GRAPH THEORY APPLICATION IN PLANNING POWER EQUIPMENT MAINTENANCE OPERATIONS

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ABSTRACT: *Maintenance scheduling is a critical stage in the lifecycle of power equipment. Equipment manufacturer sets the maintenance program and detailed operations required depending on the operation time. Maintenance operations require complete shut down of the equipment. In power systems the availability of equipment is a critical condition in ensuring the demand parameters. Therefore, planning of a power system must consider maintenance outages for various components and provide accordingly back up resources. In the same time, it is essential to minimize the planned outage caused by maintenance. Scheduling of maintenance works must be optimized in terms of both economic efficiency and timing. Graph theory is an efficient tool in optimum scheduling of power equipment maintenance, as shown in this paper.*

KEY WORDS: Maintenance; Scheduling; Graph theory

1.INTRODUCTION

Reliability analysis is in power systems is a key element in ensuring uninterrupted supply of critical consumers in industry but also civil. Reliability analysis is often employed as a decision tool in investment, maintenance scheduling and system operation. The main benefit of reliability prediction of complex systems consists not in the absolute figure predicted, but in the ability to repeat the assessment for different repair times, different redundancy arrangements in the design configuration and different values of component failure rate.

There is a strong correlation between reliability and maintenance. By maintenance it is understood the totality of actions that need to be taken in order to ensure that an equipment performs according to the design. Two types of maintenance activities exist [1]: (1) corrective and (2) preventive. While corrective maintenance is only performed when failures occur, the preventive maintenance is critical in some areas such as power industry in order to ensure that the power system/equipment guarantees the consumers demand. Improper maintenance scheduling has been shown to produce serious issues [5].

Reliability analysis boils down to failure frequency and steady-state availability [2]. A tool frequently employed to analyze quantitatively the reliability is the graph theory. Graph theory has various applications, from analysis of behavior in complex distributed systems [6] to network analysis [7].

2. GRAPH THEORY APPLIED TO SCHEDULING

The first stage of the process is defining the complete set of works required by the maintenance program. The graph vertices represent the distinct operations to be carried out and the edges represent the transition time from one operation to the next. Transition time values are usually estimated statistically and the value can be selected in an interval. The orientation of edges will be established as follows:

- If transition time from operation n to operation n+1 is less than the transition time from operation n+1 to operation n, the edge connecting vertices n and n+1 will be oriented towards vertex n+1;
- If transition time from operation n to operation n+1 is greater than transition time from vertex n+1 to vertex n, the

edge originating in vertex n+1 will be oriented towards vertex n;

- If transition time from operation n to operation n+1 are equal, the edge connecting the two vertices can be arbitrarily connected towards any of the two;
- Every vertex of the oriented graph must have a circuit (a closed loop originating and ending in the vertex).

The graph constructed in this way has an antisymmetric flow. The maximum number of possibilities to schedule the operations is equal to the number of distinct Hamiltonian paths [8] (by Hamiltonian path it is understood any path that crosses once only every vertex of the graph). The Konig's theorem states that a fully antisymmetric graph has at least one Hamiltonian path.

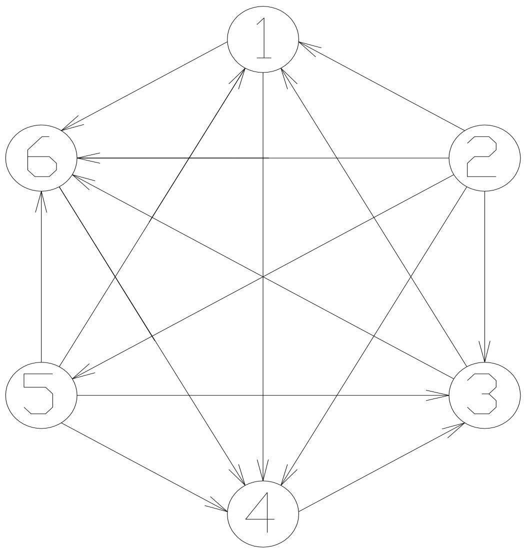

Figure 1. Example of scheduling maintenance operations by means of an oriented graph

An oriented graph example was represented in Figure 1 to demonstrate the relationships between vertices and orientation of edges. The graph represented in Figure 1 illustrates a sequence of six maintenance operations. A Boolean matrix can be defined to express the directions of edges connecting the nodes. Thus, a line will be assigned to vertex i, consisting of 1 if the edge connecting vertices i and j points to vertex j or 0 if the edge points to vertex i. The Boole matrix associated to the oriented graph in Figure 1 is:

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \qquad (1)$$

It can be noticed that for any values i and j the relationship $m_{ij} = \overline{m_{ji}}$ is valid. The matrix M was built starting with maintenance operation 1 then considering all other elements and filling the line with zeros or ones, depending on the edge direction. Thus, the element m_{12} is set to zero because the direction of the edge connecting nodes 1 and 2 is from 1 towards 2.

The next step is to create a vector consisting of the sum of matrix elements by line:

$$P = \left[\sum_{j=1}^{N} m_{ij}\right]_{i=1..N} \qquad (2)$$

$$P = [3 \quad 6 \quad 4 \quad 1 \quad 5 \quad 2] \qquad (3)$$

Sorting descendingly the elements of the vector P by their value results in the following operation sequence:

$$S = [2 \quad 5 \quad 3 \quad 1 \quad 6 \quad 4] \qquad (4)$$

To conclude, out of a total of 6! = 720 possible operation sequences, the optimum sequence is given by the vector S (Eq. 4).

Another method based on the graph theory consider the transition time from one operation to another. A transition time is constructed as in the example below:

$$T = \begin{bmatrix} & 8 & 3 & 2 & 1 & 7 & 11 \\ 4 & & 12 & 5 & 8 & 3 & 7 \\ 8 & 9 & & 1 & 2 & 4 & 3 \\ 9 & 11 & 12 & & 4 & 7 & 3 \\ 8 & 1 & 3 & 9 & & 2 & 9 \\ 7 & 9 & 3 & 12 & 11 & & 5 \\ 9 & 11 & 16 & 3 & 7 & 5 & \end{bmatrix} \qquad (4)$$

In the transition time matrix defined above, an element t_{ij} is the transition time from operation i to operation j.

Based on Konig's theorem it has been established that at least one Hamiltonian path exists. The Hamiltonian path that minimizes the total time. The sequence of steps is as follows:

1. The transition time matrix lines are considered in sequence, starting with line 1. The minimum element is selected:
   $$t_1^{min} = min\left(t_{1j}\right)_{j=1..N} \qquad (5)$$
   In the example (Eq. 4) the element is marked in red.
2. The column number corresponding to the minimum element identified above is selected. Let this index be denoted j_1^{min}.
3. The line with the index j_1^{min} is selected and the minimum element of this line is identified by the same procedure as in step 1 of the algorithm.
4. The algorithm continues returning to step 2 until all lines of the *T* matrix are considered in the process.

The shortest Hamiltonian path corresponds to the line sequence determined by the algorithm above. In the example considered in Eq. 4, the shortest path is obtained as:

$$1 – 5 – 2 – 6 – 3 – 4 – 7 \qquad (6)$$

In Eq. (6) it is important to note that the sequence above was obtained by starting with line 1 (operation 1). In the case in which the starting node is different, a different sequence is obtained.

Another interesting application of the graph theory to scheduling of maintenance/construction works is the Ford-Fulkerson algorithm. Ford-Fulkerson algorithm [8 – Cormen] is a greedy algorithm that determines the maximum length path in a flow network. The following optimization problem is defined as an application for the Ford-Fulkerson algorithm:

Let a set of maintenance operations denoted *R* and *K* a set of operators. The goal of the problem is to minimize the number of operators involved in the process of carrying out all operations in the set *R* under the constraints given in Table 1.

Table 1. The set of operations R and timing

Operation	Start time	End time
r_1	t_{12}	t_{12}
r_2	t_{21}	t_{22}
r_3	t_{31}	t_{32}
r_4	t_{41}	t_{42}
r_5	t_{51}	t_{52}
…	…	…
r_n	t_{n1}	t_{n2}

Any operator k_x can carry out a given operation r_j after having completed another operation r_i if:

$$t_{j1} = t_{i2} + \Delta t_{ij}$$

Δt_{ij} is an interval required for preparation of prerequisites of operation *j* after completing operation *i*.

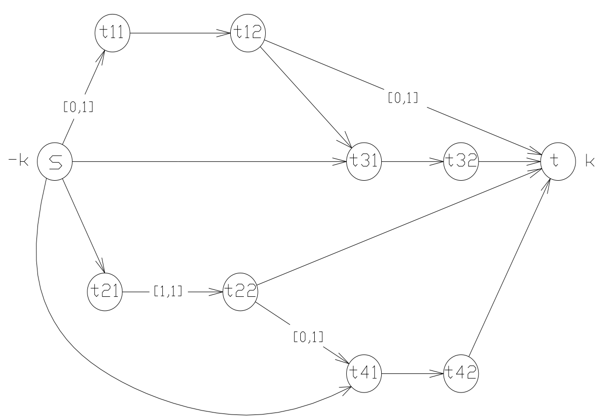

Figure 2. Example of scheduling 4 operations

The problem of maintenance operations scheduling can be formulated as a graph (Figure 2). Two special vertices must be added, *s* – source and *t* – sink. The vertex *s* will have a demand -*k* and the vertex *t* will have the demand *k*. If an edge exists between vertices t_{i1} and t_{i2} then the operation r_i can be carried out by an operator.

From Figure 2 it can be noticed that the problem can be further reformulated as a bi-partite graph (Figure 3).

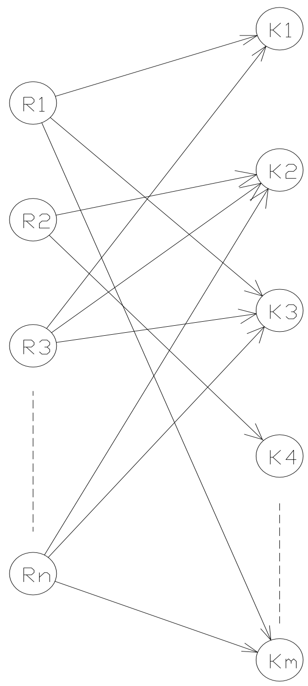

Figure 3. Bi-partite graph operations – operator

The solution of the bipartite graph in Figure 3 can be illustrated by the example shown in Figure 4.

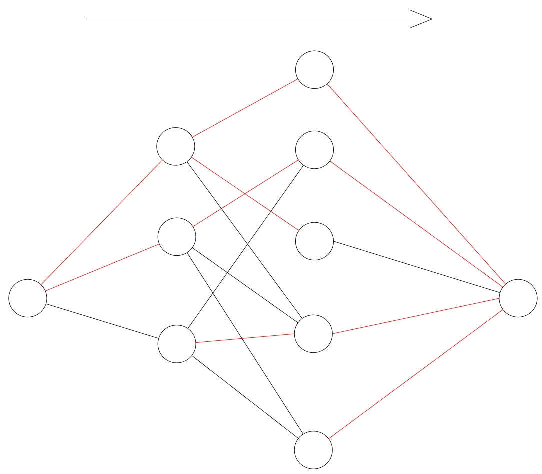

Figure 4. The matching of a bipartite graph – example with 3 operations and 5 possible operators

In Figure 4 the edges with non-zero flow (possible operations) were represented in red while the other edges represent non-possible operations. This approach is a called flow network, which consists of a directed graph where each edge has a capacity and a flow. The edges have a transport capacity and a flow. In case of scheduling problems, the capacity is 1 (and flow is also 1) if operation is possible and 0 if operation is not possible.

3. CONCLUSIONS.

Maintenance is an important concept in ensuring the desired reliability of industrial systems, especially power systems. Maintenance is a complex process involving significant logistic and organization. Due to the fact that the power equipment is out of service during the maintenance, it is extremely important that this period be minimized in order to maintain the economic efficiency parameters of the system. It is therefore essential to optimize and plan carefully each stage of the maintenance process in order to comply with the maintenance program. In cases where a large number of equipment pieces are subject to maintenance, it is important to schedule some sequences required by technological or logistic requirements. In such cases, the graph theory can offer fast and precise results and ensure the optimum conditions for the maintenance process.

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