WEIBULL / RAYLEIGH DISTRIBUTION USED TO MODEL WIND SPEEDS IN ORDER TO PREDICT WIND ENERGY PRODUCTION

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ABSTRACT: The Weibull distribution is a popular probability distribution used to model wind speeds in order to predict wind energy production. It is a continuous probability distribution with two parameters, the shape parameter *k* and the scale parameter *c*.

The Weibull distribution is often used in wind energy applications because it has a flexible shape that can fit a wide range of wind speed data. The shape parameter *k* controls the shape of the distribution, with smaller values of *k* indicating a more peaked distribution and larger values indicating a more spread-out distribution. The scale parameter *c* determines the location of the distribution along the wind speed axis.

This article will focus on the analytical expressions used for wind data assessment and on how to fit the parameters needed for them deriving analytical expressions to fit the probability density function is way to reduce the number of parameters to characterize the probability density function and the estimate wind power resource.

The Weibull and Rayleigh distributions are the most commonly used statistical functions used for wind power studies and this article have a closer look at them and reveals some commonly used method to fit their parameters.

KEY WORDS: Weibull, Rayleigh, distributions, probability, wind energy, density function.

1. INTRODUCTION

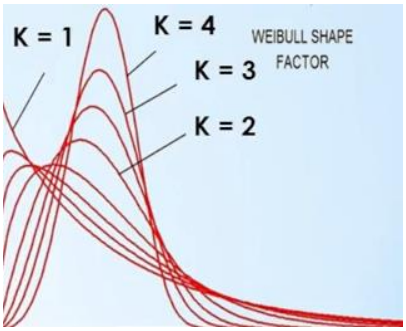
The Weibull distribution that has become a widely used standard in wind energy application due to its simplicity. It is described by the following equation

$$\varphi(v) = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}} \quad (1)$$

where $\varphi(V)$ is the probability of the wind being at speed *v*.

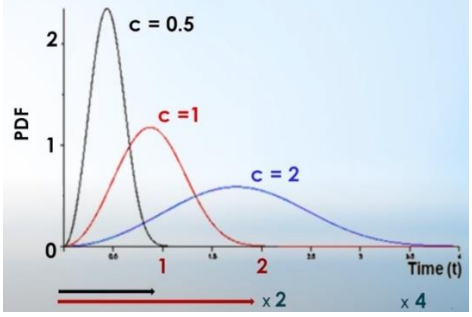
It depends on two easily estimated parameters

- *k* is known as the shape parameter and is positive and dimensionless. The higher the value of *k* the higher the median wind speed. It settles the shape of the function. Location with lots of low wind speeds as well as some very strong winds would have a shape value *k* below 2, whereas locations with fairly consistent, wind speeds around the median, would have a shape value of 3.



- *c* is known as the scale parameter it is also positive and has velocity dimension.

Looking at the example we can see why *c* is called the scale parameter, multiplying *c* by 2 stretches the scale two times but keeps the area under the curve to 1 as it is a normalized distribution.



The Weibull distribution has become a widely used standard in wind energy application due to its simplicity and there are simple analytical expressions for the moments as will be shown later. It is a reference used in wind energy software such as Wind Atlas Analysis and Application program WASP and it is included in regulations such as that on wind turbine power performance testing.
The cumulative distribution function (CDF) of the wind speed *v* evaluated at a given value v_0 is the probability that the wind speed will take a value less than or equal to the given value v_0.

Cumulative distribution function:

$$F(v) = P(v \leq v_0) = \int_0^{v_0} \varphi(v)dv \quad (2)$$

$$\int_0^{\infty} \varphi(v)dv = 1 \quad (3)$$

It is expressed as the integrand from zero to v_0, in other words it gives the area under the probability density function from 0 to v_0. In the case of the Weibull distribution, the cumulative distribution function also takes an analytical expression.

$$F(v) = 1 - e^{\left(\frac{-v_0}{c}\right)^k} \quad (4)$$

The cumulative distribution function is often used to quantify the goodness of fit of the Weibull distribution with respect to the observed probability density function.

The Rayleigh distribution is a particular case of Weibull distribution with shape parameter *k* equals 2. This is convenient as in most locations around the word the value of *k* is approximately 2. This reduces the expression of the probability density function to one single parameter.

Weibull distribution with its two parameters leads to very simple analytical expressions for the moments which are key to easily estimate the shape and scale parameters *k* and *c*. So, to adjust the *k* and *c* parameters we will need to evaluate some of the function's moments.

For example, the first moment which corresponds to the mean wind speed is the integrand from 0 to infinity of the product of the wind speed *v* by the probability density function φ .

We can obtain an analytical expression of the mean wind speed expressed as the product of the scale parameter by the gamma function of one plus one of the shape parameters, and the scale parameter can be obtained from this expression.

$$\bar{v} = \int_0^{\infty} v \cdot \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k} \cdot dv \quad (5)$$

$x = v/c$

$dv = c\,dx$

thus:

$$\bar{v} = c\int_0^{\infty} k \cdot x^k \cdot dv = \int_0^{\infty} k \cdot x \cdot x^{k-1} \cdot dx \quad (6)$$

It can be either computed numerically or tabulated to facilitate its calculation, the properties of the variable distribution and the gamma function lead to a simple expression for higher moments as well expressed by this formula.

Gamma function:

$$\Gamma_{(z)} = \int_0^{\infty} t^{z-1} \cdot \lambda_e^{-t} \cdot dt \quad (7)$$

$$\Gamma(z+1) = c_0\Gamma(z) \quad (8)$$

Results that: $\bar{v} = c \cdot \Gamma(1 + 1/k)$ (9)

$$c = \frac{\bar{v}}{\Gamma(1 + 1/k)} \quad (10)$$

Other methods are used depending of the available information we might have.

In all cases, it is important to keep in mind that the wind speed measurements should be taken under similar conditions to those used to estimate the parameters of the distribution. Additionally, the accuracy of the wind speed estimate will depend on the quality of the data and the appropriateness of the chosen distribution.

2. MOST COMMONLY USED METHODS TO PREDICT WIND SPEED

Some of the most commonly used methods when only the mean wind speed is known is the Rayleigh distribution. It is in general accurate enough and often used in wind atlas. The procedure is the same: we evaluate *c* from the mean value *v* and use the gamma function.

When more than one moment are known, as for instance the mean wind speed and the standard deviation than the methods of Moments can be used.

To compute the shape parameter *k*, this method of estimation of distribution parameters was introduced by Karl Pearson in 1894.

The power density method is also a method of moments. It ensures that the energy content of the fitted Weibull distribution equals the energy content of the observed histogram.

Wind energy is the kinetic energy of energy mass. For a constant wind speed *v* crossing a normal area *a* during a given period *t* kinetic energy contained for the air mass will be the one expressed here in the following formula.

Power Density Method.

$$k = \frac{1}{2} m_{air} \cdot v^2 = \frac{1}{2} \rho_{air} \cdot A \cdot v^2 \quad (11)$$

The wind power density for a unit of area and per unit of time is defined then as the kinetic energy per unit of area and unit of time, which in its integral continuous form it it is related to the third statistical moment

$$P_w = \frac{1}{2}\rho \int_0^{\infty} v^3 \cdot \varphi(v) \cdot dv = \frac{1}{2} \cdot \rho \cdot \bar{v} \quad (12)$$

Taking the analytical expressions of the moments and defining the energy pattern factor we obtain an expression for the shape parameter *k*. This method ensures the best estimation of wind energy potential but does not ensure the maximum likelihood with the observed distribution.

It is usually used when the mean wind speed and the wind energy density are known.

The WASP method computes these parameters from the first and third moments as well as the probability of exceeding the mean wind speed *1* - $\varphi(v)$ which must be estimated from the data.

The method focuses on the right-hand tail of the Weibull distribution which is an important part of the distribution in terms of energy.

This is why the WASP method is preferred amongst the wind energy industry.

In this method *k* and *c* are calculated by solving this equation:

$$c = \left[\frac{\bar{v}^3}{\Gamma\left(1 + \frac{3}{k}\right)} \right]^{1/3} \quad (13)$$

Popular methods also include the usual maximum likelihood estimate. The principle of the maximum likelihood originally developed by Ronald Fisher in 1912 states that the desired probability distribution is the one that makes the observed data most likely, which means that one must seek the value of the distribution parameters that maximize the likelihood.

It is a method which is similar to the graphical method but overcomes its shortfalls such as the absence of confidence interval estimate or the ambiguous choice of the initial parameter estimation.

The procedure is simple: we build the cumulative distribution function *f* and isolate the exponential and we then apply the natural logarithm twice on each side.

$$F(v_i) = 1 - e^{-\left(\frac{v}{c}\right)^k} \quad (14)$$

$$G_i = \ln[-ln(1 - F(v_i))] = k \cdot \ln(v_i) \cdot k \cdot lnc \quad (15)$$

Plotting the values and approximating them to a straight line we

will obtain the Weibull parameters *k* and *c*. The comparison of the wind speed distributions and the observational data is made on the cumulative density function for wind speed.

The real wind histograms deviate significantly from the Weibull distribution because of the complex topography of regions where the wind flow is weaker and more isotropic, but yet it is the most commonly used statistical method.

3. CONCLUSIONS

To use the Weibull distribution for wind predictions, historical wind speed data can be analyzed to determine the appropriate values of *k* and *c*. Once these parameters are determined, the Weibull distribution can be used to estimate the probability of various wind speeds occurring, which can be useful for designing wind turbines and predicting wind energy output.

It is worth noting that the Weibull distribution is just one of many probability distributions that can be used to model wind speeds, and the choice of distribution should depend on the specific characteristics of the wind data being analyzed. Additionally, it is important to validate any wind predictions using actual data to ensure accuracy.

Once the wind speed is estimated and the direction is known historically, can be predict the wind power energy using the power curve of the wind turbine. The power curve represents the relationship between the wind speed and the power output of the turbine. Typically, wind turbines start generating power at a wind speed of around 3-4 meters per second and reach maximum power output at around 12-14 meters per second. The power curve can be obtained from the manufacturer or by testing the wind turbine under different wind conditions. By using the wind speed data and the power curve, the expected energy output of the wind turbine can be predicted.

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