# STUDY OF A MECHANISM FOR GENERATING INVERSE CURVES

**Dr.eng. Mirela Cherciu,** *University of Craiova, ROMANIA*
**Dr.eng. Iulian Popescu,** Emeritus Professor**,** *University of Craiova, ROMANIA*
***Dr.eng. Liliana Luca,*** *University "Constantin Brâncuși", Tg.Jiu, ROMANIA*

**ABSTRACT:** The mathematical data of the inverse curves are given and a mechanism for generating these curves is analyzed. Actually, the goal of the paper is to check if the mechanism found is really of reversal type. The case was studied when the imposed curve is a circle, then a straight line, then a straight strophoid. Obviously, for each case there is a different mechanism, changing the driving element. It was found that the inverse curves were obtained. A problem has encountered when the imposed curve is a straight strophoid. In this case, the dimensions of the mechanism do not allow the driving element to make a complete rotation, so that the two curves have a break area, because the mechanism does not work in a certain subinterval of the cycle.

**KEYWORDS** : reversal mechanisms, inverse curves, straight strophoid

## 1. INTRODUCTION

Inverse curves are known in geometry and have a well-studied theory, such as in: [1, 2, 10]. Eric Weisstein, creator of MathWorld detailed the theory of these curves [12], and in other papers he analyzed the inverse curves of different mathematical curves. In [9] the inverse curves are analyzed too and a table is given comprising certain curves and the corresponding inverse curves. In [13] the inverse plane curves are also studied. In [8] specific problems of magnetic materials are studied using inverse curves. In [3] the inverse curves are applied to the thermal diagrams of the magnetic particles.

A mechanism for generating inverse curves is studied below.

The inverse curves are two curves $C_1$ and $C_2$ (Fig. 1), given by the equations:

$$\rho_1 = \rho_1(\varphi); \rho_2 = \rho_2(\psi) \quad (1)$$

$$\varphi - \psi = const. \quad (2)$$

$$\rho_1 \cdot \rho_2 = k^2 = const. \quad (3)$$

where k is any number, named the inversion modulus.

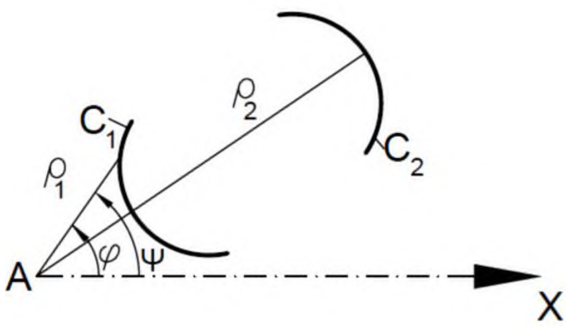


Figure 1. Inverse curves

In the literature there are also reversal mechanisms, which reverse the directions of movement of some elements, as found in steam locomotives, many machine tools and other fields. There is, therefore, a qualitative difference between the mechanisms generating inverse curves and the reversal mechanisms which reverse the directions of motion. In this way, the oscillating slider mechanism can also be considered a reversal mechanism, because the ram goes in one direction and then returns in the opposite direction.

## 2. THE MECHANISM STUDIED

In fig. 2 is given the studied mechanism.

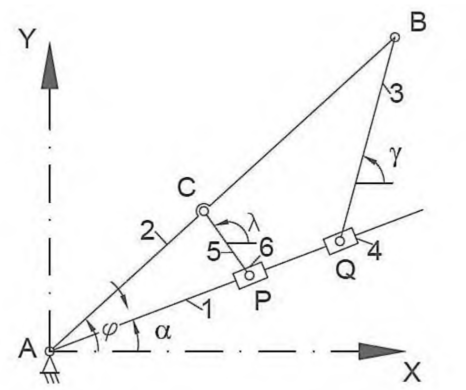


Figure 2. The mechanism studied [1]

The elements 1 and 2 are connected to the base in joint A, named the inversion center. The collinear points P and Q , also collinear with the inversion center, one draw the normal curve and the other the inverse curve. Therefore the curve on which for instance the pole P moves, must be imposed, so that the point Q has to generate the inverse curve. If the given curve is ***a circle***, being drawn by the element DP (Fig. 3), then the point Q has to describe the inverse curve.

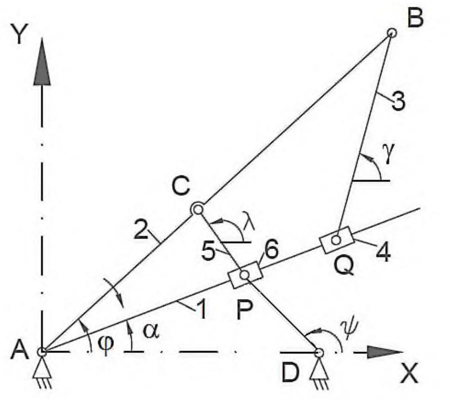


Figure 3. The curve of P is a circle

For the kinematic analysis of the mechanism, the mathematical equations of the coordinates of the different points of the mechanism are necessary. Equations are written by applying the closed-loop method, a method used in many studies to analyze mechanisms, such as [4, 5, 7]. For the mechanism in Fig. 3 the following relations result:

$$x_P = x_D + PD\cos\psi \qquad (1)$$

$$y_P = y_D + PD\sin\psi \qquad (2)$$

$$AP^2 = x_P^2 + y_P^2 \qquad (3)$$

$$\cos\alpha = \frac{x_P}{AP};\ \sin\alpha = \frac{y_P}{AP} \qquad (4)$$

$$CP = \frac{AC \cdot BQ}{AB} \qquad (5)$$

$$AP \cdot AQ = AB \cdot AC = const. \qquad (6)$$

$$x_Q = AQ \cdot \cos\alpha \qquad (7)$$

$$y_Q = AQ \cdot \sin\alpha \qquad (8)$$

Relationships (5) and (6) are imposed by the geometric considerations of drawing the inverse curve. The mechanism obtained with these relations is given in a position in Fig. 4.

Figure 4. The mechanism in a position

The successive positions of the mechanism are given in fig. 5.

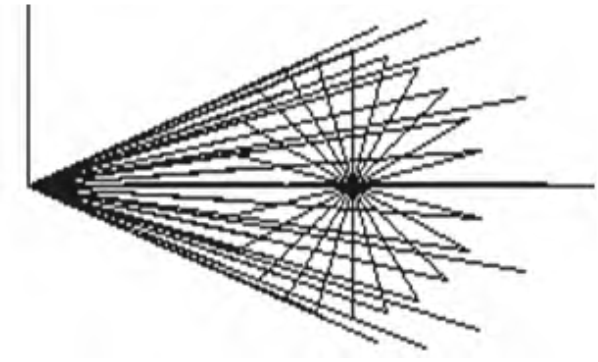

Figure 5. Successive positions

In fig. 6 it is shown the curve imposed on the point P, i.e. a circle. The following initial data were used: AB = 93: AC = 40: BQ = 40: AD = 63: PD = 26: XD = AD [mm].

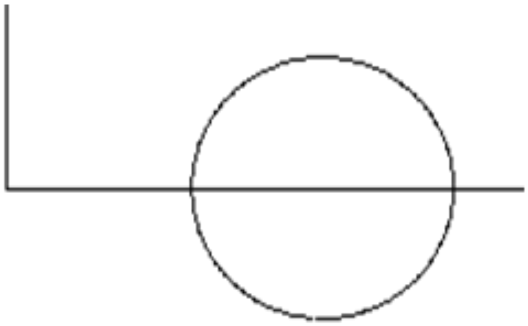

Figure 6. The curve imposed on P

The inverse curve, drawn by the point Q is given in fig. 7, being an oval shape.

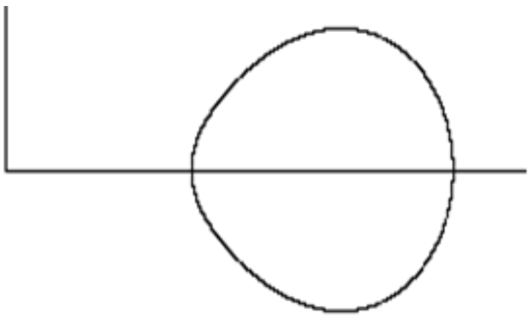

Figure 7. The inverse curve drawn by the point Q

## 3. OTHER MECHANISMS

However, if the ***curve of P is a straight line*** , the mechanism of Fig. 8 is obtained.

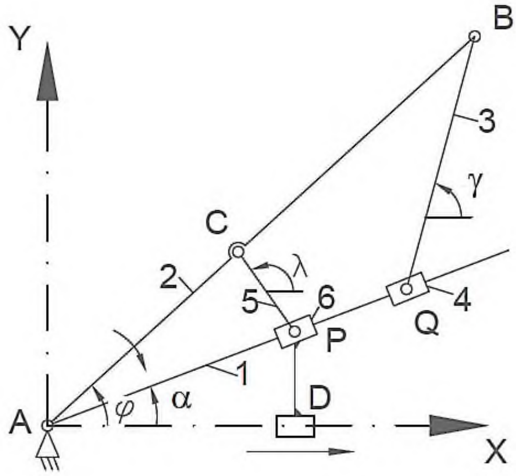


Figure 8. The mechanism with a straight line-curve for P

The relations (1) and (2) are modified, introducing XP = XD in a cycle, and YD = 12mm = const. The result is presented in Fig. 9, where the line is the trajectory of P, and the curve is the trajectory of Q, ie the inverse curve, here a circle.

Figure 9. The imposed curve - a straight line, the inverse curve - a circle.

The case in which the ***imposed curve is a straight strophoid*** was also studied.

In [6] a mechanism (Fig.10) was designed for generating the right strophoid, which was added to the previous kinematic chain, suppressing the driving slider, so that the APGE kinematic chain generates the right strophoid in Fig. 11, which becomes the imposed curve of P.

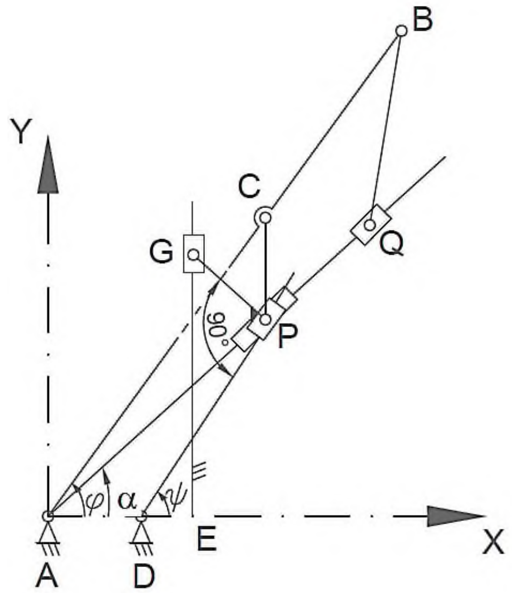


Figure 10. The mechanism with the imposed curve a straight strophoid

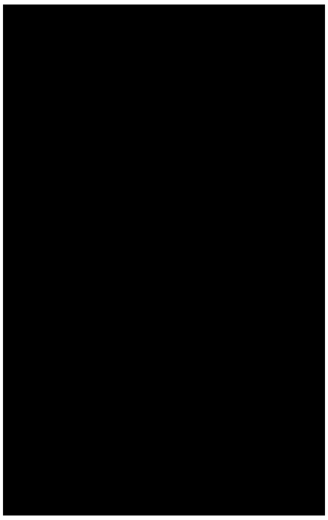

Figure 11. The straight strophoid - the curve of P

The inverse curve is an incomplete strophoid (Fig. 12). In Fig. 13 both curves are presented (the imposed is the one with the exterior branches). It is observed that neither the imposed curve is no longer complete as in Fig. 11. The reason is that the dimensions of the mechanism do not allow its movement in the subinterval $\psi = 240 \ldots 300^0$.

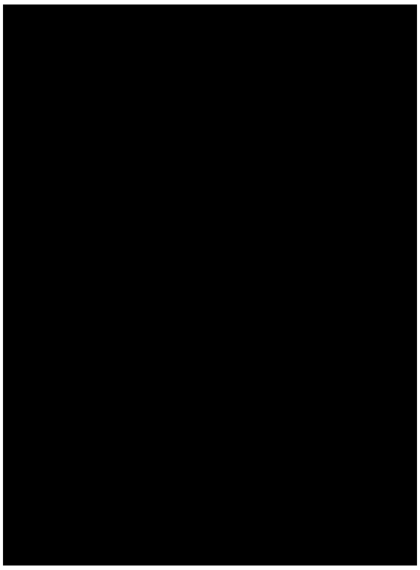

Figure 12. The inverse curve

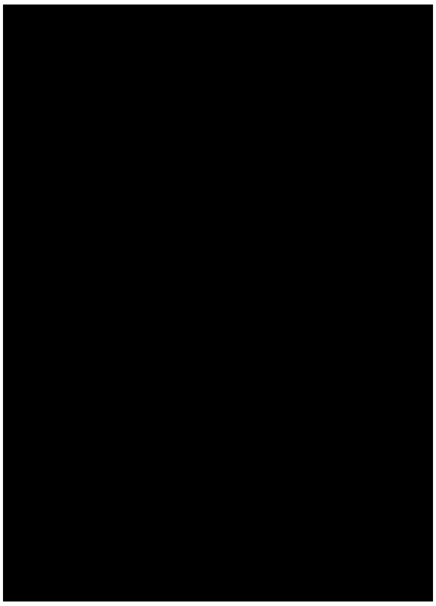

Figure 13. The imposed curve and its inverse

This situation is also observed in the diagram in Fig. 14, where in this subinterval XQ is constant, and YQ has an inclined line (the software connects the ends of the subinterval in which there are no values).

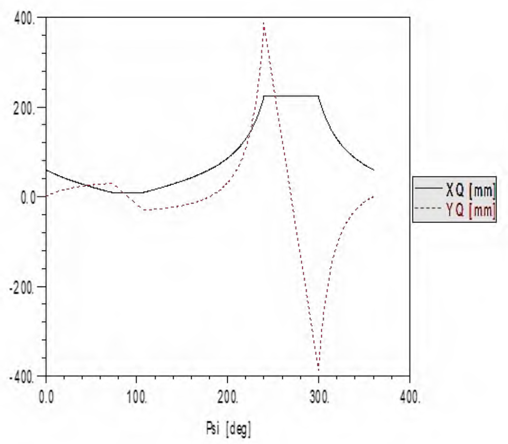


Figure 14. The diagrams XQ, YQ

## 4. CONCLUSIONS

A mechanism for generating the inverse curve of an imposed curve has been studied. The mechanism is of reversal type, quite complicated. The way of generating the inverse curve was studied for the case when the imposed curve is: a circle, a straight line, a straight strophoid. The inverse curves were obtained. In the case of the strophoid, it was found that the point that draws the imposed curve generates it completely, but when the program also calculates the inverse curve, there is a discontinuity in the mechanism's working, so that neither the imposed curve can be completely drawn.
The studied mechanism is thus proven to be a reversal mechanism.

## REFERENCES

[1] Artobolevskii, I. I. – Mehanizmî v sovremennoi tehnike. Tom II. Râciajnâe mehanizmî, Izd. Nauka, Moskva, 1971, str. 363.
[2] Artobolevskii, I. I. – Teoria mehanizmov dlia vosproivedenia ploskih crivâh. Izd. Akademii Nauk, SSSR, Moskva, 1959, str. 36.
[3] Christopher R. Pike, Andrew P. Roberts, Kenneth L. Verosub First order reversal curve diagrams and thermal relaxation effects in magnetic particles, *Geophysical Journal International*, Volume 145, Issue 3, June 2001, Pages 721–730, https://doi.org/10.1046/j.0956-540x.2001.01419.x
[4] Luca L., Popescu I., *Curves and aesthetic surfaces generated by the R-R-RTR mechanism.* Fiability & Durability, No. 1, 2013, pp. 28-34.
[5] Luca L., Popescu I., Ghimisi S., *Studies regarding generation of aesthetics surfaces with mechanisms* **.** Proceedings of the 3-rd International Conference on Design and Product Development. Montreux, Elvetia, 2012, Published by WSEAS Press, 2012, pp. 249-254.
[6] Popescu, I. Sass, L. – Mecanisme generatoare de curbe. Editura Scrisul Românesc, Craiova, 2001.
[7] Popescu, I., Luca,L., *Geometric locus generated by a mechanism with three dyads.* Annals of' Constantin Brancusi'University of Targu-Jiu. Engineering Series, Issue 3, 2015, pp. 29-37
[8] Ruta, S., Hovorka, O., Huang, P. *et al.* First order reversal curves and intrinsic parameter determination for magnetic materials; limitations of hysteron-based approaches in correlated systems. *Sci Rep* **7,** 45218 (2017), https://doi.org/10.1038/srep45218
[9] Terr, David and Weisstein, Eric W. "Inverse." From *MathWorld*--A Wolfram Web Resource, https://mathworld.wolfram.com/InverseCurve.html
[10] Tutunaru, D. - Mecanisme plane rectiliniare şi inversoare. Editura Tehnică, Bucureşti, 1969, pg. 160.
[11] Weisstein, Eric W. "Parabola Inverse Curve." From *MathWorld*--A Wolfram Web Resource.https://mathworld.wolfram.com/ParabolaInverseCurve.html.
[12] Weisstein, Eric W. „Curba inversă". De la *MathWorld* - O resursă web Wolfram. https://mathworld.wolfram.com/InverseCurve.html.
[13] *М. И. Войцеховский.* ПЛОСКАЯ ДЕЙСТВИТЕЛЬНАЯ АЛГЕБРАИЧЕСКАЯ КРИВАЯ.,https://dic.academic.ru/dic.nsf/enc_mathematics/3987/%D0%9F%D0%9B%D0%9E%D0%A1%D0%9A%D0%90%D0%AF