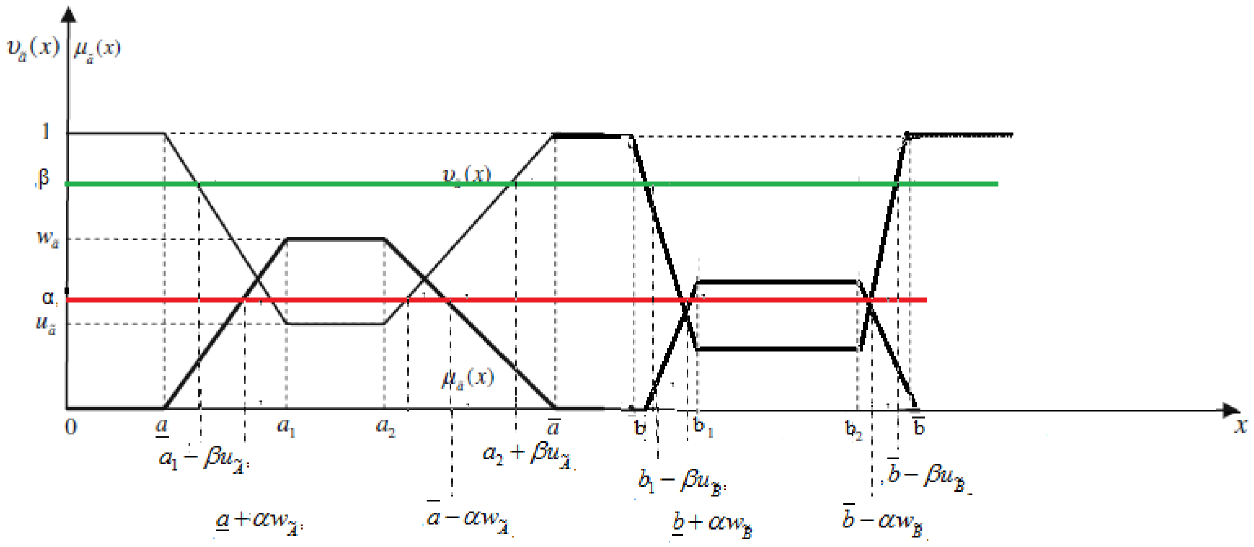

Figure 2. [5] TrIFN's $A=\left\langle\left(\underline{a},a_1,a_2,\overline{a}\right);w_A,u_A\right\rangle$ and $B=\left\langle\left(\underline{b},b_1,b_2,\overline{b}\right);w_B,u_B\right\rangle$

First determine the membership function of acceptance (membership) IFS $A\times B$ by α-cut method.

α-cut for membership function of A:

$$\mu_A(x)\geq \alpha \Rightarrow \left[\underline{a}+\alpha w_A,\bar{a}-\alpha w_A\right]\forall\alpha\in[0,1]$$
$$\Rightarrow x\in\left[\underline{a}+\alpha w_A,\bar{a}-\alpha w_A\right].$$

α-cut for membership function of B:

$$\mu_B(x)\geq \alpha \Rightarrow \left[\underline{b}+\alpha w_B,\bar{b}-\alpha w_B\right]\forall\alpha\in[0,1]$$
$$\Rightarrow y\in\left[\underline{b}+\alpha w_B,\bar{b}-\alpha w_B\right].$$

So we will have:

$z=x\times y\in$
$$\left[\underline{(a+\alpha w_A)}\left(\underline{b}+\alpha w_B\right),\left(\bar{a}-\alpha w_A\right)\left(\bar{b}-\alpha w_B\right)\right].$$

Therefore:

$$\mu_{A\times\tilde{B}}(z)=\begin{cases}0 & ,z<\underline{ab}\\ \dfrac{-B_1+\sqrt{(B_1)^2-4A_1\left(\underline{ab}-z\right)}}{2A_1}, & \underline{ab}\leq z\leq a_1b_1\\ w_Aw_B, & a_1b_1\leq z\leq a_2b_2\\ \dfrac{B_2-\sqrt{(B_2)^2-4A_2\left(\bar{a}\bar{b}-z\right)}}{2A_2}, & a_2b_2\leq z\leq \bar{a}\bar{b}\\ 0 & ,z>\bar{a}\bar{b}\end{cases}$$

Where
$$\begin{aligned}A_1&=w_Aw_B\\ A_2&=\left(\bar{a}-a_2\right)\left(\bar{b}-b_2\right)\\ B_1&=\underline{a}w_B+\underline{b}w_A\\ B_2&=-\left(\bar{b}\left(\bar{a}-a_2\right)+\bar{a}\left(\bar{b}-b_2\right)\right)\end{aligned}.$$

Next we will determine the non-membership function of acceptance (membership) IFS $A\times B$ by β-cut method.

β-cut for membership function of A:

$$\mu_A(x)\leq\beta \Rightarrow \left[a_1-\beta u_A,\bar{a}-\beta u_A\right]\forall\beta\in[0,1]$$
$$\Rightarrow x\in\left[a_1-\beta u_A,\bar{a}-\beta u_A\right].$$

β-cut for membership function of B:

$$\mu_B(x)\leq\beta \Rightarrow \left[b_1-\beta u_B,\bar{b}-\beta u_B\right]\forall\beta\in[0,1]$$
$$\Rightarrow y\in\left[b_1-\beta u_B,\bar{b}-\beta u_B\right].$$

So we will have:

$z=x\times y\in$
$$\left[\underline{(a_1-\beta u_A)}\left(b_1-\beta u_B\right),\left(\bar{a}-\beta u_A\right)\left(\bar{b}-\beta u_B\right)\right].$$

Therefore:

$$\nu_{A\times\tilde{B}}(z)=\begin{cases}1 & ,z<\underline{ab}\\ 1-\dfrac{-B_3+\sqrt{(B_3)^2-4A_3\left(\bar{a}\bar{b}-z\right)}}{2A_3}, & \underline{ab}\leq z\leq a_1b_1\\ u_Au_B & ,\ a_1b_1\leq z\leq a_2b_2\\ 1-\dfrac{-B_4-\sqrt{(B_4)^2-4A_4\left(\bar{a}\bar{b}-z\right)}}{2A_4}, & a_2b_2\leq z\leq \bar{a}\bar{b}\\ 1 & ,z>\bar{a}\bar{b}\end{cases}$$

Where
$$\begin{aligned}A_3&=u_Au_B\\ A_4&=\left(\bar{a}-a_2\right)\left(\bar{b}-b_2\right)\\ B_3&=\underline{a}u_A+\underline{b}u_B\\ B_4&=-\left(\bar{b}\left(\bar{a}-a_2\right)+\bar{a}\left(\bar{b}-b_2\right)\right)\end{aligned}.$$

Consequently $A\times B$ has a trapezoidal shape of IFN which can be approximated by TrIFS.

3. CONCLUSION

Starting from [7] and [5] we showed that membership functions and non-membership function when the TrIFN's have no particular form of [5]. Of course,
$A\times B=\left\langle\left(\underline{ab},a_1b_1,a_2b_2,\bar{a}\bar{b}\right);w_Aw_B,u_Au_B\right\rangle$
is very much influenced by the positioning of numbers $A=\left\langle\left(\underline{a},a_1,a_2,\bar{a}\right);w_A,u_A\right\rangle$ and $B=\left\langle\left(\underline{b},b_1,b_2,\bar{b}\right);w_B,u_B\right\rangle$ the system axis.

REFERENCE

[1] Atanasov, K.T., *Intuitionistic fuzzy sets*, Fuzzy Sets Syst., vol. 20, pp. 87–96, August 1986.

[2] Atanassov, K.T., *My personal view on intuitionistic fuzzy sets theory*. Stud. Fuzziness Soft Comput. 220, 23–43 (2008)

[3] Bărbăcioru, I. C., *A Note on (α,β)-cut in Intuitionistic Fuzzy Sets Theory* , Fiability & Durability 1/2014, Editura " Academica Brâncuşi", Târgu Jiu, ISSN 1844-640X, 200-206.

[4] D.-F. Li, *Decision and Game Theory in Management with Intuitionistic Fuzzy Sets*, Studies in Fuzziness and Soft Computing 308, DOI: 10.1007/978-3-642-40712-3_1, Springer-Verlag Berlin Heidelberg 2014

[5] Mahapatra1,G.S., Roy, T.K., *Intuitionistic Fuzzy Number and Its Arithmetic Operation with Application on System Failure*, Journal of Uncertain Systems, Vol.7, No.2, pp.92-107, 2013.

[6] Rezvani, S., *Multiplication Operation on Trapezoidal Fuzzy Numbers,* Journal of Physical Sciences, Vol. 15, 17-26, 2011.

[7] Taleshian, A., Rezvani, S., *Multiplication Operation on Trapezoidal Fuzzy Numbers*, Journal of Physical Sciences, Vol. 15, 2011, 17-26,ISSN: 0972-8791.

[8] Zadeh, L.A., *Fuzzy sets*, Inf. Control 8(3), 338–353 (1965).