PARALLEL ROBOTS - THE GOUGH-STEWART PLATFORM

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ABSTRACT: *According to J-P. Merlet „parallel robots, also sometimes called hexapodes or Parallel Kinematic Machines (PKM), are closed-loop mechanisms presenting very good performances in terms of accuracy, velocity, rigidity and ability to manipulate large loads" [1]. One of the most popular parallel manipulators is the general purpose 6 degree of freedom (DOF) Gough-Stewart Platform. Therefore, in the present paper, I will present a compilation of previous studies of this platform.*

KEY WORDS: *parallel robots, hexapodes, the Gough-Stewart Platform, forward and inverse kinematics.*

1. The Gough-Stewart Platform

The Gough-Stewart Platform mechanism was originally suggested by Gough [2] in 1947. He established the basic principles of a mechanism with a closed-loop kinematic structure that allows the positioning and orientation of a moving platform so as to test tire wear and tear (Figure 1). He built a prototype of this machine in 1955.

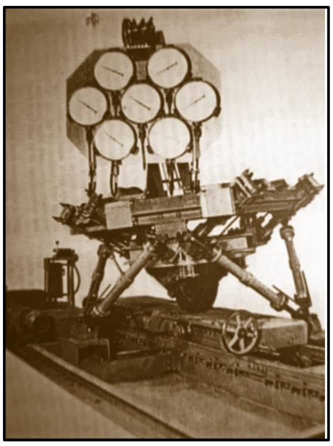


Figure 1. Gough platform [2]

In 1965, Stewart suggested the use of such a structure for flight simulators and presented it to academia [3]. Gough was the first to realize the benefits of this mechanism and research in this area was carried out after Stewart's Paper. Therefore, the combination of Stewart's and Gough's design is what is known as the Gough-Stewart Platform or Stewart Platform for short.

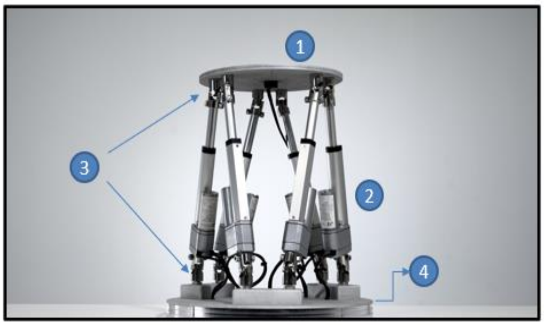


Figure 2. Main Components of a Stewart Platform (ACROME) [12]

The full assembly (Figure 2) is a parallel robotic system consisting of a rigid body top or mobile plate (1) connected through six prismatic actuators (2) to an immobile base plate (4). The actuators are connected to the two platforms through spherical or universal joints (3). This would mean the Stewart Platform is a spherical-prismatic-spherical, or SPS, robot. The actuators (legs) also have an in-built mechanism that allows changing their lengths individually.

Center of the top plate is defined by at least three coordinates, and aim is to move this center point in 3D space. The desired position and orientation of the mobile platform is achieved by combining the lengths of the six legs, transforming the six transitional DOF into three positional and three orientational ones. It can accurately position the lateral (left - right), longitudinal (forward - back) and vertical (up - down) movements and the three rotations: pitch, roll, and yaw (Figure 3).

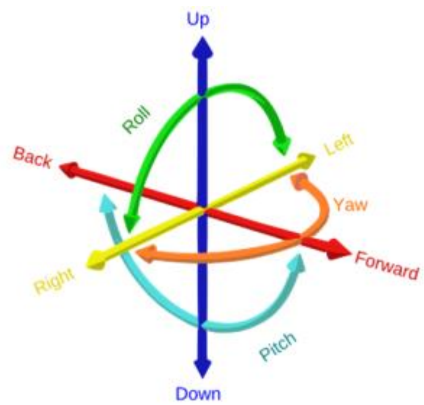
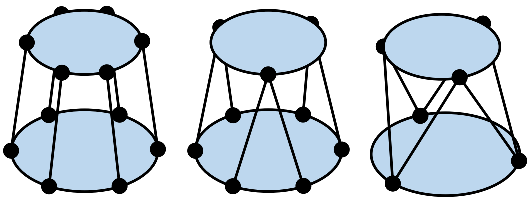


Figure 3. Stewart Platform DOF's

There are variants of the platform, but most of them have six linearly actuated legs with varying combinations of leg-platform connections. Although the joint positions can be arbitrary, evenly spacing them results in special cases.

When the joints are spaced 60° around the base and the moving platform, this is called a 6-6 configuration. When a pair of actuators share the same joint spaced 120° on the moving platform, this is called a 6-3 configuration. 120° spacing on top and bottom platforms results in a 3-3 configuration as shown in Figure 4.



(a) Type 6-6 (b) Type 6-3 (c) Type 3-3

Figure 4. Stewart Platform Configurations

In a literature survey, a true type 6-6 Stewart Platform was never found to be used in practice. Typically, platforms were designed as a mix between the 6-6 and 3-3 configuration. Instead of pairs of actuators sharing the same joint, each actuator had its own joint like in a 6-6 configuration; pairs of joints were separated by a small gap, resulting in a geometry visually similar to Type 3-3. A diagram of this geometry is shown in Figure 5.

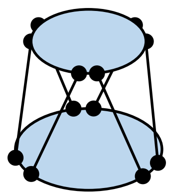


Figure 5. Typical Stewart Platform Joint Configuration

Further analysis reveals that a true 6-6 platform with 60° spacing between all joints is unstable because there is no force balance, allowing the top platform to rotate about its axis. This is illustrated in Figure 6. When a small disturbance occurs, the links cannot exert forces in opposite directions resulting in a couple about the links.

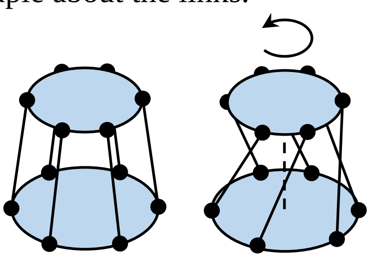


Figure 6. Unstable 6-6 Configuration

Spherical joints cannot resist this moment, leading to a collapse. This shortcoming of a true 6-6 configuration shows the value of 6-3, 3-3, and comparable geometries.

2. Kinematics of the Stewart Platform

To move upper plate, each leg must be controlled with an actuator to change the total length of the leg. By controlling each leg's length independently and with mathematical calculations called forward and inverse kinematics, the location of the top center could be changed precisely. There is no significant error to accumulate with each linkage because it is a distribution of the errors of each individual leg.

For parallel manipulators typically direct kinematics problem is challenging, but inverse kinematics is straightforward.

Inverse kinematics determines the joint values given a known or desired end-effector position and orientation. The inverse kinematics Stewart Platform problem is trivial with single solution, but when the number of kinematic chains is reduced, the number of solutions of the inverse kinematics problem increases and the problem becomes more challenging.

The following derivation is drawn from Dr. Saeed Niku's inverse kinematic derivation of the Stewart Platform. [4].

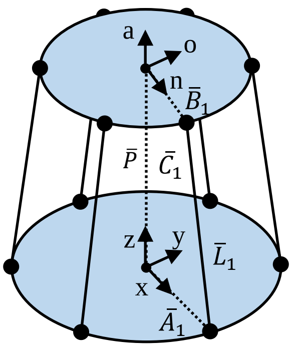


Figure 7. Vector Definition for Inverse Kinematic Derivation

A fixed reference frame *xyz* is placed at the center of the platform base. The *z*-axis is normal to the base and the *x*-axis is parallel to it, pointed towards a spherical joint. A moving reference frame *noa* is similarly placed at the center of the moving platform with the *a*-axis normal to the moving platform face and the *n*-axis pointed towards a spherical joint. The spherical joints are assumed to be connected through the linear actuator. Frame *noa* rotates by θ, ϕ, ψ respectively.

Four parts compose each kinematic chain $\bar{C}$: $\bar{A}$ connects the origin of the fixed reference frame and base spherical joint at an angle from *x*; $\bar{B}$ connects the origin of the moving reference frame and moving platform spherical joint at an angle from *n*; $\bar{L}$ connects the spherical joints; $\bar{P}$ connects the origins of the two frames, and is common for all six chains.

The purpose of the inverse kinematics is to determine the length *L* of each linear actuator, given the position *P* and orientation of frame *noa*.

The vector equation to determine each actuator length is:

$$\bar{L}_i = \bar{P} + \bar{B}_i - \bar{A}_i, \quad \text{for i = 1 ... 6.}$$

For chain $\overline{C_1}$, $\overline{A_1}$ and $\overline{B_1}$ lie along the *x*-axis and *n*-axis respectively and their angles are zero. In a Type 6-6 Stewart Platform, subsequent chains will be multiples of 60° from the *x* and *n* axes. This is not always the case, and the remaining angles can be arbitrary values; in practice, Stewart Platforms will have some degree of symmetry.

Following the vector convention Dr. Saeed Niku [4], $\bar{A_\iota}$ for the general case can be written as:

$$\overline{A_i} = \begin{bmatrix} A\cos(i_{th}) \\ A\sin(i_{th}) \\ 0 \\ 1 \end{bmatrix}$$

where the i_{th} term is the angle that $\bar{A_\iota}$ makes with *x* axis.

$\overline{A_i}$ for i = 1...6 will remain constant throughout the motion of the Stewart Platform for a given geometry.

Similarly, $\bar{B_\iota}$ can be written as:

$$\overline{B_i} = \begin{bmatrix} B\cos(i_{th}) \\ B\sin(i_{th}) \\ 0 \\ 1 \end{bmatrix}$$

for the moving platform when it is in the home position as shown in Figure 7. The home position is achieved after the robot is calibrated, and all the actuators are at their minimum length.

Because $\overline{B_i}$ is attached to moving frame *noa*, its components will change as the platform rotates. To account for this rotation, $\overline{B_i}$ must be pre-multiplied by rotation matrices. Using the variables from the moving reference frame, the rotation matrices are:

$$\mathrm{Rot}(x,\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$$

$$\mathrm{Rot}(y,\phi)=\begin{bmatrix}\cos\phi & 0 & \sin\phi\\ 0 & 1 & 0\\ -\sin\phi & 0 & \cos\phi\end{bmatrix}$$

$$\mathrm{Rot}(z,\psi)=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}$$

These *3x3* rotation matrices only account for rotation. They can be expanded to *4x4* to include position, in which the fourth row and column are zero, except for a value of one located in element 4, 4.

In order to determine the value of $\overline{B_i}$ after rotation for each kinematic chain, it must be pre-multiplied by the above rotation matrices, yielding:

$$\begin{bmatrix}B_ix\\ B_iy\\ B_iz\\ 1\end{bmatrix}_{rot}=R\cdot\begin{bmatrix}B_ix\\ B_iy\\ B_iz\\ 1\end{bmatrix}_{home}$$

where

$$R=[\mathrm{Rot}(z,\psi)][\mathrm{Rot}(y,\phi)][\mathrm{Rot}(x,\theta)]$$

Multiplying *Rot(z, ψ)* and *Rot(y, ϕ)* yields:

$$\mathrm{Rot}(z,\psi)\mathrm{Rot}(y,\psi)=\begin{bmatrix}\cos\phi\cos\psi & -\sin\psi & \cos\psi\sin\phi\\ \cos\phi\sin\psi & \cos\psi & \sin\phi\sin\psi\\ -\sin\phi & 0 & \cos\phi\end{bmatrix}$$

Multiplying the result by *Rot(x, θ)* yields:

$$\begin{bmatrix}\cos\phi\cos\psi & -\sin\psi & \cos\psi\sin\phi\\ \cos\phi\sin\psi & \cos\psi & \sin\phi\sin\psi\\ -\sin\phi & 0 & \cos\phi\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}=$$

$$=\begin{bmatrix}q_{11} & q_{12} & q_{13}\\ q_{21} & q_{22} & q_{23}\\ q_{31} & q_{32} & q_{33}\end{bmatrix}$$

where

$q_{11}=\cos\phi\cos\psi$

$q_{12}=\cos\psi\sin\phi\sin\theta-\cos\theta\sin\psi$

$q_{13}=\sin\psi\sin\theta+\cos\psi\cos\theta\sin\phi$

$q_{21}=\cos\phi\sin\psi$

$q_{22}=\cos\psi\cos\theta+\sin\phi\sin\psi\sin\theta$

$q_{23}=\cos\theta\sin\phi\sin\psi-\cos\psi\sin\theta$

$q_{31}=-\sin\phi$

$q_{32}=\cos\phi\sin\theta$

$q_{33}=\cos\phi\cos\theta$

After combining the rotation matrices yields:

$$\begin{bmatrix}B_ix\\ B_iy\\ B_iz\\ 1\end{bmatrix}_{rot}=\begin{bmatrix}q_{11} & q_{12} & q_{13} & 0\\ q_{21} & q_{22} & q_{23} & 0\\ q_{31} & q_{32} & q_{33} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}B_ix\\ B_iy\\ B_iz\\ 1\end{bmatrix}_{home}$$

$[\overline{B}_i]_{home}$ is known from the Stewart Platform geometry, and θ, ϕ, and ψ are known upon specifying the orientation of the moving platform. Therefore, $[\overline{B_i}]_{rot}$ can be solved.

Substituting the rotated $\bar{B}$ matrix into the vector equation yields:

$$\begin{bmatrix}L_{ix}\\ L_{iy}\\ L_{iz}\end{bmatrix}=\begin{bmatrix}P_{ix}\\ P_{iy}\\ P_{iz}\end{bmatrix}+\begin{bmatrix}B_{ix}\\ B_{iy}\\ B_{iz}\end{bmatrix}-\begin{bmatrix}A_{ix}\\ A_{iy}\\ A_{iz}\end{bmatrix},\quad \text{for } i=1 \ldots 6.$$

The linear actuator lengths are the desired quantity from the kinematics. The magnitude of each length is determined by:

$$|L_i|=\sqrt{(L_{ix})^2+(L_{iy})^2+(L_{iz})^2},\quad \text{for } i = 1\ldots6.$$

These calculated lengths become the setpoints for controlling the Stewart Platform.

Forward (direct) kinematics allows the position and orientation of the end effector to be known, given all joint values.

The direct kinematics problem of parallel manipulators is by far more challenging than the inverse kinematics problem since it requires solving a set of polynomial equations in the output variables.

The first one who tried to solve this problem was P. Dietmaier, in 1998. He systematically changed the geometric properties of a general Stewart manipulator and gave an example of a manipulator with 40 real solutions to the direct kinematics problem [5].

The size of the problem depends on the representation used for orientation. If a rotation matrix is used for this representation, the number of unknowns in the forward kinematic problem are 12. Position vector $\bar{P}$ introduced three unknowns *px, py, pz,* and nine other unknowns are the components of the rotation matrix.

On the other hand,

$$|L_i|=\sqrt{(L_{ix})^2+\left(L_{iy}\right)^2+(L_{iz})^2}$$

provides six nonlinear equations, for i=1...6, and the orthonormality conditions:

$$\theta \cdot \theta = \phi \cdot \phi = \psi \cdot \psi = 1$$

$$\theta \cdot \phi = \phi \cdot \psi = \psi \cdot \theta = 0$$

$$\theta \times \phi = \psi;\ \phi \times \psi = \theta;\ \psi \times \theta = \phi$$

provide the other six equations to be solved simultaneously.

Therefore, the forward kinematic problem is recast into 12 nonlinear equations with 12 unknowns. This problem is highly nonlinear and is extremely difficult to solve, and only few practical closed-form solutions have been obtained for the general Stewart platform. Moreover, even a numerical iterative procedure is not suitable for Stewart platform, because it leads to a heavy computational burden and depends highly on a good initial guess in order to converge toward the right solution. In case of using screw axis or unit quaternion representation for the orientation, the number of equations and the unknowns reduce to seven and in case of using any Euler angle representations, the number of equations reduces to six. However, the complexity of the equations significantly increases, resulting in a more expensive computational cost.

Several researchers have tried to find easier ways to solve the forward kinematic problem [6-10]. One of them was Y. Wang. He has presented a numerical method for forward kinematics of nearly general Stewart platforms, which can directly generate a unique solution [11]. This method utilizes the trivial nature of inverse kinematics of parallel manipulators, and derives a straightforward linear relationship between the small change in joint variables (legs lengths) and the resulting small motion of the platform. The solution to the forward kinematics is then obtained through a series of small changes in joint variables.

In fact, mathematical complexity of the forward kinematics of Stewart platforms has become a serious deficiency that prevents it from being used in many high-speed, real-time, and online implementations.

For finding the forward kinematics for a general Stewart platform structure an analytical solution and a numerical method were researched. The main purpose of these two representative methods was to give a scientific judgment on the performance, limitations, and implementation characteristics of these methods in practice.

3. Conclusions

This paper presented a review of the most representative studies on the Gough-Stewart Platform. As shown, the unique features of the Gough-Stewart Platform make it extremely useful for producing manipulators with a wide range of use.

The main advantage of the Stewart Platform is its load-carrying capacity - the ratio of the mass of the payload over the mass of the robot can be larger than 10.

Another advantage of the Stewart Platform is its very good positioning accuracy. This high accuracy arises from the fact that the actuators are working essentially in tension/compression and are subjected to virtually no bending – thereby leading to very small deformations – and from the fact that the errors in the internal sensors of the robot (the measurement of the lengths of the actuators) are mapped into very small errors of the platform position.

Parallel robots are also almost insensitive to scaling (the same structure can be used for large or micro robots) and they can be built using almost any type of actuator or transmission.

The main drawbacks of parallel robots are their small workspace and the singularities that can appear within the latter.

The Stewart Platform mechanism is and can be used as a basis for controlled motion with 6 degrees of freedom, such as precise manipulative tasks:

- in medical and surgical applications: in the physical therapy field helping with ankle stabilization (two Stewart platforms attached to each other) or in the surgical robotics field (as an end-effector);
- in engineering applications: fiber-optic equipment because it must be accurately positioned and oriented, enabling the light to refract and the optic signal to operate properly;
- in training and entertainment simulators for air, sea, and land vehicles;
- in astronomy: trajectory tracking for spherical radio telescopes;
- in animatronics: for imitating the body movements of a dancer (two Stewart platforms attached to each other - the lower one is controlled in forward kinematics, the upper one in inverse kinematics, assigned to follow the lower platform) etc.

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