ANALYSIS OF STUDENTS' ACADEMIC PERFORMANCE USING GENERALIZED TRAPEZOIDAL FUZZY NUMBERS AND MULTICRITERIA OPTIMIZATION METHODS

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ABSTRACT: *The paper proposes a model for identifying students' educational and financial support needs, based on the use of Generalized Trapezoidal Fuzzy Numbers (GTFN) and multicriteria optimization methods such as Fuzzy AHP and Fuzzy TOPSIS.*
The main objective is to transform the qualitative evaluations obtained from students' questionnaires—used for initial assessment—into a rigorous quantitative framework capable of capturing the uncertainty and ambiguity associated with subjective judgment.
The proposed model enables the ranking of students according to six performance criteria, integrating cognitive, social, and professional insertion dimensions.
The results highlight the stability and internal consistency of multicriteria decision-making, confirming the applicability of fuzzy methods in educational performance analysis.
The model provides a robust and extensible tool, with potential for integration into academic decision-support software platforms.

Keywords: academic performance, Fuzzy AHP, Fuzzy TOPSIS, generalized trapezoidal fuzzy numbers, multicriteria decision-making, educational analysis.

1. INTRODUCTION

The evaluation of students' academic performance represents an essential area in contemporary educational research, being closely linked to the quality of the teaching process and to the adaptation of pedagogical strategies to the individual needs of learners.
Over time, traditional evaluation methods—based on quantitative indicators such as semester averages, exam grades, or pass rates—have proven limited in their ability to capture the complexity of learning behaviors and students' motivations.
In the 1960s, with the publication of Lotfi A. Zadeh's fundamental work Fuzzy Sets (1965), a new research direction emerged in the study of uncertainty and vague knowledge representation. Fuzzy logic made it possible to express gradual membership and nuanced perceptions, providing a natural alternative to rigid models based on binary classifications.
This paradigm was later extended to multicriteria decision analysis (MCDM), offering flexible tools for evaluating alternatives under uncertain conditions.
In the 1980s, Buckley (1985) proposed extending the Analytic Hierarchy Process (AHP) into a fuzzy context, introducing Fuzzy AHP, a method that allows the assignment of weights to criteria through pairwise comparisons expressed in linguistic terms.
This contribution became a reference point for integrating uncertainty into decision models.
In parallel, Chen (2000) developed a fuzzy version of the TOPSIS method (Technique for Order of Preference by Similarity to Ideal Solution), applicable to multicriteria decision problems involving linguistic and uncertain variables.

Since the 2000s, researchers have begun applying fuzzy methods in the educational field.
Kaya and Kahraman (2011) used a fuzzy AHP approach to evaluate the quality of e-learning platforms, demonstrating the usefulness of the model in educational contexts based on subjective perceptions.
Later, Rani and Mishra (2017) proposed a fuzzy AHP method for analyzing academic performance indicators, confirming the applicability of fuzzy models in higher education.
In the last decade, interest in applying fuzzy methods to academic performance analysis has increased significantly.
Kaur and Aggarwal (2021) developed a fuzzy multicriteria model for student evaluation, showing that integrating socio-emotional and cognitive criteria leads to more balanced assessment outcomes.
At the same time, recent applied research, such as that of Hegazi, Almaslukh, and Siddig (2023), extended fuzzy modeling toward predicting academic performance, using fuzzy logic for automated reasoning and inference.
A notable contribution was made by Rakhi Bihari, Jeevaraj, and Ajay Kumar (2023), who proposed a geometric model for ranking generalized trapezoidal fuzzy numbers (GTFN), offering a precise method for ordering alternatives under high uncertainty.
More recently, Revathi, Karpagam, and Suguna (2024) applied fuzzy-based academic performance analysis to identify learning patterns and influencing factors, demonstrating the direct applicability of fuzzy logic in educational management.
The evolution of these studies shows a clear transition from purely theoretical models (based on Zadeh and Buckley) toward integrated applications in intelligent evaluation systems.
Thus, analyzing academic performance using generalized trapezoidal fuzzy numbers and multicriteria methods such as Fuzzy AHP and Fuzzy TOPSIS provides a rigorous, transparent, and extensible approach for understanding student performance under uncertainty.
Accordingly, the main objective of this study is to develop and test a fuzzy multicriteria model for evaluating academic performance, integrating cognitive, motivational, and social factors into a formal decision-making framework capable of reducing subjectivity and supporting educational decision processes.
By combining the theory of Generalized Trapezoidal Fuzzy Numbers (GTFN) with multicriteria optimization methods, such as Fuzzy AHP and Fuzzy TOPSIS, a rigorous and flexible approach is obtained for the comparative analysis of academic performance.
The purpose of the present study is to demonstrate the applicability and effectiveness of this approach in the university context, providing a reproducible and extensible model.

2. PRELIMINARIES

2.1. Introductory Concepts on Fuzzy Sets and Trapezoidal Fuzzy Numbers

The concept of a fuzzy set was introduced by Zadeh (1965) to describe phenomena characterized by uncertainty and gradual membership.
An element x belonging to a universal set X has a degree of membership μ defined over the interval [0, 1], $A = \{(x, \mu_A(x)) | x \in X\}$, which allows the modeling of linguistic expressions such as low, medium, or very good.
In the following decades, numerical fuzzy representations evolved, leading to the definition of triangular and trapezoidal fuzzy numbers (Buckley, 1985; Chen, 2000).
In the context of multicriteria evaluation, these structures enable the modeling of subjective uncertainty expressed by decision-makers.

2.2. Generalized Trapezoidal Fuzzy Numbers (GTFN)

To address situations where membership is not strictly linear, Generalized Trapezoidal Fuzzy Numbers (GTFN) were introduced. These extend the classical trapezoidal form by including two generalization parameters, α and β, which control the concavity of the left and right sides (Rakhi Bihari et al., 2023).
A generalized trapezoidal fuzzy number Ã is defined as:

(1) $\tilde{A} = (a_1, a_2, a_3, a_4; \alpha, \beta)$

where a_1, a_2, a_3, a_4 represent the fuzzy interval boundaries, and α and β are deformation parameters (left/right generalization factors) (Rakhi Bihari et al., 2023).
The associated membership function $\mu_a(x)$ is defined as:
(2)

$$\mu_A(x) = \begin{cases} 0, & x \le a_1, \\ \left(\frac{x - a_1}{a_2 - a_1}\right)^{\alpha}, & a_1 < x \le a_2, \\ 1, & a_2 < x \le a_3, \\ \left(\frac{a_4 - x}{a_4 - a_3}\right)^{\beta}, & a_3 < x \le a_4, \\ 0, & x > a_4 \end{cases}$$

(Rakhi Bihari et al., 2023; Chen, 2000).
This expression provides greater flexibility than the classical trapezoidal shape, since α and β adjust the slope of the lateral segments, enabling the representation of asymmetric subjective perceptions.

2.3. Defuzzification and the Fuzzy Mean Value

In decision-making processes, comparing fuzzy numbers requires transforming them into scalar values.
One of the most widely used techniques is the centroid method (Center of Gravity – COG), which estimates the "center of gravity" of the fuzzy shape (Chen & Hwang, 1992).
For a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, the general formula is:

(3) $$COG(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

This relationship produces a representative numerical value of Ã, used later for ranking alternatives.
The method was also applied in educational contexts by Kaur and Aggarwal (2021), demonstrating its usefulness for aggregating students' performance levels.

2.4. Operations Between Generalized Trapezoidal Fuzzy Numbers

To apply multicriteria methods such as AHP or TOPSIS in a fuzzy environment, it is necessary to define basic arithmetic operations between GTFNs.
According to Rakhi Bihari et al. (2023), the sum of two GTFNs $\tilde{A}_1 = (a_1, a_2, a_3, a_4; \alpha_1, \beta_1)$ and $\tilde{A}_2 = (b_1, b_2, b_3, b_4; \alpha_2, \beta_2)$ is defined as:
(4)
$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \alpha_1, \beta_2)$

Multiplication by a positive scalar *k* is expressed as:
(5)
$k \otimes \tilde{A} = (k \cdot a_1, k \cdot a_2, k \cdot a_3, k \cdot a_4; \alpha, \beta)$

(Revathi et al., 2024).
These operations preserve the fuzzy structure, allowing for the computation of weights, the normalization of criteria, and the aggregation of linguistic information within the Fuzzy AHP and Fuzzy TOPSIS methods.

3. METHODOLOGY AND APPLICATION

3.1. General Framework of the Fuzzy Multicriteria Method

The evaluation of students' academic performance requires the simultaneous analysis of several cognitive, motivational, and social criteria.
The proposed model integrates two established multicriteria decision-making methods within a fuzzy context:

- Fuzzy Analytic Hierarchy Process (Fuzzy AHP) – used to determine the weights of the criteria;
- Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (Fuzzy TOPSIS) – used to rank the alternatives (students).

This combination ensures consistency between subjective assessment (AHP) and objective comparison (TOPSIS), offering a comprehensive view of academic performance (Buckley, 1985; Chen, 2000).

3.2. Determining the Criteria Weights Using the Fuzzy AHP Method

Step 1: Construction of the Fuzzy Pairwise Comparison Matrix

For a set of criteria $C_1, C_2, \dots, C_n$, the fuzzy pairwise comparison matrix is denoted as:
(6)

$$A = [\tilde{a}_{ij}], \text{ where } \tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$$

and $\tilde{a}_{ij}$ represents the linguistic assessment of the importance of criterion C_i relative to C_j (Buckley, 1985).
The values are expressed as triangular fuzzy numbers (e.g., *equally important*, *slightly more important*, etc.).
Step 2: Calculation of the Fuzzy Geometric Mean for Each Criterion
(7)

$$\tilde{g}_i = (\prod_{j=1}^{n} \tilde{a}_{ij})^{1/n}$$

This step aggregates the comparative information for each criterion (Buckley, 1985; Chang, 1996).

Step 3: Calculation of the Normalized Fuzzy Weights
(8)

$$\tilde{w}_i = \tilde{g}_i \otimes (\tilde{g}_1 \oplus \tilde{g}_2 \oplus \dots \oplus \tilde{g}_n)^{-1}$$

The resulting weights are triangular fuzzy numbers.
For application in TOPSIS, they are later defuzzified using the centroid method (Equation 3), resulting in:
(9)

$$w_i = \frac{l_i + m_i + u_i}{3}$$

The Fuzzy AHP method thus allows the derivation of objective weights for each performance criterion (cognitive, motivational, social, etc.).

3.3. Evaluating the Alternatives Using the Fuzzy TOPSIS Method

After obtaining the weights, the TOPSIS method is used to rank the alternatives (students) according to their proximity to the ideal and anti-ideal solutions (Chen, 2000; Hwang & Yoon, 1981).
Step 1: Fuzzy Decision Matrix
(10)

$$\tilde{X} = [\tilde{x}_{ij}]$$

where $\tilde{x}_{ij}$ is the fuzzy evaluation of student A_i with respect to criterion C_j, expressed as generalized trapezoidal fuzzy numbers (GTFN).

Step 2: Normalization of Fuzzy Values

For benefit-type criteria (positive direction):

(11)

$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\max_i \tilde{x}_{ij}}$$

For **cost-type criteria** (negative direction), normalization is reversed:
(12)

$$\tilde{r}_{ij} = \frac{\min_i \tilde{x}_{ij}}{\tilde{x}_{ij}}$$

(Chen, 2000)

Step 3: Construction of the Weighted Fuzzy Matrix
(13)

$$\tilde{V} = [\tilde{v}_{ij}], \tilde{v}_{ij} = \tilde{r}_{ij} \otimes w_j$$

This matrix combines the performance and importance of each criterion (Hwang & Yoon, 1981).

Step 4: Determination of Ideal and Anti-Ideal Solutions
(14)

$$A^+ = \{\max_i v_{ij} \mid j \in J_b\}$$
$$A^- = \{\min_i v_{ij} \mid j \in J_c\}$$

where J_b and J_c represent the sets of benefit and cost criteria, respectively.
Step 5: Calculation of the Distances to the Ideal and Anti-Ideal Solutions
(15)

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}$$

Each distance expresses how close student A_i is to the ideal and anti-ideal solutions (Chen, 2000).

Step 6: Calculation of the Closeness Coefficient (TOPSIS Score)
(16)

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

The higher the value of C_i, the better the student's academic performance.

3.4 Practical Implementation in Excel

The complete model was implemented in the file Final_Analiza_fuzzy_AHP_TOPSIS.xlsx, which contains dedicated worksheets for:

- The input of raw data (INPUT_DATA):
-Table containing students (GT01–GT10) and criteria (C1–C6).
-Manually entered values (e.g., linguistic scores converted to numerical form).
-Additional columns for trapezoidal fuzzy numbers (a_1, a_2, a_3, a_4).
-Definition of GTFN – corresponds to Equation (1);
-Fuzzy membership function – Equation (2).
- fuzzy conversions and GTFN computations (FUZZY_mapping), corresponds to Equation (3);
- weight calculation (AHP_weights);
-Fuzzy pairwise comparison matrix (C1–C6).
-Formulas for: Fuzzy geometric mean → Equation (6), Fuzzy weight normalization → Equation (7), Defuzzification of weights → Equation (8);
- application of the TOPSIS method (TOPSIS_calculations): Fuzzy decision matrix → Equation (9), Normalization: for benefit-type criteria, Equation (10), Weighted matrix → Equation (12), Ideal and anti-ideal solutions → Equations (13)-(14), Distances → Equation (15), Closeness coefficient → Equation (16);
- sensitivity analysis:
  -Table with modified weights for $C_1 \pm 5\%, \pm 10\%, \pm 20\%$.
  - Automatic recalculation formulas for C_i.
  - Automatic chart: "Sensitivity Analysis – Figure 2";

- weighted fuzzy profiles: Weighted fuzzy profiles (GT01–GT10 × C1–C6), Automatic chart „Weighted Fuzzy Profiles" – Figure 1.

The resulting charts (Figure 1 – Weighted Fuzzy Profiles; Figure 2 – Sensitivity Analysis) provide a visual interpretation of the numerical results.

4. RESULTS AND INTERPRETATION

The integrated application of **Fuzzy AHP** and **Fuzzy TOPSIS** methods in the analysis of the academic performance of ten students generated a coherent hierarchy of overall performance, reflecting both **objective criteria** (academic achievement, digital competence) and **subjective criteria** (motivation, social integration).
The values obtained for the closeness coefficient (C_i) are summarized in the following table:

Student	C_score	Ranking
GT07	0.8068	1
GT02	0.7796	2
GT05	0.7450	3
GT01	0.6681	4
GT10	0.6135	5
GT09	0.5808	6
GT06	0.3744	7
GT03	0.3596	8
GT08	0.2311	9
GT04	0.1887	10

Table 1. Students' ranking based on the fuzzy TOPSIS closeness coefficient.

The distribution of TOPSIS scores is presented in Figure 1, confirming the concentration of the highest scores in the upper half of the sample. Higher values indicate a stronger contribution of the corresponding criterion to the overall TOPSIS score.
The results indicate that students with high values of the closeness coefficient (C_i) demonstrate a balanced profile between academic achievement, motivation, and social integration.
In contrast, those ranked lower display learning difficulties, reduced motivation, or a need for educational and financial support.

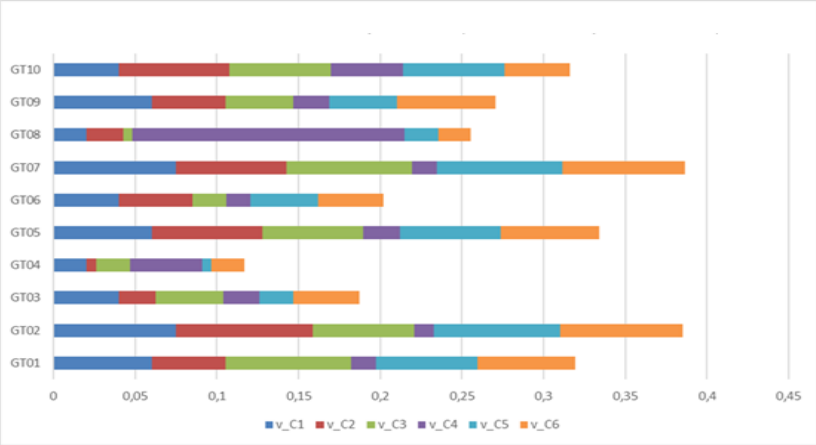

Figure 1. Weighted fuzzy profiles of academic performance for students GT01–GT10, based on the C_i values

Figure 1 illustrates the weighted fuzzy performance profiles of the ten students across the six evaluation criteria (C1–C6). Significant variations in the weighted values among students reveal the multidimensional nature of academic performance.
Students with uniform and high-value profiles (e.g., GT02 and GT07) exhibit an equilibrium between cognitive, motivational, and social dimensions.
Conversely, students with pronounced variations between criteria (e.g., GT04 and GT08) indicate specific areas where educational intervention is required.
The graphical representation confirms the robustness of the fuzzy multicriteria method, allowing for an intuitive visualization of individual differences within the analyzed sample.

4.1. Detailed Example – Student GT02

To illustrate the practical implementation of the method, the detailed calculations for student GT02 are presented below.
Raw responses (INPUT sheet): corresponding to criteria C1–C6.
Step 1 – Conversion to GTFN:

5 → (8, 9, 10, 10)
4 → (6, 7, 8, 9)

Step 2 – Calculation of Defuzzified Values (Centroids):

$$COG(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

Step 3 – Normalization and Weighting: After performing normalization and weighting, the following results were obtained:

$$COG_{GT02} = 0.7796$$

Thus, student GT02 ranks second, confirming a high level of performance and consistent motivation toward learning.

4.2. Sensitivity Analysis

The sensitivity analysis was performed on criterion C_1 – Academic Performance, which has the highest weight in the decision structure.
For this purpose, the criterion weight was incrementally modified by ±5%, ±10%, and ±20%, and for each iteration the C_i values were recalculated.
Figure 2 illustrates the stability of the ranking order as the weight of C_1 varies, with only marginal changes observed in the mid-range positions.

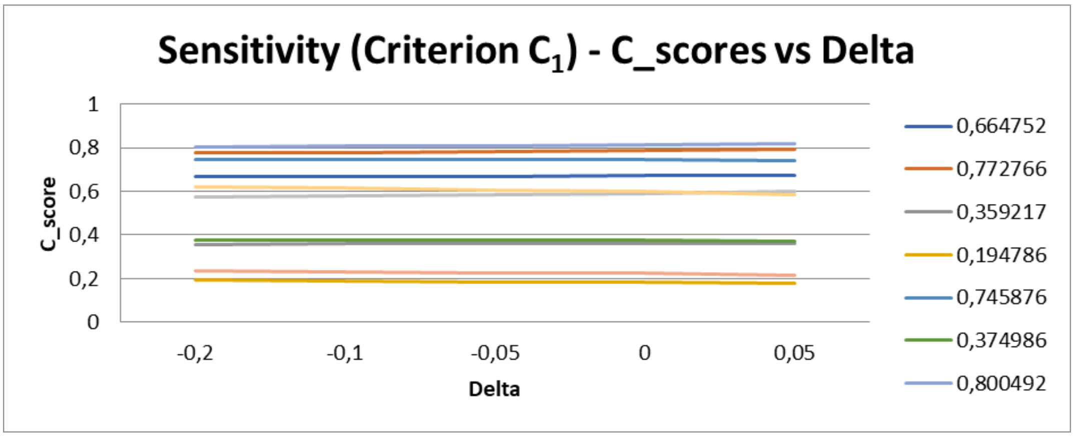

Figure 2. Sensitivity analysis of the closeness coefficients with respect to variations of the C_1 weight (±5%, ±10%, ±20%).

The results show a variation below 3% in the closeness coefficient (C_i) for the first five ranking positions, indicating a high degree of model stability and confirming the robustness of the proposed methodology.
Therefore, even when criterion weights change, the general order of students' performance remains stable, with only minor adjustments in the middle of the ranking.

5. CONCLUSIONS

The paper presented an integrated model for evaluating students' academic performance, based on Generalized Trapezoidal Fuzzy Numbers (GTFN) and multicriteria optimization methods of the AHP–TOPSIS type.
By combining fuzzy and multicriteria approaches, qualitative evaluations obtained from questionnaires and educational observations were successfully transformed into coherent quantitative values, suitable for comparative analysis and objective decision-making.

The main novelty of this study lies in the joint application of the Fuzzy AHP and Fuzzy TOPSIS methods in a real educational context, using a set of six evaluation criteria (C1–C6) that encompass both the cognitive and competence dimensions of academic performance, as well as social, motivational, and professional integration aspects.
The proposed model demonstrates that uncertainty and ambiguity inherent in subjective assessments can be effectively managed through *generalized trapezoidal fuzzy numbers,* providing a solid foundation for *ranking students according to their overall performance.*
The implementation of the model in the spreadsheet *.xlsx* confirms the practical applicability of the methodology, ensuring complete traceability of calculations and transparency in the relationships (1)-(16).
The results obtained through this instrument highlight both the consistency of the weights determined by the AHP method and the stability of the ranking generated by TOPSIS, even under controlled variations of weights (sensitivity analysis).
Figure 1 illustrates the differences in students' performance, facilitating a visual interpretation of the results and helping to identify areas requiring additional educational support.
Moreover, the sensitivity analysis (Figure 2) shows that the model is **robust**, and that changes in individual weights do not lead to significant reversals in the overall hierarchy, confirming the stability of the multicriteria decision process.

Looking ahead, the proposed model can be extended by introducing additional criteria related to extra-academic involvement, adaptability to change, and transversal competences.
It can also be further developed into a dedicated software platform integrating automated modules for data collection, processing, and fuzzy analysis of educational indicators.
Thus, the present research contributes to strengthening a rigorous scientific approach to academic performance evaluation and supports strategic decision-making in educational management.

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