STUDY ON THE USE OF IFS IN MEDICAL DIAGNOSIS

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ABSTRACT: *In this article we study Sanchez's, De et al. [3], Szmidt & Kacprzyk [12] and Philip [6] method for medical diagnosis using the notion of IFS with Fuzzy logic method.*

KEY WORDS: *Intuitionistic Fuzzy Set (IFS); Intuitionistic fuzzy relation (IFR); Fuzzy logic (FL); Intuitionistic medical diagnosis*

1. INTRODUCTION

As there are countless situations in which the data needed to make a diagnosis are rather vague, the possibility of using intuitionistic fuzzy set (IFS) to establish a diagnosis has been observed.

Using the notion of IFS, Sanchez ([8], [9]) uses the method of fuzzy logic to diagnose various diseases.

By generalizing the method of Sanchez, De et al. [3] proposed a method of applying IFS theory in establishing a diagnosis.

Szmidt and Kacprzyk [12] propose a new method for establishing a diagnosis using intuitive fuzzy sets, looking for the shortest distance [cf. Szmidt and Kacprzyk [10, 11]] between symptoms. They highlight the advantages of such a method, compared to the method presented in [3] by De, Biswas and Roy, in which the max-min-max composition rule was applied.

In [5], Own replaces IFS sets with Type II Fuzzy sets.

In [6], Philip introduce a new method which is simple to compute and improves upon the method due to De et al.

In [4] Kozae et al propose that IFS be used in the diagnosis of Covid 19.

2. PRELIMINARIES

Starting from the notion of fuzzy set,

$A=\{(x,\mu_A(x))|x\in X\}$, K.T. Atanassov [1] generalizes and introduces the notion of An intuitionistic fuzzy set (IFS) *A* in *X*

$A=\{(x,\mu_A(x),\nu_A(x))|x\in X\}$

where $\mu_A : X \to [0,1]$ is called degree of membership and $\nu_A : X \to [0,1]$ is called degree of non-membership, with the condition

$0 \le \mu_A(x)+\nu_A(x) \le 1, \forall x \in X$.

In [2], Atanassov names degrees of indeterminacy (or intuitionistic fuzzy index) of *x* to *A*, for each *A* in *X* ,the numbers:

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x), \ \forall x \in X .$$

Or each $x \in X$,

$$0 \le \mu(x)+\nu(x) \le 1$$

$A \subset B$ iff $\forall x \in X$
$[\mu_A(x) \leq \mu_B(x)\ and\ \nu_A(x)\ \leq \nu_B(x)]$

$A \supset B$ iff $\forall x \in X\ \ B \subset A$

$A = B$ iff $\forall x \in X$
$[\mu_A(x) = \mu_B(x)\ and\ \nu_A(x) = \nu_B(x)]$

$$\overline{A} = \left\{\left(x, \nu_A(x), \mu_A(x)\right) | x \in X\right\}$$

$$A \cap B =$$
$$\left\{\left(x, \min\left(\mu_A(x), \mu_B(x)\right), \max\left(\nu_A(x), \nu_B(x)\right)\right) | x \in X\right\}$$
$$A \cup B =$$
$$\left\{\left(x, \max\left(\mu_A(x), \mu_B(x)\right), \min\left(\nu_A(x), \nu_B(x)\right)\right) | x \in X\right\}$$

3. SANCHEZ'S APPROACH FOR MEDICAL DIAGNOSIS

In [7], Samuel and Balamurugan study Sanchez's approach for Medical Diagnosis using the notion Fuzzy logic and Intuitionistic Fuzzy Set. They define

$$f(x_1/x_2) = \frac{f(x_1)}{\max\{f(x_1), f(x_2)\}}$$

where x_1 and x_2 be variables defined on universe X, and we define a pair wise function $f(x_1)$ as the membership function of x_1 and $f(x_2)$ as the non-membership function of x_2.

$$\oplus(A) = \left\{\left(x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2}\right) | x \in X\right\}$$

Medical diagnosis[7]:
Suppose S is a set of symptoms, D is a set of diagnosis and P is a set of patients. Let M_1 be an Intuitionistic Fuzzy Relation M_1 $(P \to S)$ and M_2 from the set of patients to the set of symptoms S, i.e., M_2 $(S \to D)$

$$M_1 = A \cap B$$
$$M_2 = A \cup B$$
$$W_3 = \oplus(A)$$
$$W_4 = f(x_1/x_2)$$

Algorithm:
Step1: M_1 $(P \to S)$ and M_2 $(S \to D)$ IFRs are applied for the initial data and the result is a relationship $(P \to D)$.
Step2: The formula W_3 is applied to the results of type $(P \to D)$.
Step3: The formula W_4 applies to the data obtained in step 2.
Step4: Finally, we selected minimum value to the data obtained in step 3, and then we conclude that the patient P_i is suffering from the disease D_r .

4. DE, BISWAS AND ROY'S APPROACH FOR MEDICAL DIAGNOSIS

In [3], De, Biswas and Roy's study the Sanchez's approach for medical diagnosis and extend this concept with the notion of intuitionistic fuzzy set theory (which is a generalization of fuzzy set theory). In the following I will present this study.
Definition 4.1 [3] If A is an IFS of X, the *max-min-max composition* of the IFR R$(X \to Y)$ with A is an IFS B of Y denoted by $(B = R \circ A)$ and is defined by the membership function

$$\mu_{R \circ A}(y) = \bigvee_x [\mu_A(x) \wedge \mu_R(x, y)]$$

and the non-membership function

$$\nu_{R \circ A}(y) = \bigwedge_x [\nu_A(x) \vee \nu_R(x, y)]$$

$\forall y \in Y$, where $\vee = max$ and $\wedge = min$

Definition 4.2 [3] Let Q$(X \to Y)$ and R$(Y \to Z)$ be two IFRs. The *max-min-max composition* $R \circ Q$ is the intuitionistic fuzzy

relation from X to Z, defined by the membership function

$$\mu_{R\circ Q}(x,z) = \bigvee_{y}\left[\mu_Q(x,y)\wedge\mu_R(y,z)\right]$$

and the non-membership function

$$\nu_{R\circ Q}(x,z) = \bigwedge_{y}\left[\nu_Q(x,y)\vee\nu_R(y,z)\right]$$

$\forall(x,z)\in X\times Z,\ \forall y\in Y$

Proposition 4.3 [3] If *R* and *S* are two IFRs on $X \times Y$ and $Y \times Z$ respectively; then

(i) $(R^{-1})^{-1}=R$
(ii) $(S\ o\ R)^{-1}=R^{-1}\ o\ S^{-1}$

Medical diagnosis[3]
The methodology involves mainly the following three jobs:
1. Determination of symptoms.
2. Formulation of medical knowledge based on intuitionistic fuzzy relations.
3. Determination of diagnosis on the basis of composition of intuitionistic fuzzy relations.

Let *n* patients patient p_i, $i = 1 \ldots n$, in a hospital. Thus $p_i \in P$. Let *R* be an IFR ($S \rightarrow D$) and construct an IFR Q from the set of patients to the set of symptoms S. Clearly, the composition *T* of IFRs *R* and *Q* ($T = R\ o\ Q$) describes the state of patients p_i in terms of the diagnosis as an IFR from *P* to D given by the membership function

$$\mu_r(p_i,d) = \bigvee_{s\in S}\left[\mu_Q(p_i,s)\wedge\mu_R(S,d)\right]$$

and the non-membership function

$$\nu_r(p_i,d) = \bigwedge_{s\in S}\left[\nu_Q(p_i,s)\vee\nu_R(s,d)\right]$$

$\forall p_i\in P,\ \forall d\in D$

A computer-based diagnostic system can be used for this purpose (Figure 1)[3].

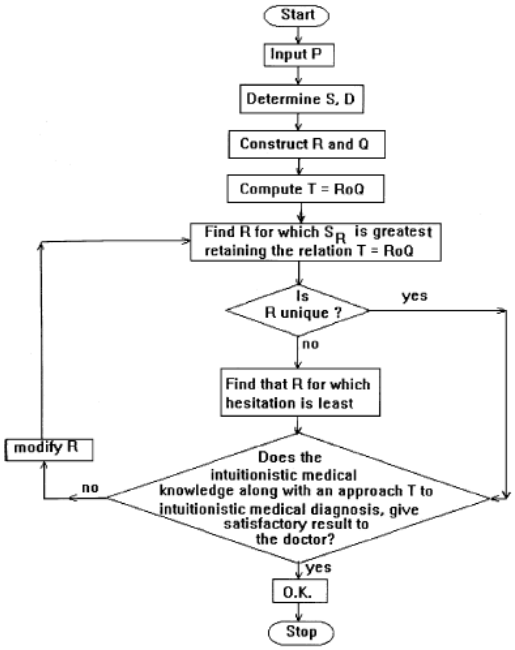

Figure 1. A brief ow-chart for IF Medical Diagnosis [3]

5. SZMIDT AND KACPRZYK APPROACH FOR MEDICAL DIAGNOSIS

Szmidt and Kacprzyk [12] note that the method given by De, Biswas and Roy only considers the dominant symptoms.
They notice that by using minimum or maximum operators only the extreme values are retained for the calculation of the degree of the membership function.
So, Szmidt and Kacprzyk [12], calculate for each patient p_i $i = 1 \ldots n$ a distance of his symptoms from a set *S* of symptoms s_j, $j = 1. \ldots m$ characteristic for each diagnosis d_k, $k = 1 \ldots r$.
The diagnosis will be considered correct for the shortest of distances.
By the way, Szmidt and Kacprzyk [10, 11] proved that "the only proper way of calculating the most widely used distances for intuitionistic fuzzy sets is to take into account

all three parameters: the membership function, the non-membership function, and the hesitation margin".
The normalised Hamming distance for all the symptoms of the *i-th* patient from the *k-th* diagnosis is equal to

$$l(s(p_i), d_k) = \frac{1}{10}\sum_{j=1}^{m}\big(|\mu_j(p_i) - \mu_j(d_k)| + |\nu_j(p_i) - \nu_j(d_k)| + |\pi_j(p_i) - \pi_j(d_k)|\big)$$

With this relationship, the distances for each patient are calculated. The lowest distance points out a proper diagnosis.
Szmidt and Kacprzyk observe that the same results (same diagnosis) are obtained when using the normalized Euclidean distance.

6. PHILIP APPROACH FOR MEDICAL DIAGNOSIS

In [6], Philip propose a new method which takes into consideration all the membership values.
For this, Philip "replace the max-min composition of De et al., by a max-prod composition and the min-max composition by a min-sum composition".
The membership values are computed as follows:

$$T^{+}(p_i, d_k) = \max_{s\in S}\big(Q^{+}(p_i, s)\otimes R^{+}(s, d_k)\big)$$

and

$$T^{-}(p_i, d_k) = \min_{s\in S}\big(Q^{-}(p_i, s)\oplus R^{-}(s, d_k)\big)$$

where $\otimes$ denotes the product operation and $\oplus$ denotes the sum operation.
T is an IFR and any diagnosis should be a crisp result.

Philip says that any of the standard methods of defusification can be used to fuzzy IFS.
The proposed method employs a max-max-prod and min-min-sum composition, to obtain the final decision.
In [6], Philip consider *P* a set of patients, *S* a set of symptoms and *D* a set of possible diagnoses for symptoms *S*.
It is obtained the IFR $Q \subseteq P \times S$, by observation.
For the above observed data, it is obtained the following IFR $R \subseteq S \times D$ showing the relation between symptoms and diseases.
The relation may be obtained from doctor's experience or from a database.
Finally, the relation *T* between the patients and the diseases given by $T = R \circ Q$, where the composition is obtained using the formulas in equations above. The IFS *T* gives the diagnosis.
It is observed that $T \subseteq P \times D$.
The final diagnosis may be obtained by considering the maximum of membership values.
When the maximum membership is not unique, the non membership also is considered and in this case, the minimum value is chosen.

7. CONCLUSION

Intuitive fuzzy sets are a generalization of fuzzy sets.
They can be useful in situations where the description of a problem by a fuzzy language variable is crude. IFSs allow us to express, for example, that the parameter of a symptom changes and other symptoms are not clear enough.
In this study we tried to expose some methods used in obtaining medical diagnoses.

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