PARTIAL CAPACITIES AND OPERATING CAPACITY OF A SHIELDED THREE-PHASE CABLE BY MEANS OF COMPLEX POTENTIAL – CASE STUDY

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ABSTRACT: The paper make a study of determination of partial capacities and operating capacity of a shielded three-phase cable by means of complex potential and the role that they have in electrical power technology.

KEY WORDS: *cable, three-phase, parameters, potential, conductor.*

Introduction

Shielded three-phase cables play a major role in electrical power technology. And the areas of application are becoming more and more diverse, because the population does not want to see any overhead lines as much as possible. In addition, they are used to connect offshore wind farms and, of course, for the energy supply of private households. Calculations of these areas of application require the presence of the cable parameters. How to get to the cable parameters will be shown below.

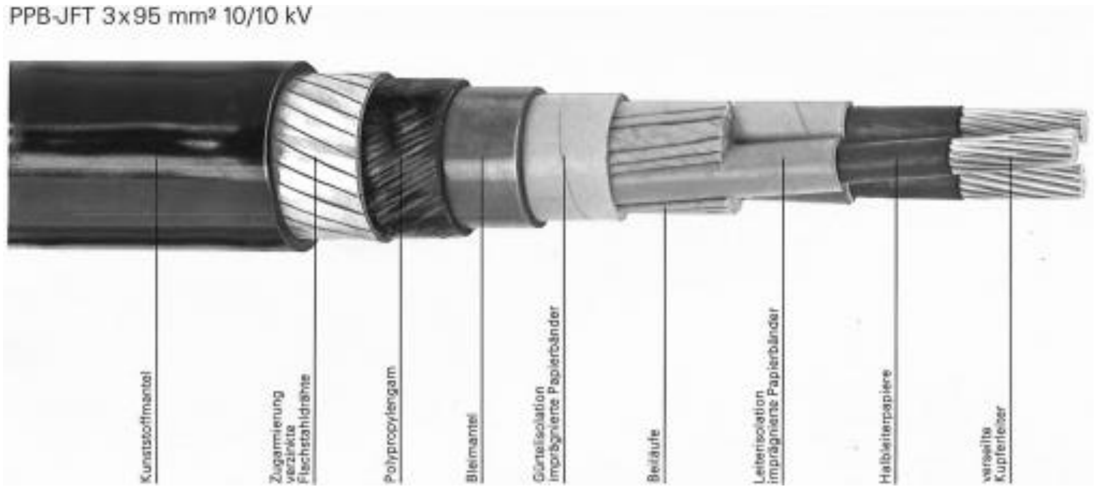

Figure 1: Belt cable PPB-JFT 3x95 mm2 [1]

Problem

A three-core cable consists of a conductive grounded cylinder jacket of the radius, in which $a$three thin cable conductors of the radii are arranged$b$ on a radius. $c \ll b$ The partial capacities $C, C_0$and the operating capacity of the cable shall be $C_B$determined.

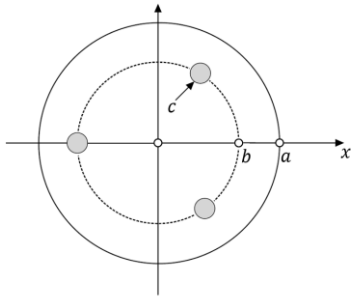

Figure 2: Problem definition

The complex magnetic Potenzial [2]

First, a line charge extended in infinity on both sides is λ_0 considered.

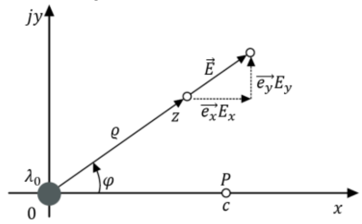

Figure 3: Line load at the origin in the complex plane

As is well known, their electrical potential is

$$V(\varrho) = -\frac{\lambda_0}{2\pi\varepsilon} \ln \frac{\varrho}{c} \qquad (1)$$

If the realvalue is replaced $\varrho$by the complex quantity in the potential $z = x + jy = \varrho e^{j\varphi}$relationship, so that the line charge under consideration is at the origin of the λ_0GAUSSian number plane drawn in Figure 3, the real potential is transferred $V = V(\varrho)$to the complex potential of the real $P = P(z)$part and $P_r$imaginary part: P_i

$$P(z) = -\frac{\lambda_0}{2\pi\varepsilon} \ln \frac{z}{c} = -\frac{\lambda_0}{2\pi\varepsilon} \ln \left(\frac{\varrho}{c} e^{j\varphi}\right) = \left[-\frac{\lambda_0}{2\pi\varepsilon} \ln \frac{\varrho}{c}\right] + j\left[-\frac{1}{\varepsilon} \lambda_0 \frac{\varphi}{2\pi}\right] = V + jP_i = P_r + jP_i \qquad (2)$$

Thus, the real part $P_r$of the complex potential is the $P = P(z)$known electrostatic potential V. With the calculation rules for complex numbers, calculations of flat fields of line loads can now be significantly simplified, as shown below.

The complex potential is now applied to several straight homogeneous line loads.

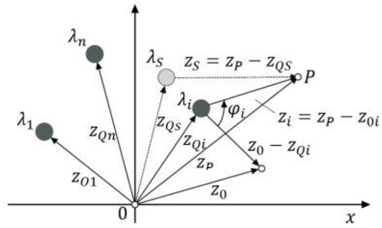

Figure 4: Several straight homogeneous line loads

If, according to the figure, there are infinitely long line charges of different homogeneous densities $\lambda_i$at the points that have the complex z_{Qi}distances from the $z_p$point, $z_i = z_p - z_{Qi}$the values of the complex potential and the complex field strength can be found by totalizing the contributions. If one also requires that the complex potential disappears at a given point z_0(e.B. earth rope), the value c must be replaced by the respective difference in the contributions: $(z_p - z_{Qi})$

$$P(z_p) = -\frac{1}{2\pi\varepsilon} \sum_{i=1}^{n} \lambda_i \ln \frac{z_p - z_{Qi}}{z_0 - z_{Qi}} \qquad (3)$$

Solution with the complex magnetic potential

In generalization of the original problem, the field of n line charges is to be used, which are arbitrarily arranged $\lambda_k$in the GAUSSian number plane of the following image at the points within a $z = x + jy$grounded conductive cable jacket.$z_k |z| = a$

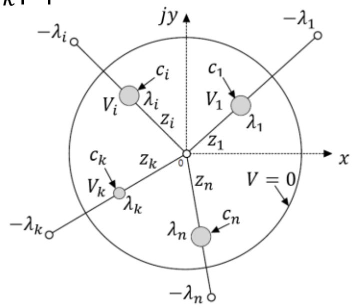

Figure 5: Cables with charges and mirror charges

In order to make the cylinder $|z| = a$an equipotential surface, mirror charges are attached to $\lambda_k$the points $\widetilde{z_k} = \left(\frac{a}{\varrho}\right)^2 z_k$witht in addition to the line $\varrho_k = |z_k|$charges. $-\lambda_k$ The complex potential P(z) in the point z of all line charges then has the value

$$P(z) = \frac{1}{2\pi\varepsilon}\left(\sum_{k=1}^{n} \lambda_k \ln \frac{z-\widetilde{z_k}}{z-z_k} + K\right)$$

(4)

, wherein the free constant K of plane fields is to be determined from the demand for the disappearance of the real part of (1) on the grounded conductive cable sheath. $|z| = a$ After the determination of the constant K carried out in this way, the following expression is then obtained for the electrostatic potential V=V(z) within the cable jacket, dit in additionto the center distances $\varrho_k$of the line charges $\lambda_k$nochtheir distances $r_k$und the distances of the mirror charges from the point$-\lambda_k$ of reference:

$$V(z) = \frac{1}{2\pi\varepsilon}\sum_{k=1}^{n} \lambda_k \ln\left(\frac{\varrho_k}{a}\frac{R_k}{r_k}\right) \text{ with } \begin{matrix} r_k=|z-z_k| \\ R_k=|z-\widetilde{z_k}| \end{matrix}$$

(5)

In the immediate vicinity of the line $\lambda_k$charge, the equipotential surfaces V(z)=const. Circular cylinders, so that the line charges can be replaced by thin cable $\lambda_k$conductors of the small radii. c_k In order to find the potential $V_i$of the i-th cable conductor, it is therefore only possible to introduce the conductor location in the potentialrelationship (5) for the $z$point coordinate, whereby the distance $z_i$for the conductor radius is to$|z_i - z_k|$ be equated. $i = k$ Thepoint distances defined in (5) thus pass into the distance of the R_{ik}considered i-th cable conductor from all mirror conductors and for the $i \neq k$distances of the r_{ik}considered i-th cable conductor from the other cable conductors, while for the $i = k$distance r_{ii}corresponds to the radius $c_i$of the considered i-th cable conductor corresponds to:

$$V_i = \frac{1}{2\pi\varepsilon}\sum_{k=1}^{n} \lambda_k \ln\left(\frac{\varrho_k}{a}\frac{R_{ik}}{r_{ik}}\right) \text{ with } \begin{matrix} r_{ik}=|z_i-z_k| \\ R_{ik}=|z_i-\widetilde{z_k}| \end{matrix} \text{ and } \begin{matrix} r_{ii}=a \\ R_{ii}=\varrho_i\left(\frac{a^2}{\varrho_i^2}-1\right) \end{matrix}$$

(6)

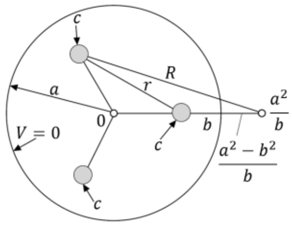

Figure 6: Cables with real distances

In the present case of the three-wire symmetrical cable with $\varrho_i = b$and $c = c$apply to the distances the relationship $R_{ik} = R$and , so that on the basis of the $r_{ik} = r$relationship (6) for the potential of the first conductor $V_1$the relationship

$$V_1 = \frac{\lambda_1}{2\pi\varepsilon}\ln\frac{a^2-b^2}{ac} + \frac{\lambda_2+\lambda_3}{2\pi\varepsilon}\ln\left(\frac{b}{a}\frac{R}{r}\right) \text{ with } \begin{matrix} r^2=3b^2 \\ R^2=b^2+\frac{a^4}{b^2}+a^2 \end{matrix}$$

(7)

receives. The expressions for the potential $V_2$and $V_3$the other two cable conductors are obtained by cyclic swapping of the indices in (7), so that as a result the three potentials with thedistances R and r defined in (7) can beexpressed by the charges as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} p_0 & p & p \\ p & p_0 & p \\ p & p & p_0 \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \text{mit } p_0 = \frac{1}{2\pi\varepsilon}\ln\frac{a^2-b^2}{ac} \text{and } p = \frac{1}{2\pi\varepsilon}\ln\left(\frac{b}{a}\frac{R}{r}\right)$$

(8)

By inversion of the system (8) one can determine, as required in the problem, the charges on the conductors and the influence charge on the $\lambda_0$cable jacket, which corresponds to the negative sum of the three conductor charges:

$$\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix} = \frac{1}{p_0-p}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} - \frac{p(V_1+V_2+V_3)}{(p_0-p)(p_0+2p)} \text{ and } \lambda_0 = -\frac{V_1+V_2+V_3}{p_0+2p} \qquad (9)$$

From the result (9) it can be inferred that, despite the symmetrical structure of the cable, the charges on the three conductorsare only proportional to the respective potentials if the potential sumdisappears, whereby the totalladung on the inside of the conductive grounded cable jacket alsoλ_0 disappears. Furthermore, the result (9) canbe achieved by introducing the potential differences to the respective considered conductor also in the form

$$\begin{matrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{matrix} = \begin{matrix}C_0V_1 + C(V_1-V_2) + C(V_1-V_3)\\ C(V_2-V_1) + C_0V_2 + C(V_2-V_3)\\ C(V_3-V_1) + C(V_3-V_2) + C_0V_3\end{matrix} \text{ with}$$

$$C_0 = \frac{1}{p_0+2p} \text{and } C = \frac{p}{(p_0-p)(p_0+2p)} \qquad (10)$$

in which the values and C then C_0represent the capacities of the conductor against earth and against the other conductor, with which one can give the capacitive replacement image given in the last image. Using theexpressions for and p given in (8), $p_0$the two capacitance values can thus be expressed by the dimensions a, b and c as follows:

$$C_0 = 2\pi\varepsilon\left[\ln\frac{a^6-b^6}{3ca^3b^2}\right]^{-1} \quad \text{and} \quad C = \frac{4\pi\varepsilon}{3}\left[\ln\frac{3b^2}{c^2}\frac{(a^2-b^2)^3}{a^6-b^6}\right]^{-1} - \frac{1}{3}C_0 \qquad (11)$$

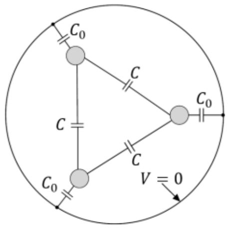

Figure 7: Capacitive replacement circuit diagram of the cable

If the considered symmetrically constructed three-wire cable is applied to a symmetricaltorsional voltage system, the voltage sum of which disappears, then after (10) for the charges of the three conductors the relationships are

$$\lambda_i = C_BV_i \text{ for } i = 1,2,3 \text{ with } C_B = C_0 + 3C = 4\pi\varepsilon\left[\ln\frac{3b^2}{c^2}\frac{(a^2-b^2)^3}{a^6-b^6}\right]^{-1} \qquad (12)$$

where the size $C_B$is referred to as operating-capacity. The for them in given expression was found when using the capacities (11). A comparison of the relationships (11) and (12) with the literature [3] results in

$$C_0 = \frac{2\pi\varepsilon l}{\ln\frac{16a(b^3-a^3)}{3D^3d}},$$

$$C_1 = \frac{2\pi\varepsilon l}{3\ln\frac{2\sqrt{3}\,a(b-a)}{d\sqrt{a^2+b^2+ab}}} - \frac{1}{3}C_0$$

with $d_{[3]} = 2c$; $D_{[3]} = 2a$; $a_{[3]} = b$; $b_{[3]} = \frac{a^2}{b}$

Figure 8C_0 and $C = C_1$from [3]

CONCLUSION

If the names in [3] are used, then the results are identical.

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[1] Kabelwerke BRUGG AG

[2] A. Nethe; H.-D. Stahlmann; Introduction to Field Theory

[3, p. 105] K. Küpfmüller; Introduction to Theoretical Electrical Engineering, 10th edition