RESEARCH CONCERNING THE SYLVESTER- KEMPE INVERTER

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Abstract. *The initial data on the geometry of inverse curves are given and the evolution over time of the research on the Sylvester-Kempe inverter is presented. Next, this inverter given in the literature is studied, the calculation relations are written by the contour method, taking into account the specific relations of the inverter. Initially the inverter is a kinematic chain, so a rotating driving element is added, the given curve being the circle generated by this element. A program is made to obtain: the position of the mechanism, the given curve and the inverse curve. Similarly, the given curve is considered as a line: parallel to the abscissa, parallel to the ordinate, bisector of the XOY axis system. The resulting inverse curves are given for these cases.*
Key words: Sylvester-Kempe inverter, reversing mechanisms, inverse curves

1. Introduction

Inverting mechanisms are intended to find the inverse curves of given curves. If a curve r1(φ) is given, then the inverse curve is r2(ψ), in relation to the pole of inversion P, common to the two radii, the condition being fulfilled: $r_1 . r_2 = k^2$, where k is the inversion modulus, the angle θ=φ - ψ constant.

Over time, these reversal mechanisms have been studied by many researchers. Thus, in [1] the geometry of some guiding and reversing mechanisms is studied. The same author studies in [2] the Peaucellier inverter generalizing it. In [3] a de-guiding mechanism is studied in which a line segment moves on a given line, the conducting element having a rotational movement (fig. 1).

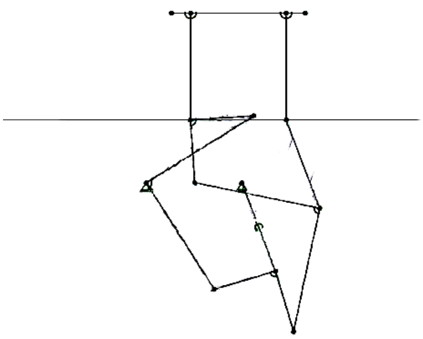

Fig. 1. Guiding mechanism [3]

Examples of given curves and their inverses are also given, extending the mechanism to the inverter type. In [4] shows a model of the Sylvester-Kempe inverter from 1875 (fig. 2).

Fig. 2. Sylvester-Kempe inverter from 1875 [4]

The geometry of the reversing mechanisms is given in [5], the English translation of the original book in Russian [7], as well as in [8]. A Sylvester-Kempe mechanism given in [6)] is studied below.

2. Studied mechanism

The mechanism is given in fig. 3.

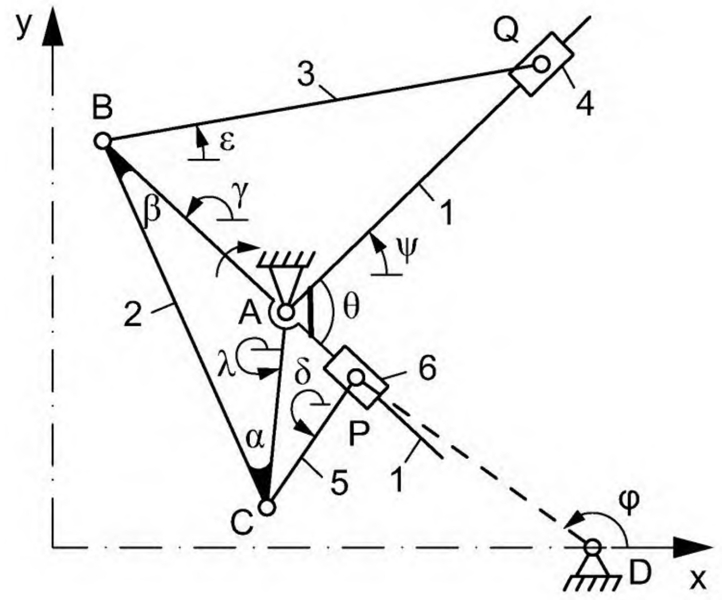

Fig. 3. The Sylvester-Kempe reversing mechanism

The inverter is without the element DP, which has been added here as a conducting element that determines the given curve of P (a circle), and the inverse curve drawn by the point Q will be found.

For the kinematic analysis of a mechanism, the mathematical equations of the coordinates of the different points of the mechanism are necessary. It is proposed to apply the contour method, a common method in the theory of mechanisms. The method has often been used in many studies of some mechanisms [9, 10, 11, 12]. This is how the following relationships are written:

$X_P = X_D + DP\cos\varphi$ (1)

$Y_P = Y_D + DP\sin\varphi$ (2)

$X_P + AP\cos\gamma = X_A$ (3)

$Y_P + AP\sin\gamma = Y_A$ (4)

$X_P + PC\cos\delta = X_C = X_A + AC\cos\lambda$ (5)

$Y_P + PC\sin\delta = Y_C = Y_A + AC\sin\lambda$ (6)

$X_B = X_A + AB\cos\gamma$ (7)

$Y_B = Y_A + AB\sin\gamma$ (8)

$X_Q = X_A + AQ\cos\Psi = X_B + BQ\cos\varepsilon$ (9)

$Y_Q = Y_A + AQ\sin\Psi = Y_B + BQ\sin\varepsilon$ (10)

$AC/AB = CP/BQ$ (11)

$AP \bullet AQ = \text{const.} = AB^2$ (12)

From (1) and (2) result the coordinates of P, and from (3) and (4) we obtain γ and AP. From (5) and (6) it follows δ, λ, XC and YC. Relationships (7) and (8) allow the determination of the coordinates of point B. From (9) and (10) results AQ, XQ, YQ. Relationships (11) and (12) are specific to the inverter.

In order to find the correlations between the angles, we use fig. 4.

The equations are obtained:

$\theta = \alpha + \beta$ (13)

$\Psi = \gamma - (\pi - \theta)$ (14)

$\lambda = \gamma + \pi - \alpha - \beta = \gamma + \pi - \theta$ (15)

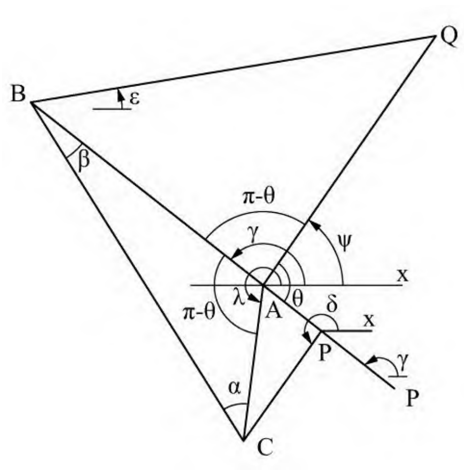

Fig. 4. Determination of angles

3. Obtained results

The following values were adopted for the dimensions of the mechanism: AB = 21:AC = 10:BQ = 36:PD = 15:XD = 48:XA = 21:YA = 20:PD = 15:α = 50:β = 25:PC = 16 (lengths in millimeters and angles in degrees). Fig. 5 shows the mechanism in one position.

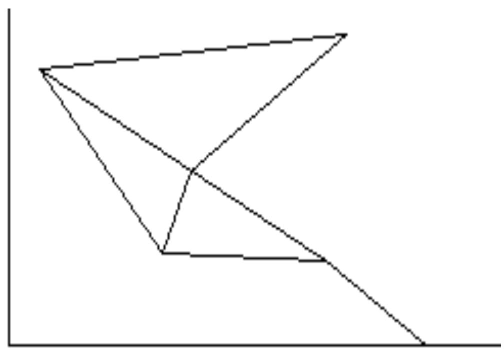

Fig. 5. The mechanism in one position

The variation of the coordinates of the Q point are given in fig. 6.

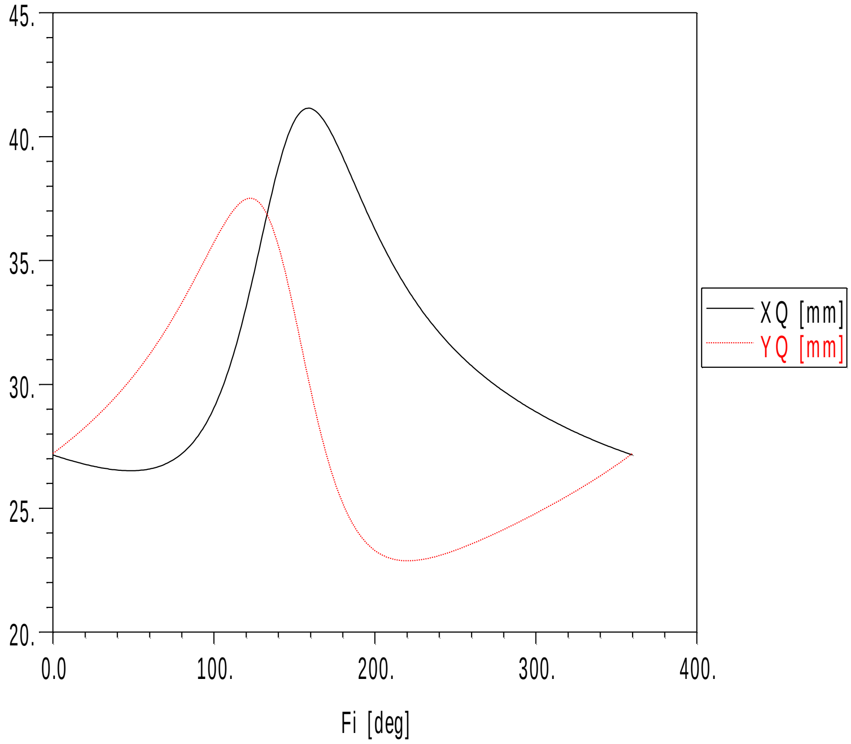

Fig. 6. The coordinates of Q

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Fig. 7 shows the given curve (bottom circle) and the inverse curve, the top circle.

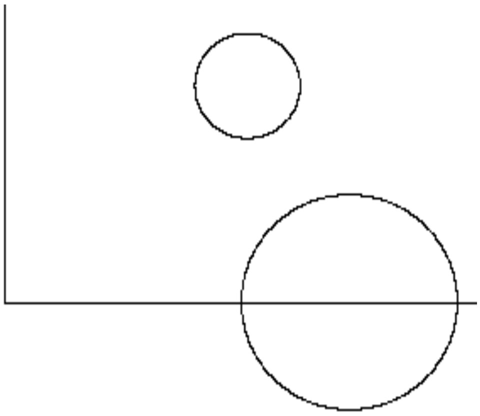

Fig. 7. The given curve and the inverse curve

4. The given curve is a straight line

Next, the given curve was considered a line parallel to the abscissa, with YP = 20 mm. In fig. 8 there are given the two lines: the parallel with the given abscissa and the inclined one - the resulting inverse curve.

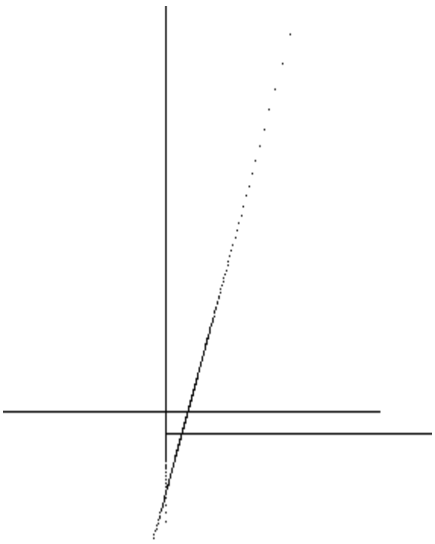

Fig. 8. The given curve is a parallel to xx

We then took the trajectory of P a straight line parallel to the ordinate, with XP = 20 mm. The curves are given in fig. 9: the parallel to the ordinate is the given one, and the arc of the circle is the resulting one.

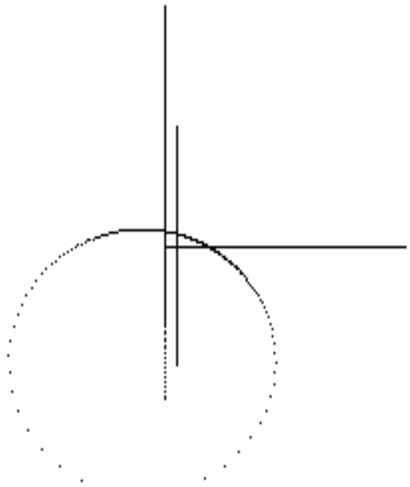

Fig. 9. The given curve is a parallel to the abscissa

We then took a line that passes through the origin and has an angle of 45 degrees with the abscissa, moving on it the point P with the step h. In this case we obtained fig. 10, where the line is the given curve and the arc of the circle is the inverse curve.

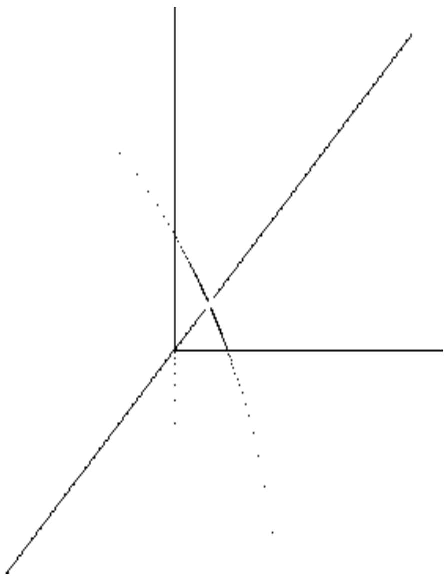

Fig. 10. The given curve is an inclined line

5. Conclusions

We started from other studies done on the Sylvester-Kempe inverter, finding that initially we were looking for guidance mechanisms, which would draw a line, or an arc of a circle, or, to move a segment from line to line. From here we came to inversion mechanisms that for a given curve generate an inverse curve. In the paper, a mechanism from the literature was studied by adding to the inverter an initial conducting element with rotational motion, which generates the given curve, so as to obtain the inverse curve. The calculation relations were written by the method of projections and a program was made that generated the inverse curve, which resulted in a circle. Next, the given curve was chosen as a line parallel to the abscissa, obtaining the inverse curve as an inclined line. Next, the given curve was considered a line parallel to the ordinate, the resulting inverse curve being an incomplete circle. We then took the curve given an inclined line passing through the origin of the axis system, obtaining the inverse curve an arc of a circle. The mechanism is therefore an inverter.

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