ON THE MOTION OF PLANAR BARS SYSTEMS WITH CLEARANCES IN JOINTS

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Abstract: On considers the kinematic chain composed by articulated bars with clearances with permanently contact. The goal is to apply the multibody methods and it need to determine the constraints matrix which for joints with clearances has a certain form. In the final part is presented a numerical application.

Keywords: matrix, differential equation, diagrams

1. Introducere

The clearances in joints influence the motion of a mechanical system by produced chocks and modifying the movement elements laws. In the most cases the chocks avoiding can be realized using the elastic supplementary links to compensate the clearances. In these cases the contacts between elements exist and the mechanical model is to introduce a supplementary element with length equal with half of clearance and inertialess.

In the following on considers a kinematics linkage with elements with clearance in joints. By applying the multibody methods, it obtains the differential matrix equation and a numerical application is done for the case of vibratory motion.

2. The constraints matrix

2.1. The case of joint without clearance.

The constraint equations of a jointed system are obtained from equality of the point $O_i$ co-ordinates (fig.1) which belongs to the element $i-1$ with point $O_i$ co-ordinates which belongs to the element $i$.

According to the multibody methods [1], it choices $\theta_i$ as generalized angles and $(X_i, Y_i)$ being the coordinates of the elements centers of gravity and if the length of the element $i$ is equal with $2 \cdot l_i$, on obtains the equality:

$$-X_{i-1} - l_{i-1} \cos \theta_{i-1} + X_i - l_i \cos \theta_i = 0$$
$$-Y_{i-1} - l_{i-1} \sin \theta_{i-1} + Y_i - l_i \sin \theta_i = 0$$

(1)

By derivation of the relations (1) by time and using the notations:
\[ [B(l)]_{i-1} = \begin{bmatrix} -1 & 0 & l_{i-1} \sin \theta_{i-1} \\ 0 & -1 & -l_{i-1} \cos \theta_{i-1} \end{bmatrix}; [B(l)]_i = \begin{bmatrix} 1 & 0 & l_i \sin \theta_i \\ 0 & 1 & l_i \cos \theta_i \end{bmatrix} \]  

(2)

on obtains the following equality:

\[ [B(l)]_{i-1} \begin{bmatrix} \dot{X}_{i-1} \\ \dot{Y}_{i-1} \\ \dot{\theta}_{i-1} \end{bmatrix} + [B(l)]_i \begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{\theta}_i \end{bmatrix} = \{0\} \]  

(3)

In the case when the joint \( O_i \) is fixed, the matrix \([B(l)]_{i-1}\) is zero matrix because don’t exists the element \(i-1\). The total constraints matrix is obtained by concatenating of the matrix from relation (2). Considering the open kinematics linkage from fig.3a and closed kinematics linkage from figure (3.a) on obtains the following constraints matrix.

\[ [B] = \begin{bmatrix} [B_1^{(1)}] & [0_{23}] & [0_{23}] \\ [B_1^{(2)}] & [0_{23}] & [0_{23}] \\ [0_{23}] & [B_2^{(3)}] & [0_{23}] \\ [0_{23}] & [B_3^{(3)}] & [0_{23}] \end{bmatrix} \]  

(4)

\[ [B] = \begin{bmatrix} [B_1^{(1)}] & [0_{23}] & [0_{23}] & [0_{23}] \\ [B_1^{(2)}] & [0_{23}] & [0_{23}] & [0_{23}] \\ [0_{23}] & [B_2^{(3)}] & [0_{23}] & [0_{23}] \\ [0_{23}] & [B_3^{(3)}] & [0_{23}] & [B_4^{(3)}] \end{bmatrix} \]  

(5)

where \([0_{23}]\) represents the zero matrix with \(m\) rows and \(n\) columns.

2.2. The case of joint without clearance

In the case of the clearance in the joint on accepts the model from [2], where a fictive element \(O_{i-1}^{(i)}O_i^{(i)}\) inertialless and with the length equal half of clearance is introduced (fig2).
Using the notation from fig.2 the following relation can be written:

\[ D_1^{(i)} = \cos \alpha_i = \frac{X_i - l_i \cos \theta_i - X_{i-1} - l_{i-1} \cos \theta_{i-1}}{r_i} \]  
\[ D_2^{(i)} = \sin \alpha_i = \frac{Y_i - l_i \sin \theta_i - Y_{i-1} - l_{i-1} \sin \theta_{i-1}}{r_i} \]  

it results

\[ (D_1^{(i)})^2 + (D_2^{(i)})^2 = 1 \]  

By derivation of the relations (8) by time and using the relations:

\[ D_3^{(i)} = (D_1^{(i)} \sin \theta_{i-1} - D_2^{(i)} \cos \theta_{i-1}) l_{i-1} \quad ; \quad D_4^{(i)} = (D_1^{(i)} \sin \theta - D_2^{(i)} \cos \theta) l_i \]  
\[ [D_{i-1}^{(i)}] = \begin{bmatrix} D_1^{(i)} & D_2^{(i)} & D_3^{(i)} & D_4^{(i)} \end{bmatrix} ; \quad [D_i^{(i)}] = \begin{bmatrix} D_1^{(i)} & D_2^{(i)} & D_3^{(i)} & D_4^{(i)} \end{bmatrix} \]  

on obtains the expression

\[ [D_{i-1}^{(i)}] \begin{bmatrix} X_{i-1} \\ Y_{i-1} \\ \theta_{i-1} \end{bmatrix} + [D_i^{(i)}] \begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{\theta}_i \end{bmatrix} = \{0\} \]  

In the case when the joint O is fixed, in the relation (11) it appears only \([D_i^{(i)}]\) matrix
because \(\dot{X}_{i-1} = \dot{Y}_{i-1} = 0 \); \(\dot{\theta}_{i-1} = 0\)

Thus for the kinematics linkage from fig.3.a on obtains the total constraints matrix for the case when in the joints \(O_2, O_3\) are clearances.

\[ [B] = \begin{bmatrix} [B_1^{(i)}] & [0_{23}] & [0_{23}] \\ [D_1^{(i)}] & [D_2^{(i)}] & [0_{23}] \\ [0_{23}] & [D_2^{(i)}] & [D_3^{(i)}] \end{bmatrix} \]  

3. The matrix differential equation of motion

Corresponding to the generalized co-ordinates \((X_i, Y_i, \theta_i), i = 1, 2, \ldots, n\) where \(n\) represents the elements number and denoting by \(m_i, J_i, i = 1, 2, \ldots, n\), masses respectively central inertia moments on obtains [1] the kinetic energy expression

\[ E_c = \frac{1}{2} \{\dot{q}\}^T [M] \{\dot{q}\} \]  

where

\[ \{q\} = (X_1 \quad Y_1 \quad \theta_1 \quad \ldots \quad X_n \quad Y_n \quad \theta_n)^T \]
Denoting by $F_{ix}$, $F_{iy}$, the exterior forces projections on the axes $O X, O Y$ and denoting by $M_i$ the resultant moment versus $C_i$ of the exterior forces which acts on the element $i$ on obtains [1], [4] the matrix differential equation.

\[
\begin{bmatrix}
[ M ] - [B]^T \cdot [\lambda_i] \\
[ B ] \cdot \begin{bmatrix} O_{j,3n} \end{bmatrix}
\end{bmatrix} = \begin{bmatrix} F_i \\
- [B] \cdot [\dot{\lambda}_i]
\end{bmatrix}
\]

(16)

where:
- $j$ is matrix $[B]$ row number
- $\lambda_i, i = 1, 2, ... j$ are Lagrange multipliers.

\[
[F] = (F_{ix} \quad F_{iy} \quad M_1 \quad \ldots \quad F_{nx} \quad F_{ny} \quad M_n)^T
\]

(17)

\[
[\lambda] = (\lambda_1 \quad \lambda_2 \quad \ldots \quad \lambda_j)
\]

(18)

\[
[R] = [\theta_j] \cdot [\dot{\theta}_j] \cdot [\dot{\theta}_j]
\]

(7)

In the case of joint without clearance the Lagrange multipliers from joint $i$ are the reactions $H_i, V_i$ and for the case of the joint with clearance the multiplier is the axial force $N_i$ along the fictive element.

To equilibrium the following expression are obtained

\[
[B]^T \cdot [\lambda] + [F] = [0]
\]

(19)

The equations (19) together with equations (1) or (8) compose the system which gives the vector $[\ddot{q}]$ for the equilibrium estate.

Making the substitution

\[
[q] = [\xi] + [\ddot{q}]
\]

(20)

on obtains the matrix differential equation of vibrations versus the précised equilibrium position.
4. Application

For the bars system from (fig.4) acted only by own elements weights the goal is to determine the time variation of the parameters \( \theta_1, \theta_2, \lambda \) knowing that the joint \( O_1 \) is with clearance and the initial condition are given below:

\[
t = 0; \theta_1 = 0; \theta_2 = \theta_2^0; X_1 = l_1; Y_1 = 0; X_2 = 2l_1 + r_2 + l_2 \cos \theta_2; Y_2 = l_2 \sin \theta_2; \dot{\theta}_1 = \dot{\theta}_2 = 0, \quad \dot{X}_1 = \dot{X}_2 = 0
\]  

(21)

Numerical application:

\( l_1 = l_2 = 0.5 \text{ m}; m_1 = m_2 = 1.2 \text{ Kg}; J_1 = J_2 = 0.1 \text{ Kg} \cdot \text{m}^2; \theta_2 = 0.3 \text{ rad}; r = 0.01 \text{ m} \)

The numerical calculus is made with the help of a routine draw up by the algorithm described in the paper and it leads to the results plotted in the diagrams from fig.5 a, b, c, d.

5. Conclusions

Fig.5.c) represents the direction variation \( \alpha \) of the fictive element and it detects that the amplitude is did not exceed the 0.25 radians value.

Fig.4
Fig. 5

Also it detects that the angle $\theta_1$ amplitude did not exceed 0.15 rad value, the angle $\theta_2$ amplitude did not exceed the 0.3 rad value and the axial force variation (fig.5.d.) is mostly 7 N.

References