MODAL ANALYSIS OF THIN CIRCULLAR PLATES WITH DIFFERENT BOUNDARY CONDITIONS

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ABSTRACT: This paper presents the comparison of the natural frequencies obtained for circular plates of different thicknesses and materials, with diverse boundary conditions. Modal analysis is performed by using the ANSYS simulation software. The results obtained in this study, have targeted the behavior of plates regarding the material, constraining methods and the eigenfrequencies in relation to the vibration modes. According to the results obtained from the modal analysis study, one can conclude that as the thickness of the steel plate decreases, its modal behavior is similar to that of a membrane.

KEY WORDS: plates, membranes, eigenvalues, boundary conditions, modes of vibrations.

1. INTRODUCTION

(TNR 12 pt)

Plates are solid bodies that have one dimension smaller than the other two and can be seen as the materialization of a surface, as a beam is the materialization of a line. A plate is generally defined [1-6] by the shape and dimensions of the median surface, and each point is considered normal if it defines the thickness, *h*, on either side of the median surface, by the values *h/2*. Numerous structures are made up of plates of different shapes and sizes, making the analysis of their dynamic response of particular importance in mechanical engineering [7-14].

It is known that plates sustain transverse loading using bending stresses and membranes can only sustain tensile loads, leading to the assumption that by analyzing the vibrational behavior the transition between the membrane and plate behavior can be depicted [15].

The development of the modern aeronautical industry has generated a new wave in the field of plate analysis. The problem of plates and plate vibrations has been investigated by many scientists [16-18] in the field of engineering. E. Reissner [19] developed a rigorous theory in which he considers plate deformation to be caused by cross-sectional shear forces.

In the present paper, we have performed modal studies to evaluate the influence of thickness, boundary conditions, and material on the dynamic response of circular plates [20-24].

The modal analysis is performed using the ANSYS simulation software, and there have been analyzed four different thicknesses for the circular plates, using three boundary scenarios: clamped 100%, 75% and 40 % on the exterior edge.

The overall dimensions in [mm] for these plates are: diameter 1000 and thickness from 0.001 to 1 mm. For the first four studies, the material used is structural steel, and for the last four studies, the plates are considered to be made of neoprene rubber.

The main objective of this study is to point out different vibrational behavior of thin plates in regards to their thickness, especially the transition between plate and membrane behavior.

Modal analysis is important in the study of plate-type structures and specifically in the case of circular plates having different boundary conditions. Recently, the use of plate-type structures in aerospace, mechanical, and automotive engineering has made this issue a key point.

2. MODAL ANALYSIS

The research aims to find the vibrational behavior of thin circular plates, by performing modal analysis for 4 steel plates and 4 neoprene rubber membranes having different thicknesses and boundary conditions. The thin geometries are generated with the help of the ANSYS modeler using shell elements, as shown in Figure 1. Shell elements isolate surface deformation from normal-direction deformation, allowing for a simple and efficient simulation of a thin structure. Shells are normally constructed by meshing a surface reflecting the structure's position and shape and combining it with section data to describe its thickness and attributes.

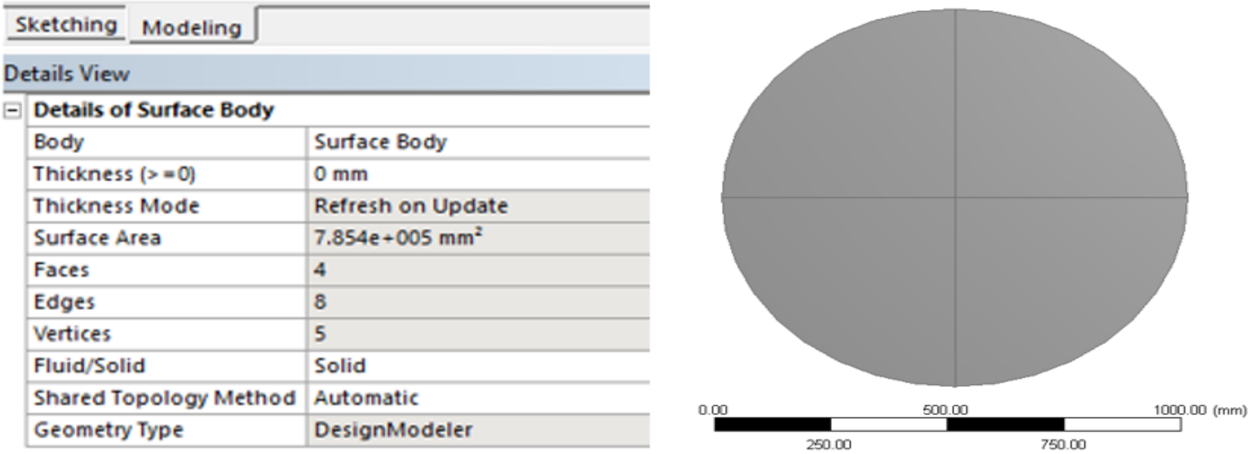


Figure 1. Geometry of the plate shell

For the modal analysis, the simulations have been carried out at a temperature of 22°C, at which we have determined the first 30-vibration mode shapes for the analyzed cases.

The studies are carried out using 4 different plate thicknesses: t_1=1 mm, t_2=0,1 mm, t_3=0,01 mm and t_4=0,001mm with three boundary scenarios resulting in 12 scenarios for the steel plate and 12 for the neoprene membrane, as shown in Table 1.

Table 1. Table caption

Material	Thickness	Material properties		
		Mass Density [kg/m³]	Young modulus [N/m²]	Poisson ratio [-]
S355JR	1	7850	$2 \cdot 10^{11}$	0.3
	0.1			
	0.01			
	0.001			
Neoprene rubber C-40	1	2000	$2{,}06 \cdot 10^{10}$	0.46
	0.1			
	0.01			
	0.001			

The applied boundary conditions are set to clamp the exterior circumference of the circular plate and the area in which they are applied for every scenario is indicated in Figure 2.

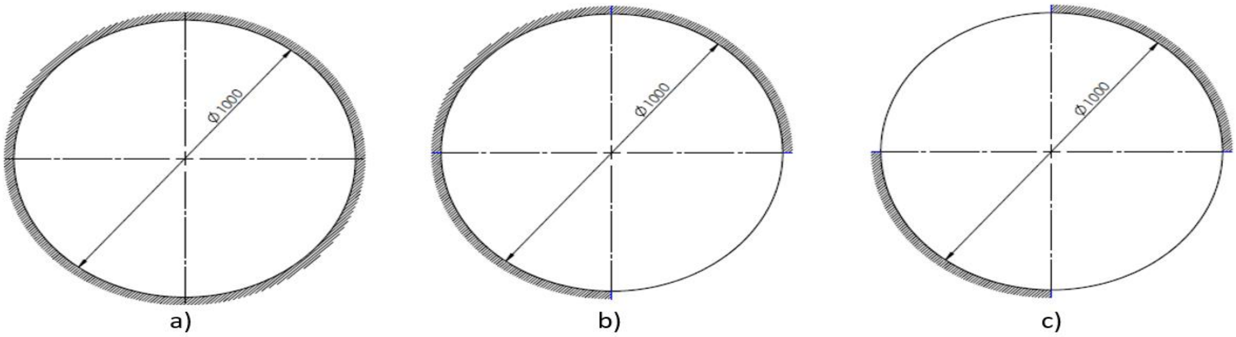


Figure 2. Imposed boundary conditions; a) 100 % clamped, b) 75% clamped, c) 50% clamped

After the geometry is defined, the constraints are imposed, and the model is ready for meshing. It was found that to be more accurate in the analysis, it is necessary to use a mesh with a maximum edge size of 5 mm, as shown in Figure 3.

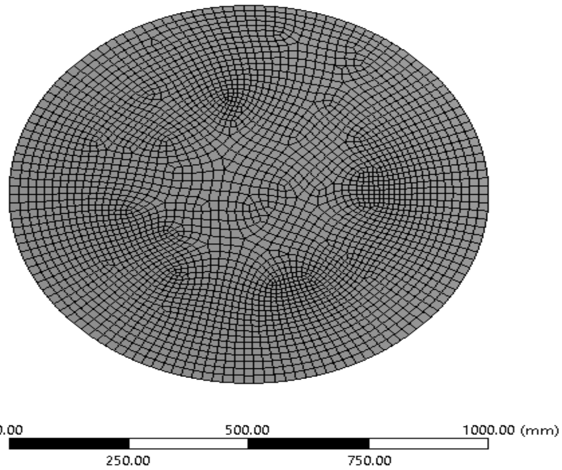


Figure 3. A zoom in on the meshed geometry of the circular plate

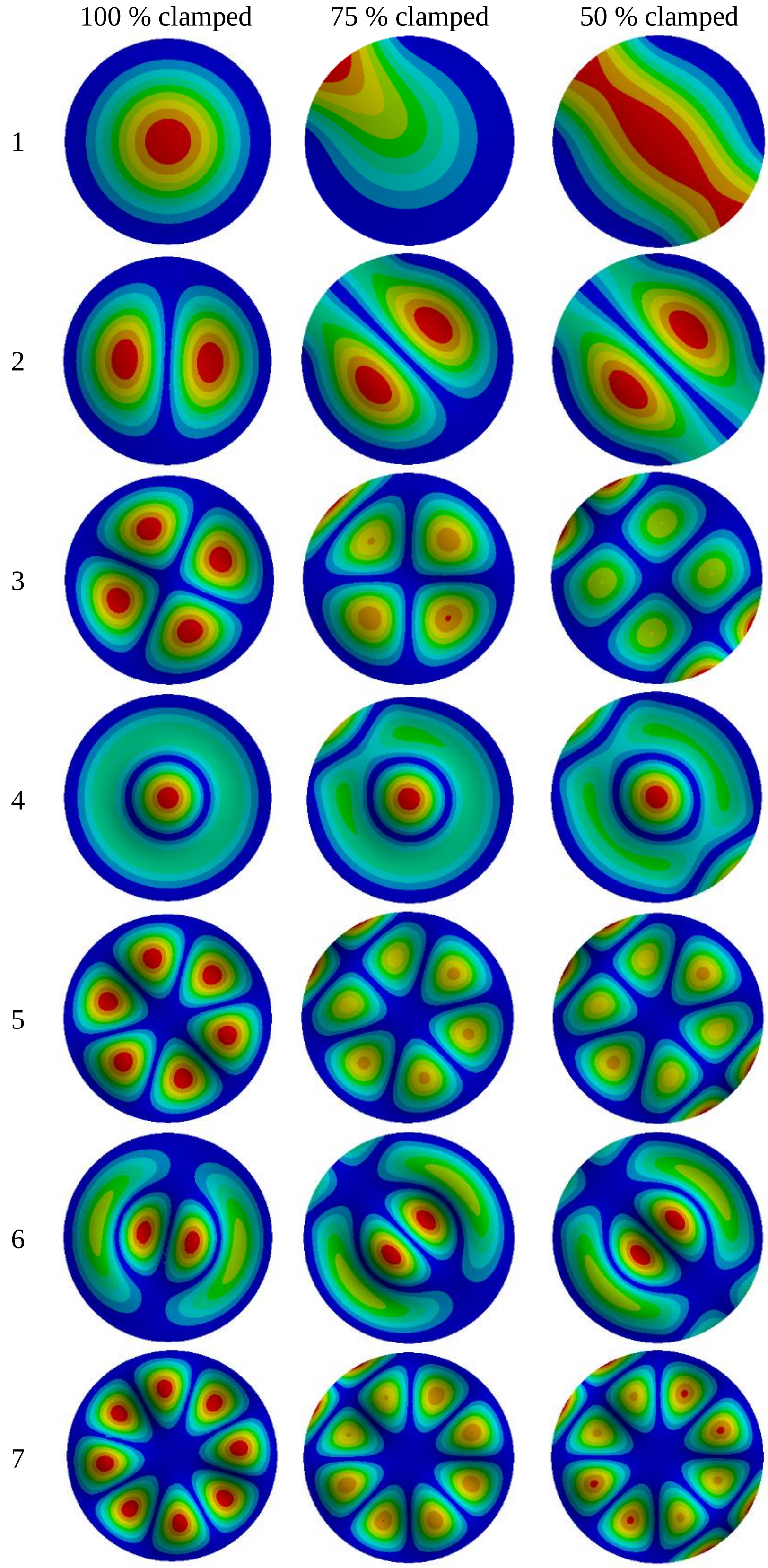
After completing the preprocessing stage, the maximum modes to find is set to 15, and the simulation is run. All results can be evaluated in the postprocessing stage with the help of the ANSYS Mul-tiphysics interface. The natural frequencies are depicted and presented in Table 2 for the scenarios containing the steel plate.

Table 2. Obtained natural frequency values for the steel plate

Mode no.	100 % clamped				75 % clamped				50 % clamped			
	Thickness [mm]				Thickness [mm]				Thickness [mm]			
	1	0.1	0.01	0.001	1	0.1	0.01	0.001	1	0.1	0.01	0.001
1	9.944	0.994	0.099	0.010	7.811	0.782	0.078	0.008	7.295	0.730	0.073	0.007
2	20.717	2.072	0.207	0.021	10.691	1.070	0.107	0.011	8.585	0.860	0.086	0.009
3	34.005	3.401	0.340	0.034	20.634	2.064	0.206	0.021	19.577	1.958	0.196	0.020
4	38.867	3.887	0.389	0.039	34.123	3.412	0.341	0.034	24.478	2.451	0.245	0.025
5	49.816	4.982	0.498	0.050	35.106	3.511	0.351	0.035	26.953	2.698	0.270	0.027
6	59.588	5.959	0.596	0.060	48.673	4.870	0.487	0.049	36.107	3.611	0.361	0.036
7	68.097	6.810	0.681	0.068	52.579	5.261	0.526	0.053	47.697	4.773	0.477	0.048
8	82.972	8.298	0.830	0.083	59.964	5.997	0.600	0.060	51.900	5.196	0.520	0.052
9	87.653	8.766	0.877	0.088	69.495	6.950	0.695	0.070	58.813	5.882	0.588	0.059
10	88.877	8.889	0.889	0.089	82.514	8.252	0.825	0.083	69.994	7.000	0.700	0.070
11	109.130	10.914	1.091	0.109	86.688	8.670	0.867	0.087	70.987	7.100	0.710	0.071
12	112.100	11.211	1.121	0.112	90.121	9.013	0.901	0.090	82.491	8.250	0.825	0.082

13	118.520	11.853	1.185	0.119	108.210	10.823	1.082	0.108	86.374	8.638	0.864	0.086
14	118.650	11.867	1.187	0.119	109.430	10.944	1.094	0.109	91.296	9.137	0.914	0.091
15	137.790	13.781	1.378	0.138	113.460	11.347	1.135	0.113	91.742	9.175	0.918	0.092

Next, vibration modes for the 3 types of boundary conditions are shown for the plate with 1 mm thickness.



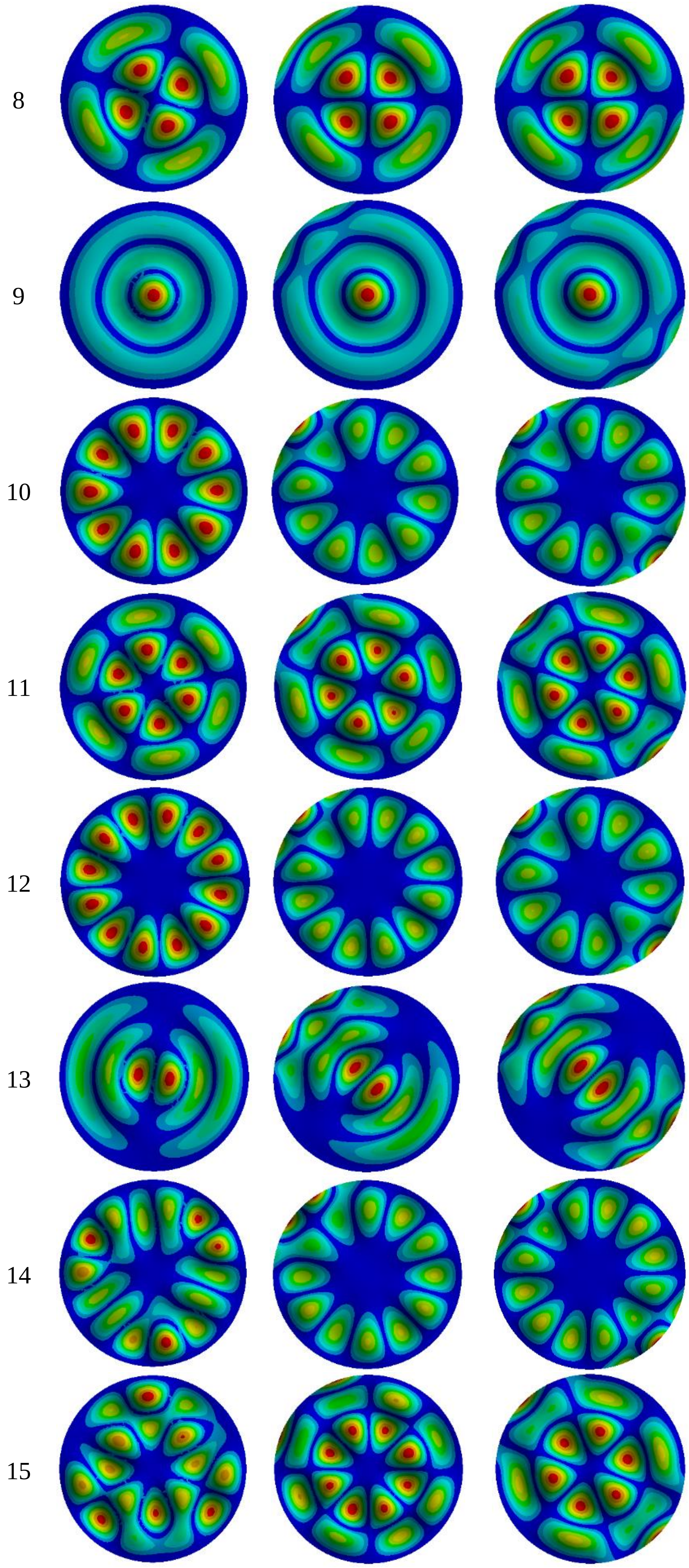


Figure 4. The first 15 modes of vibration for the plate with 1

mm thickness

Furthermore, modal analysis was performed using the same thickness, size, and boundary conditions for a circular membrane made of neoprene rubber, having its mechanical properties presented in Table 1. The obtained eigenfrequencies are shown in Table 3

Table 3. Natural frequency values obtained for the neoprene membrane

Mode no.	100 % clamped				75 % clamped				50 % clamped			
	Thickness [mm]				Thickness [mm]				Thickness [mm]			
	1	0.1	0.01	0.001	1	0.1	0.01	0.001	1	0.1	0.01	0.001
1	0.681	0.068	0.007	0.001	0.518	0.052	0.005	0.001	0.489	0.049	0.005	0.000
2	1.418	0.142	0.014	0.001	0.711	0.071	0.007	0.001	0.554	0.056	0.006	0.001
3	2.328	0.233	0.023	0.002	1.407	0.141	0.014	0.001	1.335	0.134	0.013	0.001
4	2.660	0.266	0.027	0.003	2.333	0.233	0.023	0.002	1.601	0.160	0.016	0.002
5	3.410	0.341	0.034	0.003	2.370	0.237	0.024	0.002	1.741	0.174	0.017	0.002
6	4.079	0.408	0.041	0.004	3.270	0.327	0.033	0.003	2.411	0.241	0.024	0.002
7	4.661	0.466	0.047	0.005	3.500	0.350	0.035	0.004	3.196	0.320	0.032	0.003
8	5.679	0.568	0.057	0.006	4.081	0.408	0.041	0.004	3.464	0.346	0.035	0.003
9	6.000	0.600	0.060	0.006	4.715	0.472	0.047	0.005	4.020	0.402	0.040	0.004
10	6.083	0.608	0.061	0.006	5.648	0.565	0.056	0.006	4.751	0.475	0.048	0.005
11	7.469	0.747	0.075	0.007	5.909	0.591	0.059	0.006	4.780	0.478	0.048	0.005
12	7.672	0.767	0.077	0.008	6.128	0.613	0.061	0.006	5.622	0.563	0.056	0.006
13	8.112	0.811	0.081	0.008	7.400	0.740	0.074	0.007	5.906	0.591	0.059	0.006
14	8.121	0.812	0.081	0.008	7.465	0.747	0.075	0.007	6.036	0.604	0.060	0.006
15	9.430	0.943	0.094	0.009	7.719	0.772	0.077	0.008	6.198	0.620	0.062	0.006

If in Figure 4 the mode shapes in the form of images are presented, after the study of the frequency type from the modal analysis, the results regarding the frequency values are compared to those obtained for the steel plate. The plotted frequencies for the steel plates are shown in Figure 5 for all thicknesses used.

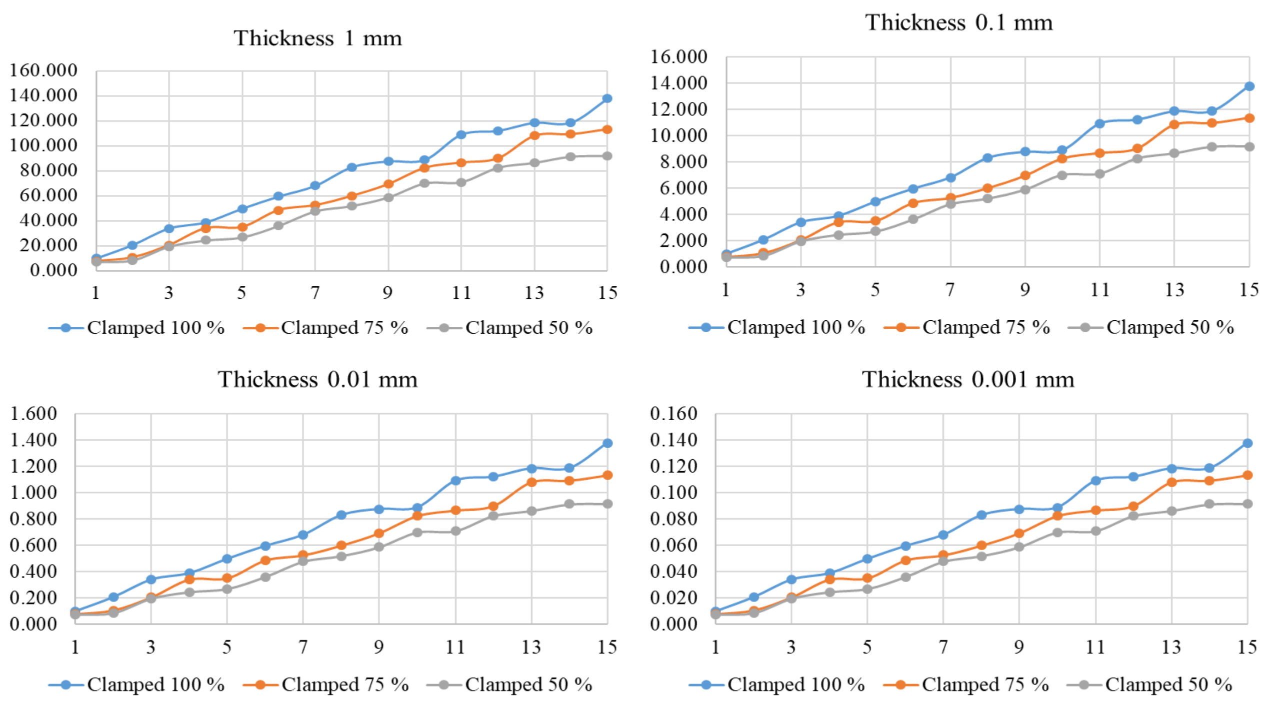


Figure 5. Plotted frequency curves for the steel circular plates

To compare the results, the frequencies for the neoprene membranes are plotted in Figure 6 for all thicknesses and boundary conditions used.

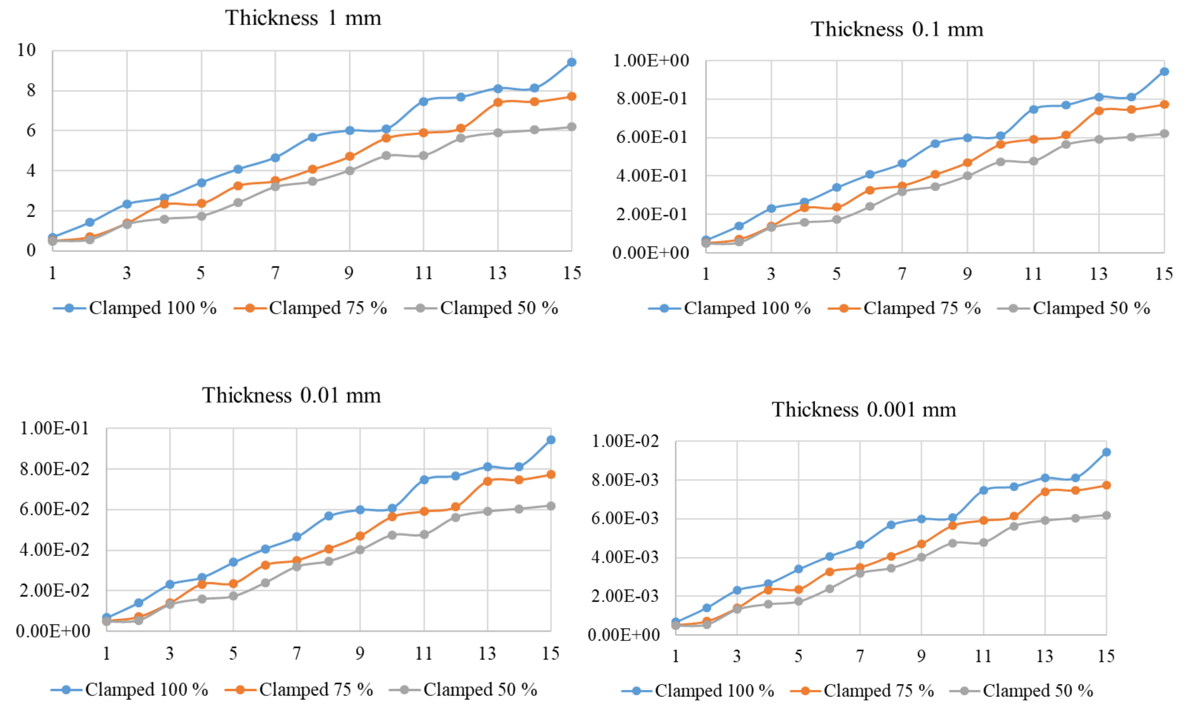


Figure 6. Plotted frequency curves for the neoprene circular membranes

From the realized study, using the ANSYS software, the results show that even if the material is different, for thin plates, the trend of the eigenfrequencies is not changed.

3. CONCLUSION

According to the results obtained from the modal analysis study, one can conclude that the steel plate differs from the neoprene one only by the value of their eigenfrequencies; the general behavior for both plates being the same. Based on the results obtained, certain mathematical relations regarding the behavior of the plate can be interpreted according to the modes of vibration.
Finally, it should be pointed out that using model update techniques, an estimation of the behavior of thin plates can be made.

REFERENCES

[1] Bratu, P. *Vibraţiile sistemelor elastice*, Ed. Tehnică, Bucureşti, 2000.
[2] Haţiegan C., Gillich G. R., Răduca E., Nedeloni M. D., Cîndea L., *Equation of Motion and Determining the Vibration Mode Shapes of a Rectangular Thin Plate Simply Supported on Contour Using MATLAB*, Analele Universităţii "Eftimie Murgu", Fascicula de Inginerie, Anul XX, Nr. 1, Reşiţa, 2013.
[3] Cornel Haţiegan, Edwald-Viktor Gillich, Ovidiu Vasile, Marian-Dumitru Nedeloni, Ioan Pădureanu, Finite *Element Analysis of thin plates clamped on the rim of different geometric forms. Part I: Simulating the* Vibration Mode Shapes and Natural Frequencies, Romanian Journal of Acoustics & Vibration, Vol.XII, Nr.1, 2015
[4] Cornel Haţiegan, Edwald-Viktor Gillich, Ovidiu Vasile, Marian-Dumitru Nedeloni, Marcel Jurcu, Petru Magheţi, *Finite Element Analysis of thin plates clamped on the rim of different geometric forms. Part II: The Absolute and Relative Variation of Natural Frequencies*, Romanian Journal of Acoustics & Vibration, Vol.XII, Nr.1, 2015
[5] L Barboni Haţiegan, C Haţiegan, GR Gillich, CO Hamat, O Vasile, MD Stroia, *Natural frequencies of thin rectangular plates clamped on contour using the Finite Element Method*, IOP Conference Series: Materials Science and Engineering, Hunedoara 2017, Volumul 294, Nr.1, 2018.
[6] Hatiegan C., Stroia M. D., Racoceanu C., Iancu V: *Using Finite Element Method to Determin Eigenmodes of Two L- Shaped Platens Assembly Embedded on a Surface. Part I*, Scientific Conference with

International Participation "CONFERENG 2018", Targu-Jiu, November 23-24, Annals of Constantin Brâncuşi University of Târgu-Jiu - Engineering Series, No. 2, 2018.
[7] Chen, J. C., Garba, J. A. On-Orbit *Damage Assessment for Large Space Structures*, AIAA Journal, Vol. 26, No. 9, 1119-1126, 1988.
[8] Haţiegan C., Gillich G. R., Gillich N., Tufoi M., Răduca M., *Modal Analysis of Thin Plates with Damage Simply Supported on all Edges*, Analele Universităţii "EFTIMIE MURGU", Fascicula de Inginerie, Anul XX, Nr. 1, Reşiţa, 2013.
[9] Marius TUFOI, Gilbert-Rainer GILLICH, Zeno-Iosif PRAISACH, Zoltan-Iosif KORKA, Cornel HATIEGAN, *Modal Analysis of a Butterfly Valve with Different Stiffening Elements*, Romanian Journal of Acoustics & Vibration, Vol.XIII, Nr.1., 2016
[10] Haţiegan C., Nedeloni M. D., Tufoi M., Protocsil C., Răduca M., *Modal Analysis Of Natural Frequencies And Mass Participation Coefficients Of Simply Supported Thin Plates With Damages*, Constanţa Maritime University Annals, Nr. XIV, Vol. 19, 2013
[11] Marius Tufoi, Gilbert-Rainer Gillich, Zeno-Iosif Praisach, Jean Loius Ntakpe, Cornel Hatiegan, *An Analysis of the Dynamic Behavior of Circular Plates from a Damage Detection Perspective*, Romanian Journal of Acoustics & Vibration, Vol.X, Nr.1, 2014.
[12] Hatiegan Cornel, Stroia Mihaela-Dorica, Popescu Cristinel, Raduca Eugen, Racoceanu Cristinel, *Study of Defects for a Rectangular Plate Freely Supported on Contour*, Analele Universităţii Constantin Brâncuşi din Târgu-Jiu - Seria Inginerie, Nr. 3, 2020.
[13] Haţiegan C, Stroia M D, Popescu C, Răduca E, Racoceanu C, *Study of Defects for a Rectangular Plate Freely Supported on Contour*, Scientific Conference with International Participation "CONFERENG 2019", Targu-Jiu, November 22-23, Annals of Constantin Brâncuşi University of Târgu-Jiu - Engineering Series, No. 3, 2019.
[14] Elishakoff, I. Sternberg, A. *Vibration of Rectangular Plates with Edge beams*, Acta Mechanica 36, 195-212, 1980.
[15 Leissa, A.W. *Singularity Considerations in Membrane, Plate, and Shell Behaviours*. Int. J. Solids and Structures 98, 3341-3351, 2001.
[16] Timoshenko, S. Woinowsky-Krieger, S. *Theory of Plates and Shells*; McGraw-Hill: New York, NY, USA, 1941; Volume 148, ISBN 0070647798.
[17] Zhou, D. Ji T. *Free vibration of rectangular plates with continuously distributed spring-mass*, International Journal of Solids and Structures, vol. 43, 2006, 6502-6520.
[18] Jiu, H.W. Liu, A.Q. Chen, H.L. Exact *Solutions for Free-Vibration Analysis of Rectangular Plates Using Bessel Functions*, Journal of Applied Mechanics, vol. 74, 2007, 1247- 1251.
[19] Mindlin, R. D. An Introduction to the *Mathematical Theory of Vibrations of Elastic Plates*, World Scientific Publishing Co. Pte. Ltd, 2006.
[20] Tang, Y.J.; Chen, J.; Huang, Y.B.; Li, D.C.; Wang, S.S.; Li, Z.H.; Zhang, W.D. *Ultra-sensitive, highly reproducible film stress characterization using flexible suspended thin silicon plates and local curvature measurements*, Micromech. Microeng. 2007, 17, 1923–1930.
[21] Cheng, T.; Dai, C.; Gan, R.Z. *Viscoelastic properties of human tympanic membrane*. Ann. Biomed. Eng. 2007, 35, 305–314.
[22] Tufoi, M. Gillich, G.R. Praisach, Z.I., Iancu, V. Furdui, H. *About the Influence of Temperature Changes on the Natural Frequencies of ClampedClamped Euler-Bernoulli Beams*, Romanian Journal of Acoustics and Vibration, year 11, no. 2, 2014, 84-87.
[23] Tufoi, M. Hatiegan, C. Vasile, O., Gillich, G.R. *Dynamic Analysis of Thin Plates with Defects by Experimental and FEM Methods*, Romanian Journal of Acoustics and Vibration, year 10, no. 2, 2013, 83-88.
[24] Hatiegan, C. Gillich, G.R. Răduca, M., Răduca, E. Budai A.M. Muntean, F. *Finite Element Analysis of Natural Frequencies and Mass Participation Coefficients for Thin Plates With Defects*, Scientific Bulletin of "Politehnica" University of Timisoara, tom 57, no. 71, 2012, 43-53.