THE COMPLEX PLANETARYMECHANISMS USED AS THE AUTOMATION TRANSMISSIONS

Ovidiu ANTONESCU [1], Margareta STRIMBEANU [2], Paun ANTONESCU [1]

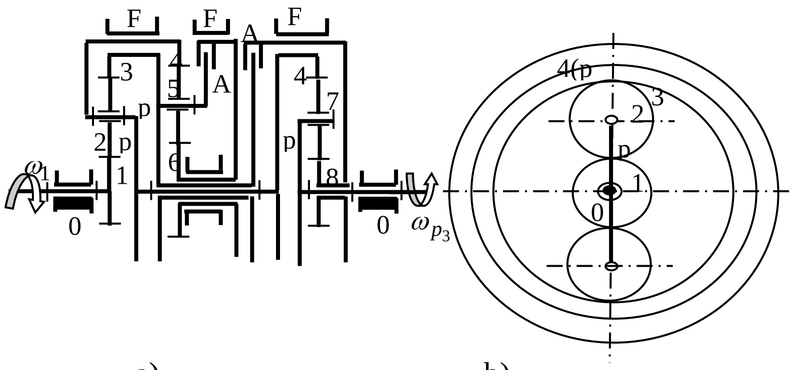
[1] Politehnica University of Bucharest, Romania
[2] High School Iuliu Maniu of Bucharest, Romania

ABSTRACT: It is considered the kinematic schema of a complex planetary mechanism, which is achieved through aggregating of three planetary units with cylindrical gearings, from the make-up of automatic transmissions used to the automotives. Known the kinematic conditions, specify those five speed stages, in paper is presented a method of synthesis of type structural – topological and kinematical following the calculation analytical expressions of the transmission functions input –output in each gear.

KEYWORDS: planetary mechanism, kinematic schema, topologic structure, transmission ratio.

1. KINEMATIC SCHEMA OF COMPLEX PLANETARY MECHANISM

Consider the kinematic schema of the planetary mechanism with cylindrical gearings [6, 7], used as the automatic transmission with 4+1 gears, in the axial projection (fig. 1a) and in the transversal projection (fig. 1b). In the transversal projection (fig. 1b) are drawn the pitch circles of the central gear 1(with external teeth), of the satellite gear 2, of the central gear 3 (with internal teeth) and the port-satellite arm p_1, which is solidarity with the central gear 4 with the internal teeth (fig. 1a).



a) b)

Fig. 1

Those 4+1 gears obtained with the help of clutches A1 and A2 and of the locking devices F1, F2 and F3, which are acted conform of some specify kinematical conditions (tab. 1).

Table 1

Gear	Clutch A1	Clutch A2	Clutch F1	Clutch F2	Clutch F3	Kinematical Conditions	Transmission Function
I		*			*	$\omega_3 = \omega_8 = 0$	$i_I = i_{1p_3} = f_I(z)$
II				*	*	$\omega_6 = 0; \omega_8 = 0$	$i_{II} = i_{1p_3} = f_{II}(z)$
III	*				*	$\omega_1 = \omega_6; \omega_8 = 0$	$i_{III} = i_{1p_3} = f_{III}(z)$
IV	*	*				$\omega_3 = \omega_6 = \omega_8$	$i_{IV} = i_{1p_3} = f_{IV}(z)$
V(MI)		*	*			$\omega_4 = 0; \omega_6 = \omega_8$	$i_V = i_{1p_3} = f_V(z)$

It is followed the calculation of geometrical mobility, the drawing of structural-topological schema [5, 6] of the complex planetary mechanism in neutral position; the kinematic analysis of the planetary mechanism through the determinate of the linear velocities distribution and the transmission ratio in each gear; the kinematic synthesis of complex mechanism, concerning the calculation of the specify transmission ratios of those three component planetary units. The study can be continued with the calculation of the cylindrical engagement, through the determination of the engagement factor through the graphical and analytical methods; the dynamic analysis of complex planetary mechanism, through the calculation of the mechanical efficiency of the complex planetary mechanism for each the speed stage and the determination of power to the driven shaft.

2. THE MOBILITY AND STRUCTURAL-TOPOLOGICAL SCHEMA OF MECHANISM

The complex planetary mechanism (fig. 1) with the cylindrical gearings, which is moved in parallel planes, is formed from three modules „planetary units" of cylindrical planetary mechanisms.

A cylindrical planetary unit (CPU) is formed from two central gears (with the fixed rotation axes), a planetary gear (satellite) with the mobile rotation axis and a crank – arm which is named the port-satellite arm (with the rotation fixed axis which is coaxial with the rotation axis of the central (sun) gears.

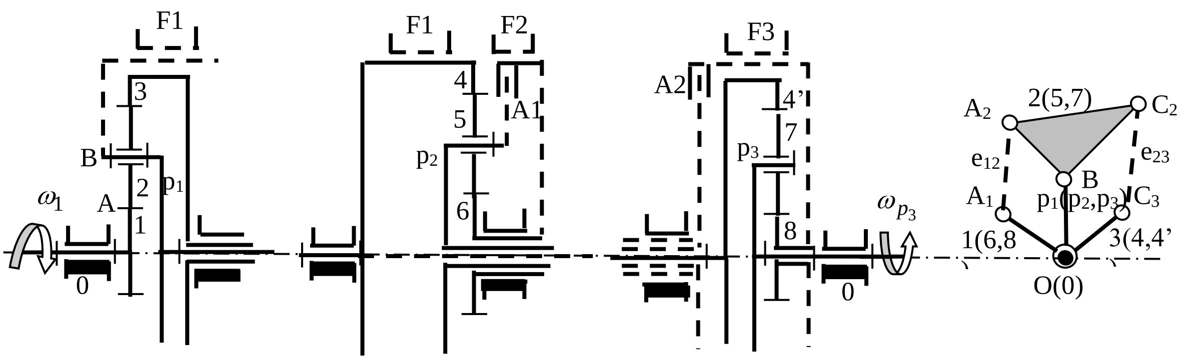
First cylindrical planetary unit (CPU1) is formed from the central cylindrical gears 1 (with the exterior teeth) and 3 (with the interior teeth), from the satellite cylindrical gear 2 (with the exterior teeth) and from the port-satellite arm p_1. In the figure 2a is shown kinematic schema of CPU1.

Second cylindrical planetary unit (CPU2) has in the make-up the central cylindrical gears 4 (p_1) with the interior teeth and 6 (with the exterior teeth), from the satellite cylindrical gear 5 (with the exterior teeth) and from the port-satellite arm p_2(3). In the figure 2b can be follow the kinematic schema of CPU2.

Third cylindrical planetary unit (CPU3) is formed from the central cylindrical gears 4' (p_1) with the interior teeth and 8 (with the exterior teeth), from the satellite gear 7 (with the exterior teeth) and from the port-satellite arm p_3, which is solid with the driven shaft.

In the figure 2c can be followed the kinematic schema of CPU3.

In each from three kinematic schemas (fig. 2a, b, c) is drawn with the dash line the kinematic elements with which are aggregated those three CPU.



a) b) c) d)

Fig. 2

The mobility of the cylindrical planetary mechanism (fig. 1), in the neutral position, calculated with formula [7, 8]:

$$M_3 = 3.n - 2C_5 - C_4 \qquad (2.1)$$

In the formula (2.1) is introduced the numerical values for the number n of the kinematic elements, the number C_5 of the rotation joints and the number C_4 of the rotation-translation joints type engagement: $n = 9; C_5 = 9; C_4 = 6.$ With these data results the numerical value of the geometrical mobility from formula (2.1):

$$M_3 = 3 \times 9 - 2 \times 9 - 6 = 3 \qquad (2.2)$$

First from these three potential mobility of the mechanism (fig. 1) is the revolute motion of the driving shaft (input shaft) on which is fixed the central gear 1. Those two potential mobility can be the rotation motions of others two the kinematic elements, as the gears $3(p_2)$, $4(p_1)$, 6, 8.

It is underlined that all fixed axes dovetails, the fixed element being represented of this fixed axis, which is common of all the central gears and of the port-satellite arms.

Because is a single driving element 1, for to transmission the motion univocal determined from the driving shaft (1) to the driven shaft (p_3), others two mobility must canceled.

The cancel those two mobility, which not correspond with the driving element, is achieved with help some couplings type clutches (A1 and A2) respectively disc brakes (F1, F2 and F3).

It is notated that a clutch solider two the mobile elements, which will turn as au single body, while a disc brake solider a mobile element with the fixed element, what involves the blocking it and the cancel of the respectively motion. The structural-topological schema is achieved in the transversal plane through the equivalence joints of rotation - translation (engagements) through all a element type bar and two the revolute joint (articulation).

We begin from the fixed element which is reduced to a fixed point in which meet more mobile elements articulated at the base, as the central gears and the port-satellite arms (fig. 2).

It is demonstrated that all those three cylindrical planetary units (fig. 2a, b, c) have same the structural-topological schema of type simple (fig. 2d) with two mobility.

$$M_3 = 3.n - 2C_5 - C_4 = 3 \times 6 - 2 \times 8 = 2 \qquad (2.3)$$

In reality the port-satellite arm p_1 is stiffened of the gears 4 and $4^{/}$, that is $p_1 \equiv 4 \equiv 4'$, while the gear 3 is stiffened with the port-satellite arm p_2, that is $3 \equiv p_2$.

Thus are introduced three structural conditions. The mobility of the complex mechanism can be calculated and with the formula of connection of mechanisms [4]:

$$M = M_{(1)} + M_{(2)} + M_{(3)} - M_l = 2 + 2 + 2 - 3 = 3 \qquad (2.4)$$

where $M_{(1)} = M_{(2)} = M_{(3)} = 2; \quad M_l = 3$.

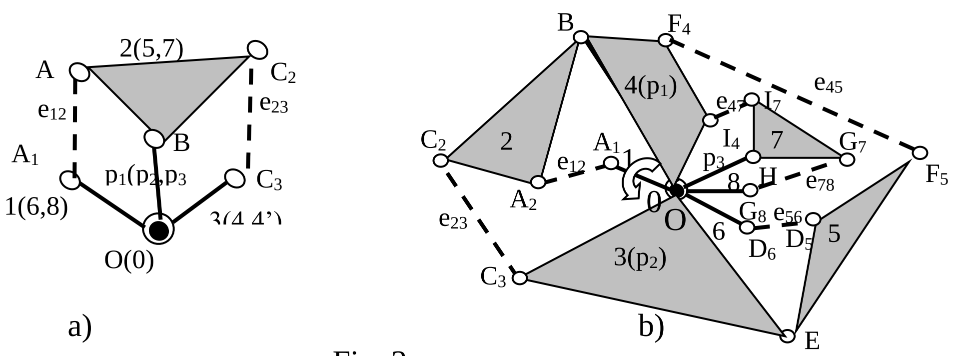


Fig. 3

Whether are introduced those three conditions of stiffening of some kinematic elements, through the superposition those three structural schemas (fig. 2d) corresponding those three planetary units, CPU1 (fig.2a), CPU2 (fig.2b) and CPU (fig.2c), can be obtained the complex structural-topological schema (fig. 3b) of whole complex planetary mechanism (fig. 1) in the neutral position, in which identifies those three identical structural schemas (fig. 2d) which was reconsidered bottom (fig. 3a).

3. KINEMATICS OF COMPLEX PLANETARY MECHANISM

3.1. *Distribution of linear velocities in gear I.*

In the gear I, conform of the table 1, are activated the clutch A2 and disc locking devise F3 (fig. 1a), what introduces the kinematic conditions which are mentioned in table: $\omega_3 = \omega_8 = 0$, that is elements 3 and 8 are fixed through stiffening with A2 and through blocking with F3.

Through introducing those two kinematic conditions, the kinematic elements 3(p_1) and 8 are stiffened and blocked to base 0. From the structural – topological schema in neutral position of mechanism (fig. 1) obtained the structural schema in gear I (fig. 4a) with 4 fixed articulations (O, C_3, E, H).

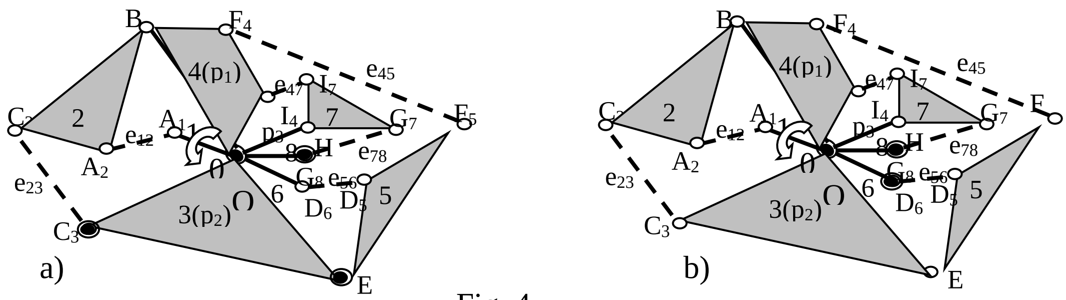


Fig. 4

The structural-topological formula of the mono-mobile mechanism, at which element 1 is driving (fig. 4a) written thus:

$$MM_I = FM(0+1) + TKC_1(e_{12} + 2 + e_{23} + p_1) + TKC_2(e_{4'7} + 7 + e_{78} + p_3) + DKC_1(e_{45} + 5) + DLc_2(e_{56} + 6) \quad (3.1)$$

By the **F**undamental **M**echanism $FM(0+1)$ is identified two **T**riadic **K**inematic **C**hain (TKC) respectively chains $TKC_1(e_{12} + 2 + e_{23} + p_1)$, $TKC_2(e_{4'7} + 7 + e_{78} + p_3)$ and others **D**yadic **K**inematic **C**hain (DKC) as $DKC_1(e_{45} + 5)$ and $DKC_2(e_{56} + 6)$. Is observed that those two dyadic chains are not on power flow from the driving element (1) until the driven element (p_3).

That is who drawn the velocities distribution only for the principal flow in which entered UPC1 through the characteristic points A, B, C and CPU3 through the characteristic points G, H, I (fig. 6a).

Because the gears 3 and 8 are blocked, the points C and G are fixed, while the velocity of each is zero, as instantaneous rotation center of the satellite gears 3 respectively 7.

For gears 1, 2 and 3 (CPU1) and gears 4', 7 and 8 (CPU3) calculated the radii of pitch circles with:

$$r_1 = r_8 = \tfrac{1}{2}\, m \cdot z_{1(8)}; \quad r_2 = r_7 = \tfrac{1}{2}\, m \cdot z_{2(7)}; \quad r_3 = r_{4'} = \tfrac{1}{2}\, m \cdot z_{3(4')}. \quad (3.2)$$

For numerical values:
$z_{1(8)} = 18; z_{3(4')} = 50; z_{2(7)} = 16; m = 2mm$,
results $r_{1(8)} = 18mm; r_{2(7)} = 16mm; r_{3(4')} = 50mm$.

On the reference line Δ_I (fig. 5b) is measured at scale following segments:
$OA = OG = 18mm; AB = BC = GH = HI = 16mm$.

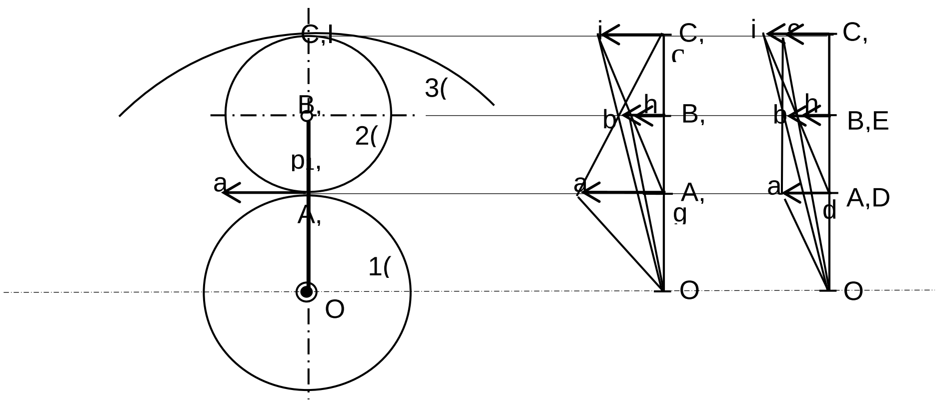
3.2. Distribution of linear velocity in gear II.

In gear II the disc locking devises F2 and F3 blocked gears 6 and 8, thus points D_6 and

G_8 are fixed, while the structural-topological schema (fig. 4b) permitted the writing equations:

$$MM_{II} = FM(0+1) + HKC(e_{12} + 2 + e_{23} + 3 + 5 + e_{56} + e_{54} + 4) + TKC(e_{4'7} + 7 + e_{78} + p_3) \quad (3.3)$$

Now all mobile kinematic elements are in the power flow on way from the driving shaft (central gear 1) to the driven shaft (port-satellite arm p_3).



a) b) c)

Fig. 5

In structural-topological schema identified conform with equation (7) a first **H**exagonal **K**inematic **C**hain (HKC) with 8 elements $HKC(e_{12} + 2 + e_{23} + 3 + 5 + e_{56} + e_{54} + 4)$ and a second **T**etradic **K**inematic **C**hain (TtKC) with 4 elements $TtKC(e_{4'7} + 7 + e_{78} + p_3)$, which contained the output element p_3.

In this case, the velocity distribution can be obtained through the method of changing of driving element, thus so that the structural schema to contain at most TKC. Thus, whether started from element p_3 as being driving (fig. 5b), formula of structural-topological composing becomes:

$$MM_{II}^{*} = FM(0+p_3) + DKC_1(7+e_{78}) + DKC_2(e_{47}+4) + TKC(e_{45}+5+e_{56}+3) + DKC_3(e_{23}+3) + DKC_4(e_{12}+1) \quad (3.4)$$

The velocities graphical distribution shown in fig. 5c, where it is started from point H, in which was chooses vector $\overrightarrow{Hh}$ orientated to left. In the final of graphical drawing is resulted vector $\overrightarrow{Aa}$ orientated to left, what verified that the transmission ratio in gear II is positive.

4. SPECIFIC TRANSMISSION FUNCTIONS FOR GEARS

On the base the imposed kinematic conditions from each gear (table 1) we follow the power flow way (fig. 1) through each from CPU.

It is written the equation of transmission ratio between the central gears, in the hypothesis of immobilizing of the port-satellite arm (Willis's method):

$$\text{CPU1: } i_{13}^{p_1} = i_{01} = \frac{\omega_1 - \omega_{p_1}}{\omega_3 - \omega_{p_1}};$$

$$\text{CPU2: } i_{46}^{p_2} = \frac{1}{i_{02}} = \frac{\omega_4 - \omega_{p_2}}{\omega_6 - \omega_{p_2}};$$

$$\text{CPU3: } i_{4'8}^{p_3} = \frac{1}{i_{03}} = \frac{\omega_{4'} - \omega_{p_3}}{\omega_8 - \omega_{p_3}}. \quad (4.1)$$

in which the transmission ratios in hypothesis of fixed axes are expressed in function of teeth numbers of the corresponding central gears:

$$i_{01} = i_{13}^{p_1} = i_{12}^{p_1} \cdot i_{23}^{p_1} = \left(-\frac{z_2}{z_1}\right) \cdot \left(\frac{z_3}{z_2}\right) = -\frac{z_3}{z_1};$$

$$i_{02} = -\frac{z_4}{z_6};\ i_{03} = -\frac{z_{4'}}{z_8}. \quad (4.2)$$

The transmission ratio i_{1p_3} in each gear expressed the ratio between the angular velocities of the driving shaft (*1*) and the driven shaft (*p₃=9*) as function of teeth numbers *z*:

$$i_{1p_3(9)} = \frac{\omega_1}{\omega_{p_3}} = \frac{\omega_1}{\omega_9} = f(z) \quad (4.3)$$

In gear *I* ($\omega_3 = \omega_8 = 0$) formulas (4.1) written in this case:

$$i_{13}^{p_1} = i_{01} = \frac{\omega_1 - \omega_{p_1}}{-\omega_{p_1}} = 1 - \frac{\omega_1}{\omega_4};$$

$$i_{46}^{p_2} = \frac{1}{i_{02}} = \frac{\omega_4}{\omega_6}; \qquad i_{4'8}^{p_3} = \frac{1}{i_{03}} = \frac{\omega_{4'} - \omega_{p_3}}{-\omega_{p_3}} = 1 - \frac{\omega_{4'}}{\omega_9}.$$

(4.4, 4.5, 4.6)

In formula (4.4) is made in evident the transmission ratio input – output (4.3):

$$i_{13}^{p_1} = i_{01} = 1 - \frac{\omega_1}{\omega_4} = 1 - \frac{\frac{\omega_1}{\omega_9}}{\frac{\omega_4}{\omega_9}} = 1 - \frac{i_{19}}{i_{49}}; \quad (4.7)$$

from which deduced, taking account of formula (4.6):

$$i_{19} = (1 - i_{01}) \cdot i_{49} = (1 - i_{01}) \cdot (1 - \frac{1}{i_{03}});$$

$$i_I = \left(1 + \frac{z_3}{z_1}\right) \cdot \left(1 + \frac{z_8}{z_{4'}}\right) \quad (4.8, 4.9)$$

In gear *II* ($\omega_6 = 0; \omega_8 = 0$) formulas (9) written thus:

$$i_{01} = \frac{\omega_1 - \omega_4}{\omega_3 - \omega_4};\ \frac{1}{i_{02}} = \frac{\omega_4 - \omega_3}{-\omega_3} = 1 - \frac{\omega_4}{\omega_3};$$

$$\frac{1}{i_{03}} = \frac{\omega_{4'} - \omega_9}{-\omega_9} = 1 - \frac{\omega_{4'}}{\omega_9}. \quad (4.10)$$

From formulas (4.10) expressed the ratio input – output (4.11)

$$i_{01} = \frac{\omega_1 - \omega_4}{\omega_3 - \omega_4} = \frac{\frac{\omega_1}{\omega_9} - \frac{\omega_4}{\omega_9}}{\frac{\omega_3}{\omega_9} - \frac{\omega_4}{\omega_9}};$$

$$\frac{1}{i_{02}} = 1 - \frac{\omega_4}{\omega_3} = 1 - \frac{\frac{\omega_4}{\omega_9}}{\frac{\omega_3}{\omega_9}};\ \frac{1}{i_{03}} = 1 - \frac{\omega_{4'}}{\omega_9} = 1 - \frac{\omega_4}{\omega_9} \quad (4.11)$$

From formulas (4.11) deduced function (4.12) in the implicit and explicit form

$$i_{II} = \frac{\omega_1}{\omega_9} = 1 - i_{03} - i_{01} \cdot \frac{1 + i_{03}}{1 - i_{02}};$$

$$i_{II} = 1 + \frac{z_8}{z_{4'}} + \frac{z_3}{z_1} \cdot \frac{1 - \frac{z_8}{z_{4'}}}{1 + \frac{z_4}{z_6}} \quad (4.12, 4.13)$$

In gear III ($\omega_1 = \omega_6; \omega_8 = 0$) transmission ratio is: $i_{III} = 1 - \frac{1}{i_{03}} = 1 + \frac{z_8}{z_{4'}}$ (4.14)

Gear IV ($\omega_3 = \omega_6 = \omega_8$) corresponded of direct transmission, without reduce: $i_{IV} = 1$ (4.15)

Gear V ($\omega_4 = 0; \omega_6 = \omega_8$) represented the returning (R), what corresponded the negative sign of transmission function:

$$i_V = i_{01}(1 - i_{03}) = -\frac{z_3}{z_1} \cdot \left(1 + \frac{z_{4'}}{z_8}\right). \quad (4.16)$$

In the case of the *kinematic synthesis* of complex planetary mechanism (fig. 1) imposed the numerical values of the transmission ratios in gears I, II and V, while from the implicit formulas (15, 19, 23) calculated the transmission ratios i_{01}, i_{02}, i_{03} specific each the cylindrical planetary unit CPU1, CPU2 and CPU3.

From equations (4.14), (4.15) and (4.16) deduced formulas for the calculation of specific ratios for CPU1, CPU2 and CPU3:

$$i_{01} = \frac{i_I - 1}{i_I - i_V} . i_V;\ i_{02} = 1 - \frac{i_{01}(1 + i_{03})}{1 - i_{03} - i_{II}};$$

$$i_{03} = 1 - \frac{i_V}{i_{01}} \qquad (4/17, 4.18, 4.19)$$

Imposing the numerical values of the transmission ratios [2] $i_I = 4{,}75;\ i_{II} = 2{,}5;\ i_V = -6{,}85$ is obtained the transmission ratios, which are specific of each cylindrical planetary unit and transmission ratio in gear III:

$$i_{01} = -2{,}21;\ i_{02} = -3{,}05;\ i_{03} = -2{,}1;\ i_{III} = 1{,}48.$$

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