UNDERSTANDING TIME-FREQUENCY REPRESENTATIONS USING EXCEL

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ABSTRACT: *We present a method to create representations that permit observing the frequency evolution in time, which is often necessary in industrial applications. The algorithm is implemented in the MS Excel and a generated signal with known parameters for observing the conformity between real and expected results is analyzed. The use of this algorithm is simple and it can be easily understood because limited data is involved. We also highlight how the signal is represented on the portions close to the moment in which the frequency change occurs*.

KEY WORDS: Signal processing, frequency estimation, time-frequency representation.

1. INTRODUCTION

In a series of engineering applications, it is necessary to know the temporal evolution of the frequency of some machines. Usually, frequency estimation methods are based on the use of discrete Fourier transform (DFT). Its precision is related to the signal acquisition time, which for accurate results must be a whole number of periods *T*, hence a whole number of cycles of the targeted frequency component [4]. The precision of the frequency estimation can be improved if involving several interpolation algorithms [13]. Among them, we distinguish between interpolation methods that use two or three points from the complex values of spectrum [1], [3], [10], [14] or from the module spectrum [5], [9], [11], [16]. However, these methods do not indicate when the transition from one frequency to another occurs, thus it is impossible to observe the changes in the system's state [14].

In this paper we show how the frequency evolution in time can be observed, and consequently how the operation of a machine or the state of a structure can be assessed, se for instance papers [8], [17], [18]. It is also shown herein how the data for the time-frequency representation can be easily calculated. Such a simple procedure proposed in by the authors permits for sure a deep understanding of this mathematical approach.

2. THE DFT ALGORITHM

This section presents how the DFT algorithm is used to represent a signal in the frequency domain if we know the evolution in the time domain. Therefore, we consider the discrete signal $x[n]$ formed by a sequence of N values, where the moments in time are n=0…N-1. This signal is written:

$$\{x[n]\} := x_0, x_1 ... x_{N-1} \quad (1)$$

To find out its frequency components, we commonly apply the DFT algorithm. Involving this algorithm, we create a series of N-1 values

$$X_k = \sum_{n=0}^{N-1} x[n]\left[\cos\left(2\pi \frac{n}{N} k\right) - j\sin\left(2\pi \frac{n}{N} k\right)\right] \quad (2)$$

In Eq.(2) we denoted $j^2 = -1$. The whole series of values, which is the representation of the discrete signal in the frequency domain, is:

$$\{X[k]\} := X_0, X_1 \ldots X_{N-1} \quad (3)$$

with each of the individual values belonging to a spectral line k.
If we consider the real part

$$\operatorname{Re} X_k = \sum_{n=0}^{N-1} x[n]\left[\cos\left(2\pi \frac{n}{N} k\right)\right] \quad (4)$$

and the imaginary part

$$\operatorname{Im} X_k = -\sum_{n=0}^{N-1} x[n]\left[\sin\left(2\pi kn / N\right)\right] \quad (5)$$

we can write

$$X_k = \operatorname{Re} X_k + j \operatorname{Im} X_k \quad (6)$$

Now, the modulus of X_k that belong to the spectral line k is given by:

$$\left|X_k\right| = \sqrt{\left(\operatorname{Re} X_k\right)^2 + \left(\operatorname{Im} X_k\right)^2} \quad (7)$$

Because $|X_k| = |X_{N-k}|$, for reasons of symmetry it is sufficient to consider only the half of the spectrum [7].

3. BASICS OF TIME-FREQUENCY REPRESENTATIONS

Usually the frequency spectrum is used in signal analysis. But, as previously mentioned, this spectrum does not provide us with any information about the time period in which some phenomena characterized by certain frequencies occur. For this reason, a time-frequency representation is needed.
To study the temporal evolution of the signal frequency we created an application in MS Excel, in which a sinusoidal signal

$$x[n] = a\sin(2\pi f \frac{n}{N} \Delta t + \varphi) \quad (8)$$

is generated with a certain number of samples N, defining the signal length t_S. In Eq.(8) we denoted with Δt the time resolution, that is

$$\Delta t = \frac{t_S}{N-1} \quad (9)$$

Also in Eq.(8) a is the amplitude of the signal, f is the frequency and φ is the initial phase. In order to observe the frequency evolution over time, we consider three sequences that have the frequencies, f_1, f_2 and f_3 and the phases φ_1, φ_2 and φ_3. The amplitude is maintained constant, in order to clearly notice its evolution over time.

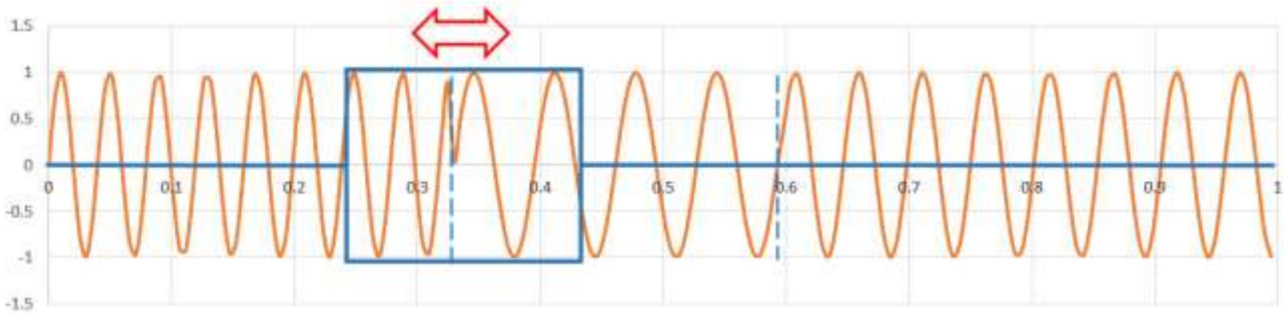

Figure 1. Analyzed signal

The application calculates the instant values for the sinusoids for the resulted signal $x[n]$ for all time steps n having values between 0 and N-1. An example of a signal generated with three segments as those described above is given in Figure 1.

Further, the application calculates, for each time moment, the real and the imaginary values with Eq.(4) and Eq.(5), respectively. In consequence, we obtain two square matrices with N rows (one for each time moment n) and N columns (one for each spectral line k). The sum of each column represents the contribution of the spectral line to the total amount of energy contained in the signal.

The module $|X_k|$ for the k=1…N spectral lines is obtained by extracting the square root of the sum of the squared values obtained for the real and the imaginary part. Summarizing the values for each spectral line k, the amplitudes of the DFT are obtained. The amplitude values are associated with the spectral lines or with the frequencies calculated for the spectral lines, by employing the relation:

$$f_k = k\Delta f = k\frac{1}{t_S} \qquad (10)$$

The representation of the modules X_k for the relevant spectral lines is presented in Figure 2. From this figure we can notice that the frequencies can be (coarsely) estimated, but without any information regarding the evolution in time.

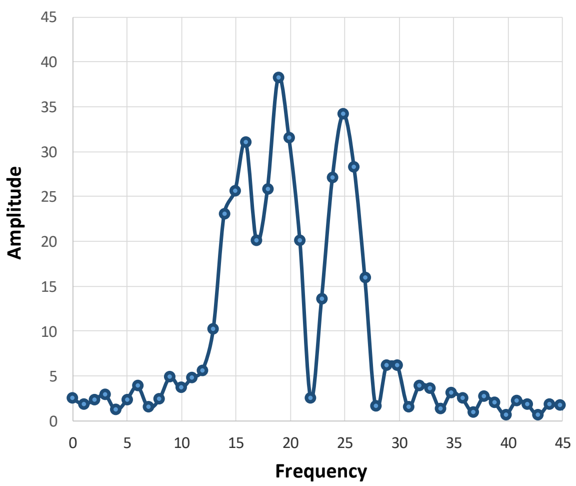

Figure 2. Typical DFT representation for a three-component signal

To properly identify the three signal segment with the different three frequencies, we apply rectangular windows to the original signal, as shown in Figure 1. Outside this window, all values for the signal are zero, resulting in a zero-padded signal with the same length as the original signal. In consequence, the frequency resolution will be the same for the original and the windowed signal, namely $\Delta f = 1/t_S$, and the spectral lines maintain their position in the spectrum.

The window is removed sample-by-sample along the original signal, and the X_k values are calculated and associated with the time corresponding to the center of the window. In this way, we achieve a 3D representation, on the abscissa being the time, on the ordinate the frequency, and the amplitude is represented in the vertical direction. In the case of a 2D representation, the amplitude is indicated by a color scheme.

Obviously, for the ends of the time axis the number of samples involved in the applied window are smaller as that of the genuine window because the center of the window approaches end of the time axis. For these parts of the representation the energy contained in the window decreases, so the amplitude of the individual DFTs decrease in consequence.

4. NUMERICAL EXAMPLE

It this section we consider the signal with the time length of 1 second, generated with N=197 samples.

In the analyzed time period, the signal will pass from one frequency to another. The first frequency is f_1=25.17 Hz, the second is f_2=15.17 Hz and the third is f_3=19.33 Hz.

The frequency changes from f_1 to f_2 at t_1=0.33s, and from f_2 to f_3 at t_2=0.6s, see Figure 1. Here, the segment limits are marked with vertical dashed lines.

The initial phases for the first two signal segment is φ_1=φ_2=0. These phase values correlated with the first segment's time lengths ensure a sudden transition. The phase φ_3 is selected to ensure a smooth transition. In this way, we can observe the specific segments of the signal characterized by different frequencies, but also find out how the time-frequency representation looks like when the signal presents sudden or smooth frequency transition.

It is worth mentioning that the three segments of the signal are observable in Figure 1 because there is no noise and single harmonic components are generated. Dissimilar, for noisy signals and/or for those containing several harmonic components, this frequency change is no longer evident.

The rectangular window consists of 40 samples that sweep the entire time interval, so that the middle of the interval covers the entire interval from 0s to 1s.

For the frequency of f_1=25.17 Hz, the DFT for the signal portion located in the time interval between t_1=0.015s and t_2=0.213s is calculated and the spectrum in Figure 3 is obtained.

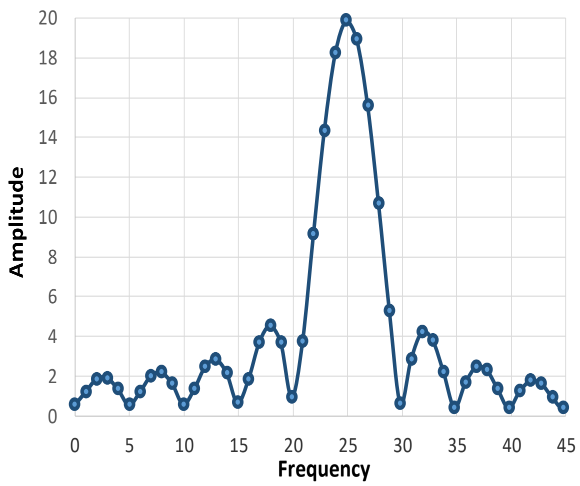

Figure 3. The spectrum for the signal segment that has just f_1=25.17 Hz

We now move on the time interval between t_1=0.355s and t_2=0.543s and for the frequency f_2=15.17Hz the spectrum is determined and represented graphically, see Figure 4.

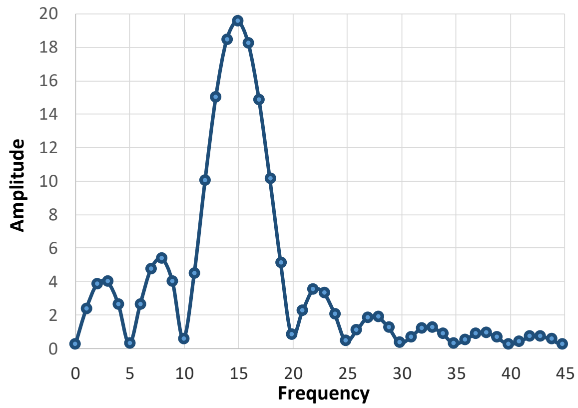

Figure 4. The spectrum for the signal segment that has just f_2=15.17 Hz

Figure 5 shows the spectrum determined for a signal portion of the time interval between t_2=0.609s and t_3=0.807s, for the frequency f_3=19.33 Hz.

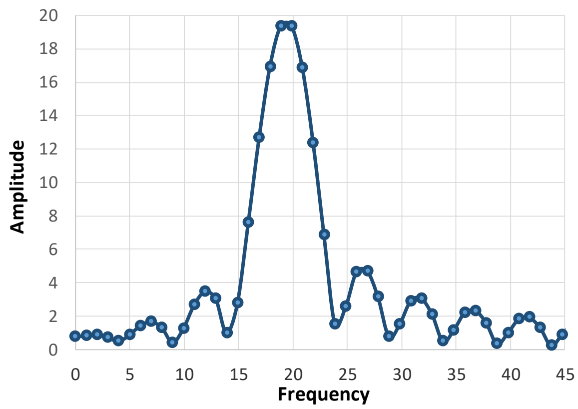

Figure 5. The spectrum for the signal segment that has just f_3=19.33 Hz

In each of the three cases presented a normal spectrum is obtained, and the frequency is quite accurately indicated. From the three figures we can observe the right frequency, but without knowing at what moment the estimation was made.

Let us see now what happens if the window contains a segment with two different frequencies. For the portion of the signal in which the transition from frequency f_1=25.17 Hz to frequency f_2=15.17 Hz is made, the spectrum is represented in Figure 6. Here, a sudden disruption of the signal is generated, which is usually not present in real-life systems.

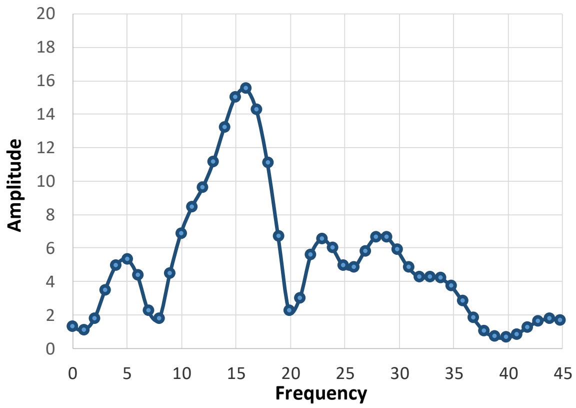

Figure 6. The spectrum for the signal segment that has portions with frequencies f_1=25.17 Hz and f_2=15.17 Hz as well

In Figure 6 we observe a component with around 15 Hz and a possible component at 25 Hz which is less evident.

For a continuous signal transition, we obtain the spectrum like that presented in Figure 7. Here, the apparent frequency of around 17 Hz is indicated, which corresponds to an intermediate value, framed by the two real frequencies.

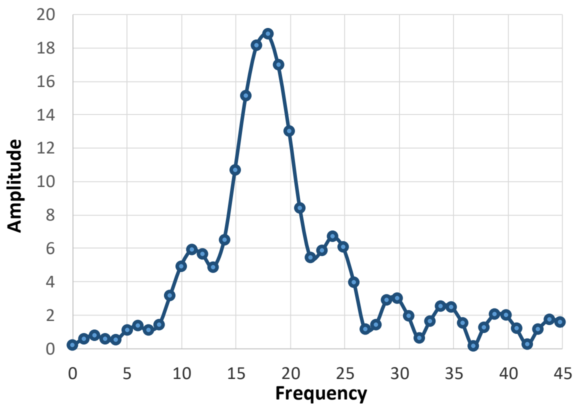

Figure 7. The spectrum for the signal segment that has portions with frequencies f_2=15.17 Hz and f_3=19.33 Hz as well

When analyzing the amplitudes, we observe that for the first three cases, that of a windowed signal portion with unchanged frequency, the amplitude is quite good indicated. This means that for the signal with 40 samples, the amplitude is

$$A = \frac{2}{N} \max\left\{X_k\right\} \tag{11}$$

Dissimilar, for the sudden transition we observe that the amplitudes are lower as expected, one being indicated more precisely, while the second is barely visible.

To clearly observe when a machine or a structure changes its state, 3D time-frequency representations are made. The lateral view of the representation is depicted in Figure 8, where we have the frequencies on the abscissa and the amplitudes at the ordinate. The top view of the time-frequency representation is illustrated in Figure 9. We distinguish the three portions of the signal with different frequencies, and the regions in which the frequencies change.

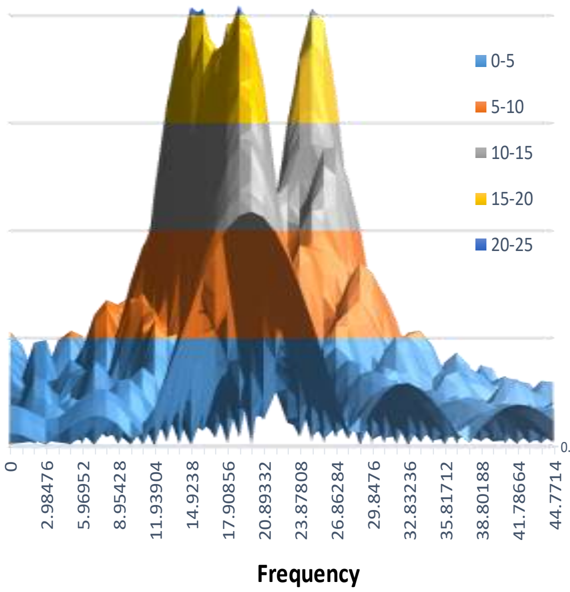

Figure 8. Lateral view of the 3D time-frequency representation for the signal scanned with a rectangular window

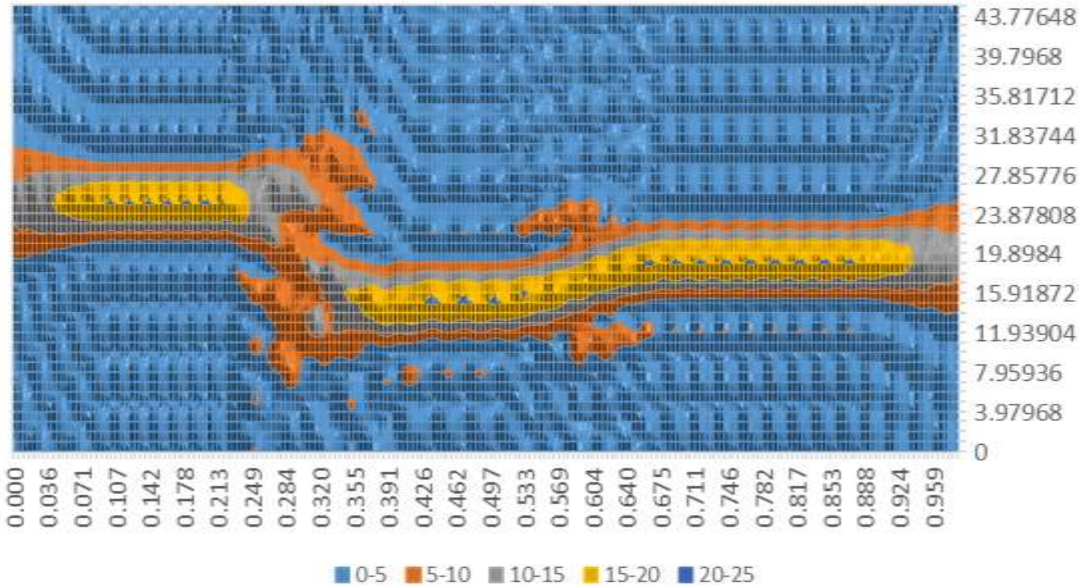

Figure 9. Top view of the 3D time-frequency representation

CONCLUSION

In industrial applications it is important to accurately estimate frequencies and know when these frequencies manifest. To this aim we perform a time-frequency analysis.

This paper presents how to determine the temporal evolution of the frequency. Once the phenomenon is identified, the frequencies can be accurately extracted.

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