THE EFFECT OF THE INTERMEDIATE SUPPORT POSITIONS ON THE DYNAMIC BEHAVIOR OF A CONTINUOUS BEAM WITH 3 SPANS AND FREE ENDS

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ABSTRACT: The paper presents the modifications of the eigenvalues for a continuous beam with three spans, free at the ends. The analysis model studied is based on Euler-Bernoulli's theory of the continuous medium. By imposing the boundary conditions for each characteristic point of the beam and solving the resulting system of equations the characteristic relationship was established analytically, whose solutions represent the eigenvalues for the calculation of natural frequencies and the vibration mode shapes. For the first six vibration modes, the modifications of the eigenvalues are illustrated in 3D format, considering that the intermediate supports can occupy any position on the beam.

KEY WORDS (TNR 10 pt Bold): continuous beam, eigenvalues, natural frequency.

1. INTRODUCTION

A multi span beam supported on hinges is known as continuous beam [1]. A continuous beam is a statically indeterminate multi-span beam on hinged support [2]. The end spans may be clamped, free or simply supported. In contrast to a simply supported beam, which has supports at each end, a continuous beam is much stiffer and stronger [3].
A structure which have mass and elasticity is capable of oscillatory motion [4]. From the point of view of dynamics, the phenomenon of vibrations involves an alternative exchange of potential energy with kinetic energy and kinetic energy with potential energy [5].
In this paper, the authors describe the evolution of the eigenvalues and natural frequencies for a three-span beam free at both ends. Two intermediate support hinges have been added and iteratively moved from one beam end to the other.
The beam is homogeneous and isotropic, that means the beam material follows Hooke’s law. The eigenvalues can be used to get the continuous beam natural frequencies, of interest being the first six weak-axis bending modes are considered.

2. THE FREQUENCY EQUATION

It is considered a continuous beam, free at the ends with two intermediate supports (fig. 1).
To determine the eigenvalues of a beam with two intermediate supports, the distances between each pair of them are considered as a separate beam.
The beam is considered of normalized length, respectively $l_1 + l_2 + l_3 = 1$ and having constant cross section.

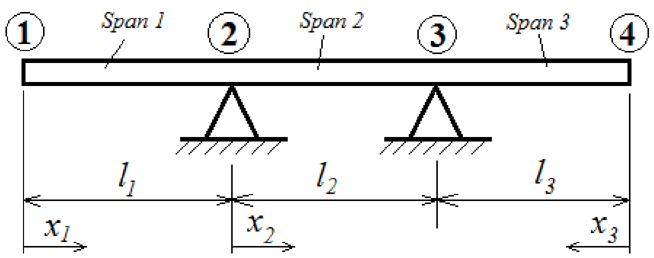

Figure 1.Continuous beam with 3 spans, free at the ends.

Based on the Euler-Bernoulli theory, for each characteristic point on the beam (1, 2, 3, 4), the following boundary conditions can be written:

$$\begin{cases} \langle 1\rangle \begin{cases} W_1''(0)=0 \\ W_1'''(0)=0 \end{cases} \\ \langle 2\rangle \begin{cases} W_1(l_1)=0 \\ W_2(0)=0 \\ W_1'(l_1)=W_2'(0) \\ W_1''(l_1)=W_2''(0) \end{cases} \\ \langle 3\rangle \begin{cases} W_2(l_2)=0 \\ W_3(l_3)=0 \\ W_2'(l_2)=-W_3'(l_3) \\ W_2''(l_2)=W_3''(l_3) \end{cases} \\ \langle 4\rangle \begin{cases} W_3''(0)=0 \\ W_3'''(0)=0 \end{cases} \end{cases} \quad \textbf{(1)}$$

where,

$$W_i(x_i)=A_i\sin(a_n x_i)+B_i\cos(a_n x_i)+ \\ +C_i\sinh(a_n x_i)+D_i\cosh(a_n x_i) \quad \textbf{(2)}$$

is the normal mode of span;
i = 1, 2, 3 represents the number of spans;
n = n^{th} vibration mode number;
A_i, B_i, C_i, D_i are the integration coefficients;
a_n - the eigenvalues;
$x_1\in[0,\ l_1],\ x_2\in[0,\ l_2],\ x_3\in[0,\ l_3]$.

By solving the system (1) and introducing the notations:

$$\begin{cases} Z_{11}=2\dfrac{1+\cos(al_1)\cosh(al_1)}{\cos(al_1)+\cosh(al_1)} \\ Z_{12}=-2\dfrac{\cos(al_1)\sinh(al_1)-\sin(al_1)\cosh(al_1)}{\cos(al_1)+\cosh(al_1)} \end{cases} \quad \textbf{(3)}$$

$$\begin{cases} Z_{21}=1-\cos(al_2)\cosh(al_2) \\ Z_{22}=\cos(al_2)\sinh(al_2)-\sin(al_2)\cosh(al_2) \\ Z_{23}=2\sin(al_2)\sinh(al_2) \end{cases} \quad \textbf{(4)}$$

$$\begin{cases} Z_{31}=2\dfrac{1+\cos(al_3)\cosh(al_3)}{\cos(al_3)+\cosh(al_3)} \\ Z_{32}=-2\dfrac{\cos(al_3)\sinh(al_3)-\sin(al_3)\cosh(al_3)}{\cos(al_3)+\cosh(al_3)} \end{cases} \quad \textbf{(5)}$$

which are constant for a certain location of the intermediate supports, the frequency equation (6) is obtained:

$$(Z_{12}\cdot Z_{21}+Z_{11}\cdot Z_{22})\cdot Z_{32}+ \\ +(Z_{12}\cdot Z_{22}+Z_{11}\cdot Z_{23})\cdot Z_{31}=0 \quad \textbf{(6)}$$

The solutions (6) represent the eigenvalues a_n for a continuous beam with three spans.
Expression (6) is the generalized form for calculating eigenvalues for a beam with three openings.
For its use, only the boundary conditions at the ends of the beam must be taken into account, respectively the relations (3) and (5) specified for each type of support must be used. In this situation, relations (3) and (5) represent the free end of the beam.
Relationship (4) is the same, regardless of how the beam is supported at the ends.

2. THE EFFECT OF INTERMEDIATE SUPPORT POSITIONS

In order to analyze the effect of the location of the intermediate supports on the dynamic behavior of the beam, it was considered that the intermediate supports can be in any location on the beam.
This means $l_1\in(0,\ 1),\ l_2\in(0,\ 1),\ l_3=1-(l_1+l_2)$.

The results obtained for the first six eigenvalues of the vibration mode are illustrated in 3D format in figure 2, for a continuous beam with three spans and free at the ends.

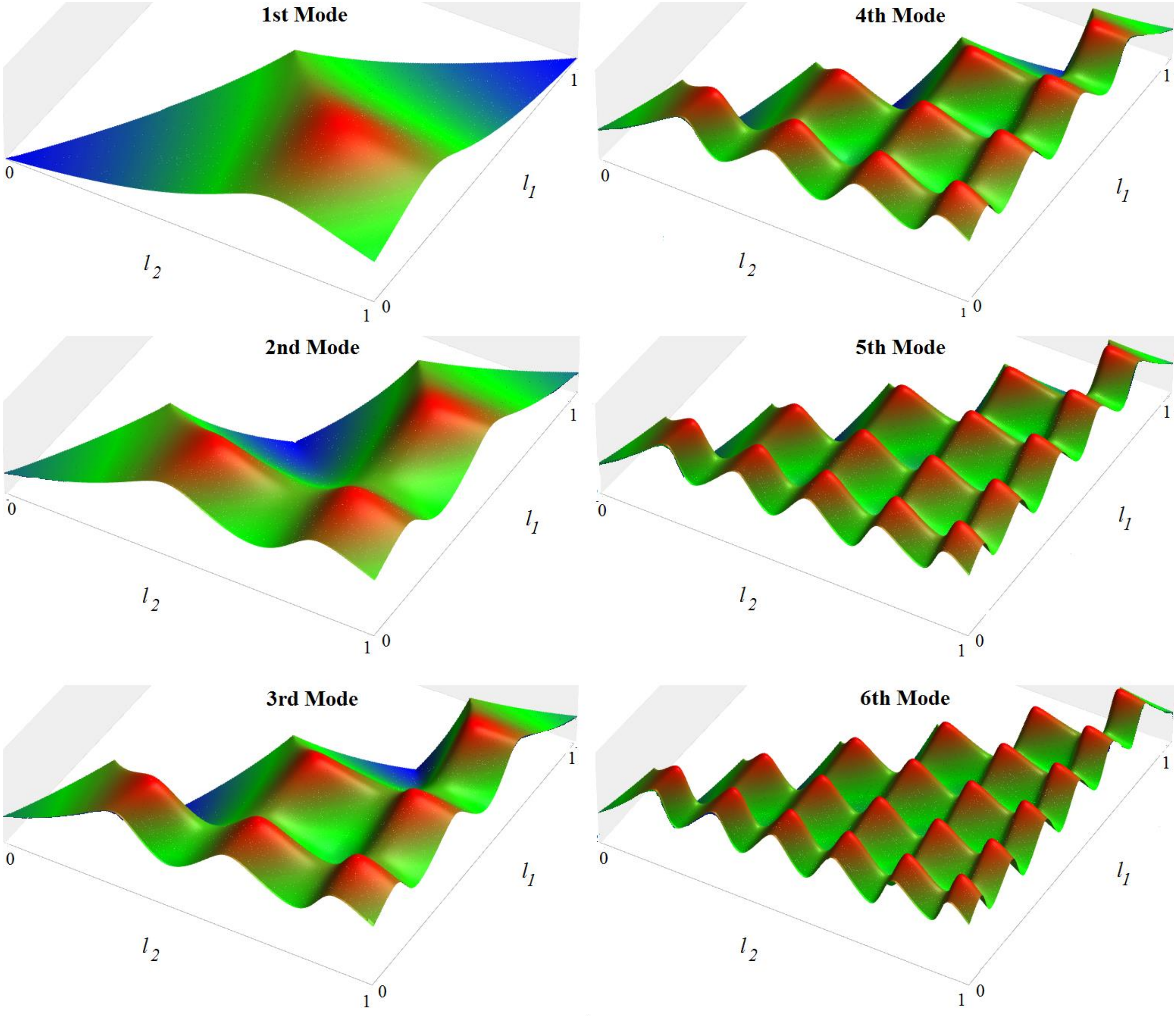

Figure 2. Eigenvalues for the first six vibration mode for a continuous beam with 3 spans, free at ends, and intermediate supports moved from one end to the other end.

The obtained surfaces presented in figure 2 gives us a general image on the evolution of the eigenvalues, respectively on the evolution of the natural frequencies, depending on the position that the intermediate supports can have.

It is known from literature that the natural frequencies for the n^{th} vibration mode are given by the equation:

$$f_n = \frac{a_n^2}{2\pi}\sqrt{\frac{E \cdot I}{m \cdot L^4}} \quad \textbf{(7)}$$

where:

f_n [Hz] is the natural frequency;
E [N/m^2] is the elasticity modulus;
I [m^4] is the moment of inertia;
m [kg] is the beam mass;
L [m] is the beam length.

Relationship (7) allows us to determine the natural frequencies for the healthy continuous beam.

Knowing the eigenvalues and natural frequencies for the undamaged beam, we can apply relationships related to the severity of the transverse crack and the local bending moment in the slice around the crack and we can predict the frequencies of the damaged beam [6, 7].

3. CONCLUSION

Knowing the eigenvalues can be determined analytically the natural frequencies for continuous beams with three spans.
From the analysis of figure (2) for the first six vibration modes, it can be seen that depending on the location of the intermediate supports, the eigenvalues influence the values of the natural frequencies.
Knowing the eigenvalues, one can analytically determine the modal shape functions. Having the modal shape functions, it is very easy to obtain the curvature of the modal shape function, respectively the deformation energy [8].
From the literature, it is known that the deformation energy is directly proportional to the eigenfrequencies of the damaged beam [9, 10], which allows us to calculate the eigenfrequencies of the damaged beam for any location of the damage on the beam.

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