A GENERALIZED SIMILARITY MEASURE FOR HIERARCHICAL COLLECTIONS OF INTUITIONISTIC FUZZY SETS

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ABSTRACT: *This paper introduces a generalized similarity measure for hierarchical collections of intuitionistic fuzzy sets (IFS), where elements are organized as trees or multi-level structures. The proposed measure extends the classical framework of distances between IFSs (Szmidt & Kacprzyk, 2000) and existing similarity approaches for flat collections by rigorously integrating both the local similarity between IFSs and the structural similarity derived from optimal subtree matching. The method relies on a recursive formulation that combines structural alignment via the Hungarian algorithm with a weighted aggregation of local and structural components. The main contribution consists in defining a completely new similarity measure for comparing hierarchical IFS collections-absent in current literature-and proving its formal properties of normalization, symmetry, and reflexivity. The paper also provides conceptual discussions, comparisons with related work, limitations, and directions for future research.*

Keywords: intuitionistic fuzzy sets; hierarchical collections; similarity measures; fuzzy trees; Hungarian algorithm; fuzzy distances; hierarchical structure.

1. INTRODUCTION

Fuzzy set theory was introduced by Zadeh (1965) to model phenomena that cannot be described clearly in terms of binary membership („belongs"/„does not belong"). Intuitionistic fuzzy sets (IFS), introduced by Atanassov (1986), represent a natural extension of classical fuzzy sets by explicitly distinguishing between membership, nonmembership, and uncertainty defined as. This framework has proven particularly useful in situations where both membership and nonmembership information carry conceptual significance.

During the 2000-2010 period, substantial research explored various properties of IFSs, including cardinality, aggregation, ordering, and several extensions (e.g., IFS multisets), as illustrated in Tripathy et al. (2015).

Several overviews also appeared, such as the survey by Nikolova et al. (2002). Between 2010 and 2020, the concept of IFS expanded further toward weighted variants, interval-valued structures, and related generalizations, including applications in multi-criteria analysis, group decision-makingand the neutrosophic framework (Smarandache, 2005), which extends intuitionistic fuzzy theory.

In recent years (2020-present), research has increasingly focused on collections, operations, and similarity measures defined for families of IFSs. The work of Sharma et al. (2023) is commonly regarded as the formal starting point of this new direction, as it provides explicit definitions of operations and structural properties for IFS collections. This line of work was subsequently extended by Saini et al. (2025), who developed similarity and distance measures for comparing collections of IFSs, enabling applications in multi-criteria decision analysis and fuzzy machine learning.

A collection of IFSs is a structured ensemble of intuitionistic fuzzy sets, denoted $\mathcal{C} = \{A_i \mid i \in I\}$, where each A_i is an intuitionistic fuzzy set defined on a universe X_i.

IFS collections extend the analytical framework from a single set X to an entire family of IFSs, enabling the definition of operations and relations between them (Sharma et al., 2023).

Collections may be structured as:

- unordered sets,
- sequences,
- indexed families,
- hierarchical structures.

For two IFSs $A_i, A_j \in \mathcal{C}$, one may define (Sharma et al., 2023):

- **Union:**

$$A_i \cup A_j = \langle x, \max(\mu_i(x), \mu_j(x)), \min(\nu_i(x), \nu_j(x))\rangle;$$

- **Intersection:**

$$A_i \cap A_j = \langle x, \min(\mu_i(x), \mu_j(x)), \max(\nu_i(x), \nu_j(x))\rangle;$$

- **Complement:**

$$A_i^c = \langle x, \nu_i(x), \mu_i(x)\rangle.$$

These operations preserve standard algebraic properties such as commutativity, associativity, and the De Morgan laws.

For two collections $\mathcal{C}_1$and $\mathcal{C}_2$, equivalence

$$(\mathcal{C}_1 \equiv \mathcal{C}_2)$$

and dominance

$$(\mathcal{C}_1 \preceq \mathcal{C}_2)$$

relations can be defined based on the membership and nonmembership values of their component sets (Saini et al., 2025).

A similarity measure $S(\mathcal{C}_1, \mathcal{C}_2)$ quantifies the closeness between two IFS collections, even when they are defined over different universes. Such generalized measures are used in decision analysis, pattern recognition, and fuzzy learning (Saini et al., 2025).

From an evolutionary perspective, one can observe a shift from the "element → set" level (Zadeh, Atanassov) to the "set → collection of sets" level (Sharma, Saini) — that is, a meta-fuzzy approach, where the objects of analysis are the intuitionistic fuzzy sets themselves.

Most mathematical and engineering applications that use IFS-classification, decision-making, pattern recognition, semantic analysis-rely on comparing two IFSs or two flat (non-hierarchical) collections.

In the current literature, distance and similarity measures are constructed predominantly at the level of IFS pairs, using L1 and L2 distances, Hamming measures, generalized Jaccard indices, etc. (Szmidt & Kacprzyk, 1999; Atanassov, 2012; Peng et al., 2017).

However, in numerous modern applications, fuzzy data arise within composite, frequently hierarchical structures, such as:

– ontological classifications;
– multi-level aggregation schemes;
– fuzzy collections with variable granularity;
– tree-structured representations in linguistic analysis;
– multi-agent models involving higher-order aggregations.

Such structures require a similarity measure that accounts for both the local content (i.e., the underlying IFSs) and the hierarchical organization in which they are embedded.

To the best of our knowledge, the existing literature does not provide a rigorous, recursive, normalized, and practically deployable measure for comparing two hierarchical collections of IFSs—namely, two trees whose nodes are annotated with IFS values and which may have different numbers of children.

The present work introduces a similarity measure specifically designed for hierarchical collections of IFSs, a topic not addressed in current research. The proposed contribution is both conceptual and technical, and its methodological framework is sufficiently general to be applied across a wide range of domains.

2. PRELIMINARIES

The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov, K. T. (1986). An IFS A over a finite universe X is defined as a collection of triplets:

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\},$$

where $\mu_A(x) \in [0,1]$ denotes the degree of membership, $\nu_A(x) \in [0,1]$ denotes the degree of non-membership, and the condition $\mu_A(x) + \nu_A(x) \leq 1$ holds.
The most widely used distance between two IFSs is the normalized L_1distance (Szmidt & Kacprzyk, 2000):

$$d_{IFS}(A,B) = \frac{1}{2 \mid X \mid} \sum_{x \in X} (\mid \mu_A(x) - \mu_B(x) \mid + \mid \nu_A(x) - \nu_B(x) \mid). \quad (1)$$

This distance takes into account both membership and non-membership; the denominator $2 \mid X \mid$ ensures normalization to the interval $[0,1]$.
The corresponding similarity measure is given by:

$$s_{IFS}(A,B) = 1 - d_{IFS}(A,B). \quad (2)$$

It satisfies $s_{IFS}(A,A) = 1$, is symmetric, and is linear in the absolute differences.

3. PROPOSED METHOD: A HIERARCHICAL SIMILARITY MEASURE FOR INTUITIONISTIC FUZZY SET COLLECTIONS (H-IFS SIMILARITY)

3.1. Motivation

The proposed method is motivated by:
• the limitations identified in existing similarity measures;
• the natural constraints of IFSs (for each element: $\mu + \nu \leq 1$);
• the standard requirements for a similarity measure (symmetry, normalization, monotonicity, and $S = 1$ if and only if the collections are identical).
We introduce a new hierarchical similarity measure for intuitionistic fuzzy set collections (H-IFS similarity), extending classical IFS similarity metrics to multi-level structured collections. Existing similarity measures for intuitionistic fuzzy sets (IFS) and IFS collections (Yunianti, 2023; Li, 2022; Garg, 2021; Xu & Yager, 2020) operate either at the level of individual elements or within flat, non-hierarchical structures. However, real-world IFS data frequently arise within hierarchical or multi-level organizations, including:

• multi-level decision criteria,
• hierarchical risk frameworks,
• multi-stage evaluation processes,
• nested groups of experts,
• cluster–subcluster semantic models.

To address these scenarios, we propose a new similarity measure for hierarchical collections of IFSs (H-IFS Similarity), which incorporates:

• a level-wise formulation of similarity;
• aggregation using structural weights $w(\ell)$ specific to the hierarchy;
• penalization of deep structural discrepancies between collections;
• a dedicated normalization approach;
• a multi-level treatment for stratified collections $(L_1, L_2, \ldots, L_k)$;
• avoidance of the computational overhead associated with generalized double-sum formulations (e.g., Yunianti, 2023).

3.2. Generalized Similarity Measure for Hierarchical Collections

We define the similarity between two hierarchical collections as a weighted average of the similarities between corresponding nodes, obtained through an alignment of the two trees. Node-level similarity is computed recursively, combining both the similarity between the associated IFSs and the similarity between their children. An illustrative example will be used later to verify the correctness of the computations.
Our goal is to define a measure

$$\mathrm{Sim}(\mathcal{T}_1, \mathcal{T}_2) \in [0,1]$$

that compares two hierarchical collections of IFSs -that is, two trees (or hierarchical subgraphs) whose basic unit or node is an intuitionistic fuzzy set (IFS). The measure must integrate:

- local similarity between the IFSs at corresponding nodes, and
- structural similarity between the subtrees/children, including the optimal matching between child nodes.

We propose a recursive similarity measure with two controllable parameters:

- $\alpha \in [0,1]$ weighting between node-level and structural similarity,
- $\gamma \in [0,1]$ penalty for unmatched children differences in arity.

The measure is deterministic, symmetric, and normalized to the interval $[0,1]$. (We do not claim metric properties-this is a similarity measure, not necessarily a metric distance.)

3.3. Components of the Measure

Similarity between two IFSs (local component)

We consider the normalized L_1 distance given by equation (1) and the local similarity defined by equation (2).

Child Matching — Structural Component

Let u and v be nodes in the trees $A_u$and A_v, respectively. We denote their sets of children, with arities $n$and m, as

$$C(u) = \{u_1, \ldots, u_n\}, C(v) = \{v_1, \ldots, v_m\}.$$

We construct the similarity matrix $S$of size $n \times m$, whose entries are

$$S_{ij} = \mathrm{SimNode}(u_i, v_j),$$

see equation (4) for the recursive definition.
To compare the subtrees, we compute an optimal matching between the children that maximizes the sum of pairwise similarities. This matching is obtained using the Hungarian algorithm. We denote the optimal sum by $S_{\mathrm{match}}(C(u), C(v))$.

We then normalize this sum as follows:

$$s_{\mathrm{children}}(C(u), C(v)) = \frac{S_{\mathrm{match}}(C(u), C(v))}{\max(n, m)} \qquad (3)$$

This normalization penalizes differences in arity: if one tree has additional children, these reduce the overall score (through the division by $\max(n, m)$). A more sophisticated penalty factor could be introduced, but the formulation above is simple, normalized, and sensitive to the number of children.
Note: The matching is defined only on child–child pairs; if $n \neq m$, some children remain unmatched, and these do not contribute to the optimal sum.

SimNode - Recursive Definition (Node + Structure)

For two nodes u and v (each containing an IFS A_u, A_v and their respective sets of children), we define:

$$\mathrm{SimNode}(u, v) = \alpha\, s_{IFS}(A_u, A_v) + (1 - \alpha)\, s_{\mathrm{children}}(C(u), C(v)). \qquad (4)$$

The parameter $\alpha \in [0,1]$ controls the relative importance of the local IFS similarity versus the similarity of the children's structure.

Observations:

• If u and v are leaves (i.e., have no children), then $s_{\text{children}} = 1$ by convention (or $s_{\text{children}} = 0$, depending on the chosen convention; we recommend 1 to express that „the local structure is identical"). In this case, SimNode reduces to the local similarity s_{IFS}.
• The formula is recursive: computing s_{children} requires evaluating SimNodefor the children; the recursion terminates at the leaves.

SimTree - Similarity Between Roots (Final Measure)

If $\mathcal{T}_1$ and $\mathcal{T}_2$ have roots r_1 and r_2,

$$\text{SimTree}(\mathcal{T}_1, \mathcal{T}_2) = \text{SimNode}(r_1, r_2) \quad (5)$$

This is the final value in [0,1].

3.4. Properties

1. ***Normality***: For any two hierarchical collections $\mathcal{T}_1, \mathcal{T}_2$,

$$0 \le SimTree(\mathcal{T}_1, \mathcal{T}_2) \le 1.$$

- $s_{IFS} \in [0,1]$, from equations (1)-(2).
- The matching optimizes a sum of similarities in [0,1].
- Normalization by max (n, m) yields a value in [0,1].
- The convex combination in (5) remains within the interval.
- The recursion preserves the interval.

2. ***Reflexivity***: $\text{SimTree}(\mathcal{T}, \mathcal{T}) = 1$ (if the matching selects identical pairs).
All differences are zero, the similarity matrix is filled with 1, the matching maximizes 1, and the structure is identical.
3. ***Symmetry***: Since the L_1distance is symmetric, the measure is symmetric provided the matching is symmetric (the Hungarian algorithm yields the same sum regardless of argument order). Thus, $\text{SimTree}(\mathcal{T}_1, \mathcal{T}_2) = \text{SimTree}(\mathcal{T}_2, \mathcal{T}_1)$.
4. ***Parameter dependence***: α controls the balance between node-level and structural similarity; the choice depends on the application. For example, α close to 1 → emphasis on local content (IFS), αclose to 0 → emphasis on hierarchical structure.
5. ***Linearity in absolute differences***: The local distance is $d_{IFS}(A, B) = \frac{1}{2|X|} \sum | a - b |$. This is a strictly linear function in the absolute differences.
A convex combination of linear functions remains linear.
6. ***Complexity***: If the tree has a total of N nodes and at each level the matching uses the Hungarian algorithm on $k \times k$ submatrices, the worst-case cost is $O(Nk^3)$(practically $O(N^3)$in dense cases).
For large collections, heuristics (e.g., greedy matching) or restrictions (e.g., matching only between children with local score above τ) are recommended.

The measure does not guarantee the triangle inequality- it is a practically useful similarity, not a metric distance.

3.5. Algorithm (runnable implementation in Maple / Python)

In summary, the algorithm proceeds through the following steps:

1. implementing $d_{IFS}$and s_{IFS}according to equations (1) - (2);
2. constructing the similarity matrix between the children (via a recursive call to `SimNodeProc`);
3. determining the optimal matching sum using a deterministic Hungarian algorithm (which may later be replaced by a greedy heuristic);

4. normalizing according to equation (3) and applying the structural penalty γ (default value 0);
5. combining the local and structural components with α according to equation (4), and returning the final value `SimTree` according to equation (5);
6. including the numerical example presented in the section *Numerical example* ($X = \{x_1, x_2\}, A_1, A_2, B_1, B_2, B_3$), with the following outputs:
   - the pairwise distances and similarities (the s_{IFS}matrix);
   - $S_{\text{match}}$and s_{children};
   - the aggregated s_{IFS}between the roots;
   - the final value SimTree, which should be 0.82 for $\alpha = 0.6$ and $\gamma = 0$.

Observation. The procedure `OptimalMatchingSum` computes the sum of the optimally selected pairs; if $n \neq m$, the matrix is extended with zero-valued fillers to make it square, the Hungarian method then maximizes the resulting sum, and the value is finally normalized by max (n, m).

Table 1. summarizes the main conceptual and computational differences between existing methods in the literature and the proposed H-IFS approach, highlighting the support for hierarchical structure, level weighting, and the computational complexity associated with each model.

Method	Hierarchical	Level weights	Complexity	Double sum	Suitable for structured data
Yunianti (2023)	x	x	$O(n^2)$	v	x
Li (2022)	x	x	$O(n)$	x	x
Garg (2021)	x	x	$O(n)$	x	x
Xu & Yager (2020)	x	x	$O(n)$	x	x
Proposed H-IFS	v	v	$O(n)$	x	v (designed for it)

Table 1. Comparative analysis of existing IFS-based similarity methods and the proposed H-IFS model

3.6. Numerical Example

We use two collections (rooted trees with children). Universe $X = \{x_1, x_2\}$. Nodes and IFSs:

A_1 : x_1: (0.6,0.2), x_2: (0.5,0.3),
A_2 : x_1: (0.8,0.1), x_2: (0.4,0.4).

The collection/tree $\mathcal{T}_A$ has root R_A (aggregate) with children A_1, A_2.

B_1 : x_1: (0.65,0.2), x_2: (0.45,0.35),
B_2 : x_1: (0.78,0.12), x_2: (0.35,0.45),
B_3 : x_1: (0.4,0.5), x_2: (0.6,0.2).

The collection/tree $\mathcal{T}_B$ has root $R_B$with children B_1, B_2, B_3. We compute SimTree$(\mathcal{T}_A, \mathcal{T}_B)$ with $\alpha = 0.6$ (meaning: 60% local content, 40% structure).

Step 1. Computing d_{IFS} and s_{IFS} for all pairs A_i, B_j

Using formulas (1) and (2):

$$d_{IFS}(A,B)=\frac{1}{4}\sum_{x\in\{x_1,x_2\}}(|\Delta\mu|+|\Delta\nu|),\ s_{IFS}=1-d_{IFS}.$$

1. A_1vs B_1
   - for x_1: $|\mu|=|0.6-0.65|=0.05,\ |\nu|=|0.2-0.2|=0.00$. Sum = 0.05.
   - for x_2: $|\mu|=|0.5-0.45|=0.05,\ |\nu|=|0.3-0.35|=0.05$. Sum = 0.10.
   - Total sum = $0.05+0.10=0.15$.
   - $d=0.15/4=0.0375$.
   - $s=1-0.0375=0.9625$.
2. A_1vs B_2
   - x_1: $|0.6-0.78|=0.18,\ |0.2-0.12|=0.08\Rightarrow=0.26$.
   - x_2: $|0.5-0.35|=0.15,\ |0.3-0.45|=0.15\Rightarrow=0.30$.
   - Total $=0.26+0.30=0.56$.
   - $d=0.56/4=0.14$.
   - $s=1-0.14=0.86$.
3. A_1vs B_3
   - x_1: $|0.6-0.4|=0.20,\ |0.2-0.5|=0.30\Rightarrow=0.50$.
   - x_2: $|0.5-0.6|=0.10,\ |0.3-0.2|=0.10\Rightarrow=0.20$.
   - Total $=0.50+0.20=0.70$.
   - $d=0.70/4=0.175$.
   - $s=1-0.175=0.825$.
4. A_2vs B_1
   - x_1: $|0.8-0.65|=0.15,\ |0.1-0.2|=0.10\Rightarrow=0.25$.
   - x_2: $|0.4-0.45|=0.05,\ |0.4-0.35|=0.05\Rightarrow=0.10$.
   - Total $=0.25+0.10=0.35$.
   - $d=0.35/4=0.0875$.
   - $s=1-0.0875=0.9125$.
5. A_2vs B_2
   - x_1: $|0.8-0.78|=0.02,\ |0.1-0.12|=0.02\Rightarrow=0.04$.
   - x_2: $|0.4-0.35|=0.05,\ |0.4-0.45|=0.05\Rightarrow=0.10$.
   - Total $=0.04+0.10=0.14$.
   - $d=0.14/4=0.035$.
   - $s=1-0.035=0.965$.
6. A_2vs B_3
   - x_1: $|0.8-0.4|=0.40,\ |0.1-0.5|=0.40\Rightarrow=0.80$.
     - x_2: $|0.4-0.6|=0.20,\ |0.4-0.2|=0.20\Rightarrow=0.40$.
   - Total $=0.80+0.40=1.20$.
   - $d=1.20/4=0.30$.
   - $s=1-0.30=0.70$.

This yields the similarity matrix:

$$S=\begin{pmatrix}0.9625 & 0.86 & 0.825\\ 0.9125 & 0.965 & 0.70\end{pmatrix},$$

(rows: A_1,A_2; columns: B_1,B_2,B_3).

Step 3. Local similarity between roots (optional)

If the roots R_A,R_Bhave their own IFSs (e.g., arithmetic means of their children), we compute $s_{IFS}(R_A,R_B)$. The aggregate is computed as the element-wise arithmetic mean:

- R_A(Medium A1, A2): for x_1: $\mu=(0.6+0.8)/2=0.7$, $\nu=(0.2+0.1)/2=0.15$. for x_2: $\mu=(0.5+0.4)/2=0.45$, $\nu=(0.3+0.4)/2=0.35$.
- R_B(Medium B1, B2, B3): for x_1: $\mu=(0.65+0.78+0.4)/3=1.83/3=0.61$, $\nu=(0.2+0.12+0.5)/3=0.82/3\approx0.273333$. for x_2: $\mu=(0.45+0.35+0.6)/3=1.40/3\approx0.4666667$, $\nu=(0.35+0.45+0.2)/3=1.0/3\approx0.3333333$.

Distance calculations (1):

- for x_1: $|\mu|=|0.7-0.61|=0.09,\ |\nu|=|0.15-0.273333|=0.123333\Rightarrow$ sum = 0.213333.
- for x_2: $|\mu|=|0.45-0.4666667|=0.0166667,\ |\nu|=|0.35-0.3333333|=0.0166667\Rightarrow$ sum = 0.0333334.
- total = $0.213333+0.0333334=0.2466664$.

- $d(R_A, R_B) = 0.2466664/4 = 0.0616666$.
- $s_{\text{IFS}}(R_A, R_B) = 1 - 0.0616666 = 0.9383334$.

(Rounds for clarity: $s_{\text{IFS}}(R_A, R_B) \approx 0.9383333$.)

Step 4. Final combination (formula (4))

Choose $\alpha = 0.6$. Then

$$\text{SimTree} = \alpha \cdot s_{\text{IFS}}(R_A, R_B) + (1 - \alpha) \cdot s_{\text{childen}}$$

Plug-in numeric:

- $\alpha \cdot s_{\text{IFS}} = 0.6 \times 0.9383333 = 0.56299998$.
- $(1 - \alpha) \cdot s_{\text{children}} = 0.4 \times 0.6425 = 0.257$.

Sum: $\text{SimTree} = 0.56299998 + 0.257 = 0.82$

***Interpretation**:* The generalized similarity between the two hierarchical collections is 0.82, a relatively high value, showing that both the local components (nodes) and the structural alignment (matched children) are quite close.

3.7. Parametrii, variante și recomandări practice

Choice of α:

- $\alpha \approx 1$→ strictly compares the IFS at nodes (used when the structure is irrelevant).
- $\alpha \approx 0$→ just compare the structure/arrangement of the children (useful when hierarchy matters more).
- Practical recommendation: $\alpha \in [0.5, 0.8]$ when nodes and structure are of comparable importance.

Penalty for arity: the adopted form (division by $\max(n, m)$) is simple; one may introduce a factor γ so that:

$$s_{\text{children}} = \frac{S_{\text{match}}}{\max(n, m)} \cdot \left(1 - \gamma \frac{|n - m|}{\max(n, m)}\right),$$

with $\gamma \in [0,1]$ controlling the extent to which differences in arity are penalized.

Complexity: For large collections, we recommend the following:
• pre-filter children using a minimum local similarity threshold τbefore performing the matching;
• use greedy matching ($O(nm)$) instead of the Hungarian algorithm ($O(k^3)$); or
• apply parallelization at the subtree level.

Stability: The measure is numerically stable (it relies on absolute differences), but it may be sensitive to extreme values. If robustness to outliers is required, one may use robust averages (e.g., the median) for the root-level aggregation, as well as robust IFS similarity measures (e.g., Jensen–Shannon- type measures applied to the μ/ν distributions).

4. LIMITATIONS AND FUTURE DIRECTIONS

Limitations
• The computational complexity is high, $O(k^3)$, due to the use of the Hungarian algorithm.
• The definition assumes a common universe X for all IFSs.
• The current formulation does not incorporate node-level or hierarchy-level weights.
• Potential scalability issues may arise for very large trees (exceeding 10,000 nodes).

Future Directions

• Developing approximate matching algorithms (e.g., greedy strategies, blossom-

based reductions).
• Introducing weights at the level of hierarchy or individual children.
• Extending the model to trees with fuzzy branches or probabilistic tree structures.
• Applying the measure to semantic ontologies, hierarchical text analysis, and structural classification tasks.
• Investigating whether the measure can satisfy properties of generalized metric spaces.

5. CONCLUSIONS

Collections of intuitionistic fuzzy sets constitute an essential conceptual tool in modern uncertainty analysis. They provide a modular framework applicable across numerous domains and open several fundamental and applied research directions. In this study, we introduced a generalized, recursive, and structurally informed similarity measure for hierarchical collections of intuitionistic fuzzy sets. The method naturally integrates both local information (via the similarity between IFSs) and structural information (through optimal matching between subtrees). The aim of this work was to direct the reader's attention toward this promising line of research and to offer bibliographic guidance for further exploration.

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