ON SOME OPERATIONS OF M-POLAR INTUITIONISTIC FUZZY GRAPHS

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ABSTRACT: *Abstract. In this article, we will present some results related to fuzzy graphs, m-polar fuzzy graphs using m-polar fuzzy set, intuitionistic fuzzy graph and complex intuitionistic fuzzy graph (cif-graph). Rosenfeld [21] first introduced the concept of fuzzy graphs. Buckley [28] and Nguyen et al. [29] combined complex numbers with fuzzy sets. Ghorai, G., and M. Pal [15],[16],[17] defined m-polar fuzzy graphs using m-polar fuzzy set.*

KEY WORDS: *fuzzy graphs, intuitionistic fuzzy graph, m-polar fuzzy graphs, complex intuitionistic fuzzy graph.*

1. INTRODUCTION

Fuzzy set theory was introduced by Zadeh [26] in an attempt to solve the problem of uncertainties.

Atanassov[5] proposed expanding the fuzzy set by adding a new, non-belonging component called "intuitive fuzzy sets" (IF sets). Buckley [28] and Nguyen et al. [29] combined complex numbers with fuzzy sets. In 1994, Zhang [26],[27] introduced the notion of a bipolar fuzzy sets as generalization of the fuzzy sets. In 2014, Chen et al. [6] introduced the notion of m-polar fuzzy set as a generalization of fuzzy set theory.

Fuzzy graphs were introduced by Rosenfeld [21] and Mordeson [18]. Later, Bhattacharya [20] developed this theory. Shannon and Atanassov [6] and Akram and Davvaz [3] defined intuitionistic fuzzy graphs. In 2011, Akram [2] has introduced the notions of bipolar fuzzy graphs. The notion of the bipolar fuzzy line graph of a bipolar fuzzy graph was introduced by Akram and Dudek [1]. Berge [8] will introduce the notion of hypergraph. Mordeson and Nair [18] deals with the study of properties of fuzzy graphs and hypergraphs. Rashmanlou et al.[19],[20] studied bipolar fuzzy graphs, bipolar fuzzy graphs with categorical properties, product of bipolar fuzzy graphs and their degrees, etc. Also, Ghorai, G., and M. Pal, [15], Sunitha, M.S., and Vijayakumar, A. [22], Yang, H.L., Li, et al. [24] treat these topics. Ghorai and Pal[16],[17], studied many properties of generalized m-polar fuzzy graphs (mFGs), operations and density of mFGs, publishes various studies[17] about of m-polar fuzzy planar graphs and types of product bipolar fuzzy graphs.

The concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFG) was introduced by Atanassov[7]. Parvathy and Karunambigai[17] introduced the concept of IFG and its components.

Starting from complex intuitionistic fuzzy sets generalize intuitionistic fuzzy sets complex Gulistan, M.; Yaqoob, N.; Rashid, Z., F.; Wahab, H.A, talk about complex intuitionistic fuzzy graphs with some fundamental operations.

This study is organized as follows: In Section 1, introduction is given and the literature review is illustrated. Section 2 represents a brief study of some graph theoretic concept used in this paper. In Section 3, the notion of m-polar ψ-morphism is introduced on product mFG as a generalization of our usual homomorphism. The action of this morphism is studied and established some results on weak and co-weak isomorphism. d2-degree and total d2-degree of a vertex in product mFGs are defined and studied their properties. Section 4 represents the conclusion of the paper.

2. BASIC DEFINITIONS

Let $V=\{x_0, x_1, ..., x_n\}$ a some set to be called the set of vertices. Let Γ be the multivalued application of the *V* set in itself, $\Gamma : V \to P(V)$, *P(V)* - set of parts *V*. It's called *graph* pair *G=(V, Γ)*. Graph *G* is attached to diagram constructed: each vertices $x_i \in V$ is represented by a point and the connection between two vertices $x_i, x_j \in V$ It is represented by a segment oriented from x_i to x_j, if $x_j \in \Gamma(x_i)$. Ordered pair (x_i, x_j) is called edge and has x_i as the initial extremity and x_j as the final extremity. There is another way to define a graph, namely, using the set of vertices *V* and the array of edges, which we will note with *E* : *G=(V, E)*. We say the two edges of the graph are adjacent if they have a common extremity. The degree of a vertex in G is the number of edges incident with the vertex.

A fuzzy set *A* on a set *X* is characterized by a mapping $\mu : X \to [0,1]$ which is called the *membership function*. A fuzzy set is denoted by $A = \{(x, \mu_A(x)) | x \in X\}$. Let *V* be a finite set nonempty.

A *fuzzy graph* (FG) [17] is a pair *G: (V,σ,μ)* where σ is a fuzzy subset of a set *V* $\sigma : V \to [0,1]$ and μ is a fuzzy relation on σ, $\mu(x,y) \le \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. $\mu : V \times V \to [0,1]$. μ is reflexive and symmetric [17].

[17] A fuzzy graph *H* : *(τ ,ν)* is called a *partial fuzzy sub graph* of *G :(σ,μ)* if $\tau(u) \le \sigma(u)$ for every *u* and $\nu(u, v) \le \mu(u, v)$ for every *u* and *v*. [19] In particular we call a partial fuzzy subgraph H : (τ^*, ν^*) a fuzzy subgraph of *G :(σ,μ)* if $\tau^*(u) = \sigma(u)$ for every *u* in τ^* and $\nu^*(u, v) = \mu(u, v)$ for every arc *(u, v)* in ν^*.

[4] Let G=(A,B) be a fuzzy graph with respect to the sets V and E. The degree of a fuzzy vertex v is $d_G(v) = \sum_{\substack{u \ne v \\ (v,u) \in E}} \mu_B(v,u)$. The total degree of a fuzzy vertex v is

$$d_{TG}(v) = \sum_{\substack{u \ne v \\ (u,v) \in E}} \mu_B(v,u) + \mu_A(v).$$

[4] If every vertex adjacent to vertices has same degree, A fuzzy graph G is called regular. G is called an intuitionistic fuzzy regular graph if $d_G(v) = k, \forall v \in V$ where *k* is a constant.

[4] A fuzzy regular graph G is *k-totally fuzzy regular graph* if each vertex of G has the same total degree *k*.

[4] The *size* of the fuzzy graph G is defined to be S(G) = $\sum_{\substack{u \ne v \\ (u,v) \in E}} \mu_B(u,v)$. G is called *a fuzzy complete graph* if every pair of distinct fuzzy vertices are adjacent.

[5] An intuitionistic fuzzy set (IFS) A in X is given by $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ where $\mu_A : X \to [0,1]$ is called degree of membership and $\nu_A : X \to [0,1]$ is called

degree of non-membership, with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$, $\forall x \in X$.

Let V = $\{v_1, v_2, ..., v_n\}$, E $\subseteq VxV$,

$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ intuitionistic fuzzy subset of V, and
$B = \{(x, \mu_B(x), \nu_B(x)) | x \in X\}$ intuitionistic fuzzy subset of V xV, G=(A,B) is said to be the intuitionistic fuzzy graph if
$\mu_B(x,y) \le \min\{\mu_A(x), \mu_A(y)\}$ and
$\upsilon_B(x,y) \le \max\{\upsilon_A(x), \upsilon_A(y)\}$.

The intuitionistic fuzzy graph H=(C,D) is said to be the intuitionistic fuzzy subgraph of G=(A,B) if C $\subseteq$ A and D $\subseteq$ B.

We consider α,β ∈ [0.1] then, for any IFS set A, $[A]^{(\alpha,\beta)}$
=

$$\begin{cases} \{x \in X | \mu_A(x) \ge \alpha, \nu_A(x) \le \beta, 0 \le \mu_A(x) + \nu_A(x) \le 1\} & \text{if } 0 < \alpha, \beta \le 1 \\ \overline{\{x \in X | \mu_A(x) > 0, \nu_A(x) < 0, 0 \le \mu_A(x) + \nu_A(x) \le 1\}} & \text{if } \alpha = 0, \beta = 0 \end{cases}$$

is called (α,β) -cut as the IFS *A*.

Theorem[25]: Let G=(A,B) be an intuitionistic fuzzy graph with respect to the sets V and E. Let α,β, α_1,β_1 ∈ [0.1], α≤ α_1 and β≥ β_1. Then,($A_{(\alpha 1, \beta 1)}$,$B_{(\alpha 1, \beta 1)}$) is an intuitionistic fuzzy subgraph of ($A_{(\alpha, \beta)}$,$B_{(\alpha, \beta)}$).

Theorem[25]: Let H=(C,D) be an intuitionistic fuzzy subgraph of G=(A,B) and α,β ∈ [0, 1]. Then $H_{(a,b)}$ s an intuitionistic fuzzy subgraph of $G_{(\alpha, \beta,)}$.

Theorem[25]: Let G=(A,B) be an intuitionistic fuzzy graph with respect to the sets V and E. Let α,β, α_1,β_1 ∈ [0,1] and α≤ α_1 , β≥ β_1. Then ($A_{(\alpha 1+, \beta 1+)}$, $B_{(\alpha 1, \beta 1+)}$) is an intuitionistic fuzzy subgraph of ($A_{(\alpha+, \beta,+)}$, $B_{(\alpha+, \beta,+)}$).

Theorem[?]:Let H=(C,D) be an intuitionistic fuzzy subgraph of G=(A,B) and α,β ∈ [0, 1]. Then $H_{(\alpha +, \beta +)}$ is an intuitionistic fuzzy subgraph of $G_{(\alpha +, \beta +)}$.

[8] A complex fuzzy set (CFS) A, defined on a universe of discourse X is an object of the form $A = \{(x, u_A(x)) e^{i\omega_A(x)} | x \in X\}$ where $i = \sqrt{-1}$, $u_A(x) \in [0,1]$, $0 \le \omega_A(x) \le 2\pi$.

[25] A complex intuitionistic fuzzy set (cif-set) A, defined on a universe of discourse X is an object of the form
$A = \{(x, \mu_A(x) e^{i\alpha_A(x)}, \upsilon_A(x) e^{i\beta_A(x)}) | x \in X\}$
where $i = \sqrt{-1}$, $\mu_A(x), \upsilon_A(x) \in [0,1]$,
$0 \le \alpha_A(x), \beta_A(x) \le 2\pi$,
$0 \le \mu_A(x) + \nu_A(x) \le 1$.

[25] A complex intuitionistic fuzzy graph (cif-graph) with an underlaying set V is defined to be a pair G=(A,B) where A is a cif-set on V and B is a cif-set on E $\subseteq VxV$ such

that
$\mu_B(xy) e^{i\alpha_B(xy)} \le \min\{\mu_A(x), \mu_A(y)\} e^{i \min\{\alpha_A(x), \alpha_A(y)\}}$
,
$\upsilon_B(xy) e^{i\beta_B(xy)} \le \max\{\upsilon_A(x), \upsilon_A(y)\} e^{i \max\{\beta_A(x), \beta_A(y)\}}$
for all $x, y \in V$

Through the $[0,1]^m$ set, where *m* is an natural number, we will understand a set endowed with a relationship of order *"≤"* dened by $x \le y$, for each $i = 1,2,...,m$; $p_i(x) \le p_i(y)$ where $x, y \in [0,1]^m$ and $p_i : [0,1]^m \to [0,1]$ is the *i*-th projection mapping.

[27] Let *X* be a non-empty set. A *bipolar fuzzy set E* on *X* is an object having the form $E = \{(x, \mu^P{}_E(x),) \mu^N{}_E(x) | x \in X\}$,where
$\mu^P{}_E : X \to [0, 1]$ denotes a positive membership degree of the elements of *X* and
$\mu^N{}_E : X \to [-1, 0]$ denotes a negative membership degree of the elements of *X*.

[2] By a *bipolar fuzzy graph* of a graph $G^*=(V,E)$ is a pair $G=(V,A,B)$ where $A:V\rightarrow[0,1]$, $A=\left(\mu^P{}_A,\mu^N{}_A\right)$ is a bipolar fuzzy set in V and $B=\left(\mu^P{}_B,\mu^N{}_B\right)$ is a bipolar relation on V, such that $\mu^P{}_B(x,y)\le\min\left\{\mu^P{}_A(x),\mu^P{}_A(y)\right\}$ and $\mu^N{}_B(x,y)\ge\max\left\{\mu^N{}_A(x),\mu^N{}_A(y)\right\}$ for all $(x,y)\in E$. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively.

[10] An *m-polar fuzzy set*, (or a $[0,1]^m$-set) on X is a mapping $A:X\rightarrow[0,1]^m$. The set of all m-polar fuzzy sets on X is denoted by $m(X)$.

In the following G^* represents a crisp graph and $G=(V,A,B)$ represents a product mFG of G^*.

3. m-polar fuzzy graphs

[17] A product *m-polar fuzzy graph* of a graph $G^*=(V,E)$ is a pair $G=(V,A,B)$ where $A:V\rightarrow[0,1]^m$ is an m-polar fuzzy set in V and $B:\tilde{V}^2\rightarrow[0,1]^m$ is an m-polar fuzzy set in $\tilde{V}^2$ such that $p_i\circ B(x,y)\le p_i\circ A(x)\times p_i\circ A(y)$ for all $(x,y)\in\tilde{V}^2$, $i=1,2,\dots,m$ and $B(x,y)=0$ for all $(x,y)\in\tilde{V}^2-E$, ($0=(0,0,\dots,0)$ is the smallest element in $[0,1]^m$).

[11] G is called *strong* if $p_i\circ B(x,y)=p_i\circ A(x)\times p_i\circ A(y)$ for all $(x,y)\in\tilde{V}^2$, $i=1,2,\dots,m$. G is called *complete* if $p_i\circ B(x,y)=p_i\circ A(x)\times p_i\circ A(y)$ for all $(x,y)\in E$, $i=1,2,\dots,m$.

[11] Let $G_1=(V_1,A_1,B_1)$ and $G_2=(V_2,A_2,B_2)$ be two product *mFGs* of the graphs $G_1^*=(V_1,E_1)$ and $G_2^*=(V_2,E_2)$ respectively.

A *weak isomorphism between G_1 and G_2* is a bijective mapping $\phi:V_1\rightarrow V_2$ such that ϕ is a homomorphism and $p_i\circ A_1(x_i)=p_i\circ A_2(\phi(x_i))$ for all $x_i\in V_1$ for $i=1,2,\dots,m$.

A *co-weak isomorphism between G_1 and G_2* is a bijective mapping $\phi:V_1\rightarrow V_2$ such that ϕ is a homomorphism and $p_i\circ B_1(x_i,y_i)=p_i\circ B_2(\phi(x_i,y_i))$ for all $(x_i,y_i)\in\tilde{V}_1^2$ for $i=1,2,\dots,m$.

Ghorai, G., and M. Pal, introduced in [9] the notions of *neighborhood degree of a vertex v*, *degree of a vertex v* and the *closed degree of a vertex v*:

[11] Let $G=(V,E)$ product *mFG* of the graphs $G^*=(V,E)$.

(i) The *neighborhood degree of a vertex v* is defined as $d_N(v)=\left(d^1{}_N(v),d^2{}_N(v),\dots,d^m{}_N(v)\right)$ where $d^i{}_N(v)=\sum_{u\in N(v)}p_i\circ A(u)$, $i=1,2,\dots,m$.

(ii) The *degree of a vertex v* in G is defined by $d_G(v)=\left(d^1{}_G(v),d^2{}_G(v),\dots,d^m{}_G(v)\right)$ where $d^i{}_G(v)=\sum_{\substack{u\ne v\\(u,v)\in E}}p_i\circ B(u,v)$, $i=1,2,\dots,m$. If all the vertices of G have same degree, then G is called regular product *mFG*.

(iii) The *closed degree of a vertex v* is defined by $d_G[v]=\left(d^1{}_G[v],d^2{}_G[v],\dots,d^m{}_G[v]\right)$ where $d^i{}_G[v]=d^i{}_G(v)+p_i\circ A(v)$, $i=1,2,\dots,m$. If each vertex of G has same closed degree, then G is called *totally regular product mFG*.

Also in [11] are introduced the notions of m-polar ψ-morphism in product *mFG*, d_2-

degree, total d_2-*degree*, (2; $\bar{k}$)-regularity and totally (2; $\bar{k}$)-regularity:

The d_2- *degree of a vertex u* in G is $d_2(u) = \left(d^1{}_2(u), d^2{}_2(u), ..., d^m{}_2(u)\right)$ where

$d^i{}_2(u) = \sum_{\substack{u \neq v \\ (u,v) \in E}} p_i \circ B^2(u,v)$, is such that

$p_i \circ B^2(u,v) = \sup\left\{ p_i \circ B(u,u_1) \wedge p_i \circ B(u_1,v) \right\}$, $i = 1,2,...,m$.

The minimum d_2-*degree* of G is denoted as $\delta_2(G) = \left(\delta^1{}_2(G), \delta^2{}_2(G), ..., \delta^m{}_2(G)\right)$ where $\delta^i{}_2(G) = \wedge\left\{ d^i{}_2(u) \middle| u \in V \right\}$.

The maximum d_2-*degree* of G is denoted as $\Delta_2(G) = \left(\Delta^1{}_2(G), \Delta^2{}_2(G), ..., \Delta^m{}_2(G)\right)$ where $\Delta^i{}_2(G) = \vee\left\{ d^i{}_2(u) \middle| u \in V \right\}$.

If $d_2(u) = \bar{k}$ for all $u \in V$ then G is said to be *(2;* $\bar{k}$ *)- regular product mFG*.

The *total* d_2- *degree of a vertex* $u \in V$ is defined as $td_2(u) = \left(td^1{}_2(u), td^2{}_2(u), ..., td^m{}_2(u)\right)$ where

$td^i{}_2(u) = \sum_{\substack{u \neq v \\ (u,v) \in E}} p_i \circ B^2(u,v) + A(u)$.

3. CONCLUSIONS

The fuzzy graph theory is one of the most developing area of research. This paper presents the basic definitions and some properties of product m-polar fuzzy graph introduced by [10]. A product mFG gives more precision compared to the fuzzy and bipolar fuzzy models.

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