STUDIES REGARDING THE DETERMINATION OF THE ADEQUATE ASSEMBLY TOLERANCES DEPENDING ON THE MANUFACTURING COSTS OF THE PARTS PROCESSED ON CLASSIC MACHINE TOOLS

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Abstract: *The paper deals with a study case regarding the determination of adequate assembly tolerances. The study concerned the assembly of two bearings on an axle. Determining the adequate tolerances of the sizes chains is based on Lagrange's multipliers use. The optimization criterion for determining the assembly tolerances is the minimization of the product achievement cost. The relation between tolerances and cost will be revealed in a graphic component.*

Keywords: assembly, tolerances, costs.

1. Introductory notions.

In order to determine and allocate the adequate assembly tolerances and not only, a significant factor is the function between cost and tolerance. In time, several algebraic functions that make the connection between cost and tolerance have been described.

The mathematic models for approximating the tolerance – cost relation are:

- energetic pattern (1969): $C = A + B\,T^k$
- exponential pattern (1972): $C = A \cdot e^{-B\,T}$
- mutual quadratic pattern (1973): $C = A + B/T^k$
- mutual energetic pattern (1975): $C = A + B\,T^{-k}$
- mutual pattern (1990): $C = A + B/T$
- combined exponential-linear pattern (1991): $C = A + B + B_1T + B_2T^2 + B_3{}^{12}T^3$

All the relations described above could not consider all the factors that contribute to manufacturing costs, referring here to the cost of materials, excise costs, cutting tool costs, etc. Therefore, in time various objective functions have been proposed as they include various types of costs. Unfortunately, all the information regarding a part manufacturing cost is not published by companies, because even the companies that use the same machine may have various costs from one section to another (manpower, materials, devices, etc). A study between cost and tolerance is very necessary for cutting processes for various values of nominal sizes.

The study regarding the processing cost depending on tolerances is approached in this paper for classic machine tools and this is why it cannot be applied for modern machine tools.

In order to be able to determine the minimal cost depending on elements tolerances, Lagrange's method has been developed, supposing a cost function of the form:

$$C = A + B/T^2 \qquad (1)$$

It was extended, using the cost function as follows:

$$\frac{\partial}{\partial T_i}(func.\cos t) + \lambda \frac{\partial}{\partial T_i}(func.\ tol.) = 0 \qquad (i = 1,\dots,n) \qquad (2)$$

$$\frac{\partial}{\partial T_i}\left(\sum\left(A_j + \frac{B_j}{T_j^{k_j}}\right)\right) + \lambda \frac{\partial}{\partial T_i}\left(\sum T_j^2 - T_{asm}^2\right) = 0 \qquad (i = 1,\ldots,n)$$

$$\lambda = \frac{k_i B_i}{2T_i^{(k_i+2)}} \qquad (i = 1,\ldots, n)$$

Eliminating λ, expressed by term T_1 (arbitrarily selected):

$$T_i = \left(\frac{k_i B_i}{k_1 B_1}\right)^{1/(k_i+2)} \cdot T_1^{(k_1+2)/(k_i+2)} \tag{3}$$

Replacing for every T_i in the assembly tolerance sum, we get:

$$T_{asm}^2 = T_1^2 + \sum\left(\frac{k_i B_i}{k_1 B_1}\right)^{2/(k_i+2)} \cdot T_1^{2(k_1+2)/(k_i+2)} \tag{4}$$

A similar derivation based on the algebraic method, that is the product of allocated tolerances sum, is the equation:

$$T_{asm} = T_1 + \sum\left(\frac{k_i B_i}{k_1 B_1}\right)^{1/(k_i+1)} \cdot T_1^{(k_1+1)/(k_i+1)} \tag{5}$$

The unknown in this relation is T_1. The value of T_1 will repeat itself until both parts of the equation are equal, achieving tolerances with minimal costs.

2. Case study.

In order to determine adequate tolerances using Lagrange's method, we present, as a case study, the assembly method of bearings on an axle, fig.1. Two brackets maintain the distance between the bearings in order for them to be able to fix on the axle. Tolerances accumulation in the assembly results in the end in the variation of the compensating element. In the case sizes chain solving is made through the compensation method. For this case positive compensation is necessary.

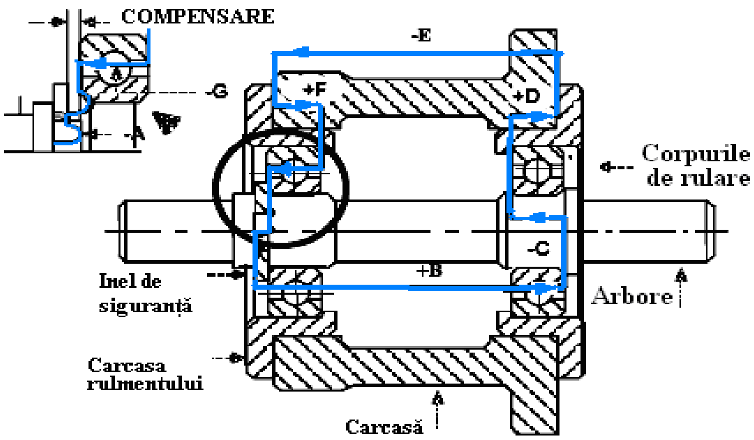

Fig. 1. Bearings assembly on an axle.

The initial tolerances for parts B, D, E, and F are selected from tolerance guides, like for instance those presented in table 1 (tolerances taken from the specialized literature or from the STAS for certain accuracy classes and various values of nominal sizes).

The histogram indicated the typical tolerance field for several technological operations for making the parts: facing, boring, enlarging, reaming, milling, broaching, rectification, etc. Every row of the table corresponds to a different field of values of the nominal size. For instance, a part is faced at a nominal size of Ø30mm it can be created at a tolerance field from 0,03 mm to 0,20 mm. The way these tolerances are achieved depends on the number of passing, the type of the machine tool, its roughness, the fastening devices of the part, the cutting tool, etc.

It is indicated that tolerances be initially chosen at the middle of the tolerance field for every processed size, then to be adopted within the design limitations suitable, therefore reducing manufacturing costs.

Table 2 presents the information on this problem. The safety ring ***(A)*** and the two bearings ***(C)*** and ***(G)*** assembled on the axle are purchased parts, therefore their tolerances are fixed and cannot be changed during the tolerances allocation process. The other parts are processed in the factory. Depending on parts role and importance, during their design process, tolerances values are initially given for parts B, D, E and F.

Table 1. Sizes field of nominal values and their tolerances.

Sizes field, in mm		**Tolerance, in mm**								
from	**to**									
0	**15**	0,004	0,005	0,008	0,013	0,020	0,030	0,050	0,075	0,13
15	**25**	0,004	0,006	0,010	0,015	0,025	0,040	0,065	0,12	0,15
25	**40**	0,005	0,008	0,013	0,020	0,030	0,050	0,075	0,13	0,20
40	**70**	0,006	0,010	0,015	0,025	0,040	0,065	0,12	0,15	0,25
70	**110**	0,008	0,013	0,020	0,030	0,050	0,075	0,13	0,20	0,30
110	**190**	0,010	0,015	0,025	0,040	0,065	0,12	0,15	0,25	0,40
190	**355**	0,013	0,020	0,030	0,050	0,075	0,13	0,20	0,30	0,50
355	**500**	0,015	0,025	0,040	0,065	0,12	0,15	0,25	0,40	0,65
Processing type										
Lapping and honing		xxxxxxxxxxxxxxxxxx								
Diamond rectification and facing		xxxxxxxxxxxxxxxxxxxxxxxxx								
Broaching			xxxxxxxxxxxxxxxxxxxxxxx							
Reaming				xxxxxxxxxxxxxxxxxxxxxxxxxxxxx						
Facing, enlarging, planing						xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx				
Milling							xxxxxxxxxxxxxxxxxxxxxxxxxx			
Boring								xxxxxxxxxxxxxxxxxxx		

These tolerances are initially selected from table 1, supposing a value that is at the middle of the tolerance field they are part of. The compensation that will appear will be represented by the clearance between the axle and the bearing support ring which is determined with the tolerance accumulation in the assembly.

Tab. 2. Description of initial tolerances

Component elements	**Nominal size, mm**	**Initial tolerance, in mm**	**Processed tolerances limitations, in mm**	
			Minimal tolerance	**Maximal tolerance**
A	1,3	0,038– fixed tolerance	-	-
B	203,2	0,2	0,075	0,50
C	12,95	0,06– fixed tolerance	-	-
D	10,2	0,05	0,012	0,130
E	195,9	0,15	0,065	0,40
F	10,2	0,05	0,012	0,130
G	12,95	0,06– fixed tolerance	-	-

The average compensation is given by the sum of nominal sizes of the parts included in the assembly cycle and is given by the relation:

Average compensation=-A+B-C+D-E+F–G=-1,3+203,2-12,95+10,2–195,9+10,2-12,95=0,5mm

Necessary compensation = 0,5±0,2 mm

The algebraic method is used to achieve the compensated tolerance by adding the initial tolerances of the chain elements, as follows:

$$\mathrm{T}_{\Sigma} = \mathrm{T}_A + \mathrm{T}_B + \mathrm{T}_C + \mathrm{T}_D + \mathrm{T}_E + \mathrm{T}_F + \mathrm{T}_G = 0.038 + 0.2 + 0.06 + 0.05 + 0.15 + 0.05 + 0.06 = 0.608 \text{ mm}$$

In order to apply the minimal cost algorithm we have to: $T_{\Sigma} = T_{Asm} - T_{Fix}$

And replacing T_D, T_E and T_F in T_B, we have the equation:

$$T_{Asm} - T_A - T_C - T_G = T_B + \left(\frac{k_D B_D}{k_B B_B}\right)^{1/(k_D+1)} \cdot T_B^{(k_B+1)/(k_D+1)} + \left(\frac{k_E B_E}{k_B B_B}\right)^{1/(k_E+1)} \cdot T_B^{(k_B+1)/(k_E+1)} + \left(\frac{k_F B_F}{k_B B_B}\right)^{1/(k_F+1)} \cdot T_B^{(k_B+1)/(k_F+1)} \tag{6}$$

$$0.35 - 0.038 - 0.06 - 0.06 = \mathrm{T}_B + \left(\frac{0.46823 \cdot 0.07202}{0.43899 \cdot 0.15997}\right)^{1/1.46823} \cdot \mathrm{T}_B^{(1.43899/1,46823)} + \left(\frac{0.46537 \cdot 0.12576}{0.43899 \cdot 0.15997}\right)^{1/1.46537} \cdot \mathrm{T}_B^{1.43899/1.46537} + \left(\frac{0.46823 \cdot 0.07202}{0.43899 \cdot 0.15997}\right)^{1/1.46823} \cdot \mathrm{T}_B^{1.43899/1.46823} \tag{7}$$

The values of k and B for every nominal size were achieved from the cost-tolerance functions for every operation, being described in tables from the specialized literature. Following the mathematic solution, of the above relation, we get the value of T_B. The resulted value of T_B will be replaced in the individual expressions in order to get the related values of T_D, T_E and T_F and forecasted cost.

$$\mathrm{T}_B = 0.06\text{mm}$$

$$\mathrm{T}_D = \mathrm{T}_F = \left(\frac{0.46823 \cdot 0.07202}{0.43899 \cdot 0.15997}\right)^{1/1.46823} \cdot \mathrm{T}_B^{1.43899/1.46823} = 0.038\text{mm} \tag{8}$$

$$\mathrm{T}_E = \left(\frac{0.46537 \cdot 0.12576}{0.43899 \cdot 0.1599}\right)^{1/1.46537} \cdot \mathrm{T}_B^{1.43899/1.46537} = 0.056\text{mm}$$

Table 3. Allocated tolerances and their cost.

Data on the tolerance cost					Tolerances allocation	
Size	Assembly A	Coefficient B	Exponent k	Initial tolerance, in mm	Algebraic method	Statistic method
A	-	-	-	0,038	0,038	0,038
B	1	0,15997	0,43899	0,2	0,06	0,18
C	-	-	-	0,06	0,06	0,06
D	1	0,07202	0,46823	0,05	0,038	0,12
E	1	0,12576	0,46537	0,15	0,056	0,18
F	1	0,07202	0,46823	0,05	0,038	0,12
G	-	-	-	0,06	0,06	0,06
Tolerances variation during assembly				0,608 0,276	0,35	0,35
Cost				9,34UM	11,07UM	8,06UM
Acceptance ratio					1,00	0,99737
Real cost					11,07UM	8,08UM

The assembly cost is given by coefficient A. The installation cost does not influence optimization. For this example, assembly costs are chosen equal, in this way they will not mask the effect of the allocated tolerance. In this case, only the cost of 4,00 monetary units is added to the assembly for every case, in order not to influence the final cost.

Parts A, C and G are supplied (purchased), and then their tolerances are fixed and their price cannot be changed by reallocating the tolerances, and therefore no cost information is included in the table.

The allocated tolerance resulted by calculation with the help of the statistic method was similarly achieved, using equation (1). In this assembly example, as we can notice in table 3, the cost grows when tolerances allocation is calculated through the algebraic method. The initial tolerances, when added through the algebraic method, give an assembly variation of 0,608 mm.

This exceeds the specific tolerance assembly limitation of 0.35mm. Therefore, it was necessary to decrease the allocated tolerances, by increasing costs.

When the tolerance is achieved through the statistic method, the assembly variation was 0,275mm. This value is below the specified assembly tolerance limitation. Therefore, the allocated tolerances increased by decreasing costs. A graphic comparison is presented in fig. 2. The real cost, in table 4, is defined as being the total cost of an assembly divided to benefits.

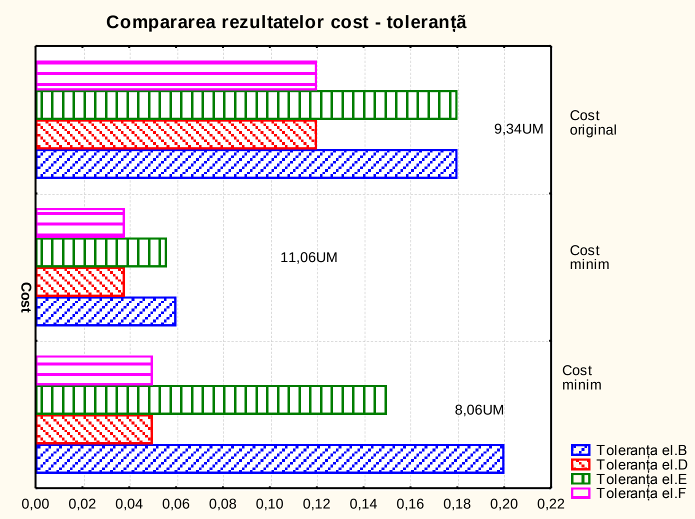

Fig. 2. Comparison of the results between tolerance and cost

Tab. 4. Minimal real cost.

Cost model	Assembly cost	z	Optimal acceptance ratio	Real cost
A + B/tol^k	**4UM**	**2,03**	**0,9576**	**7,67UM**
A + B/tol^k	**8UM**	**2,25**	**0, 9756**	**11,82UM**

Therefore, the total adopted cost also includes a part of the assembly cost. The total cost is adjusted in order for a part of it to include the cost of assembly waste. Anyway, it does not include some parts that can be recovered or the cost of waste parts as individual component parts.

A very important thing is to calculate the optimal acceptance ratio, meaning waste rate resulting in the minimal real cost. This requires an iterative solution. For the presented case, the results are recorded in table 4. The results indicate that the increase of tolerances will save manufacturing costs money, but will increase waste cost. The iterative solution applied to the acceptance ratio allows to find the value that minimizes the combined cost between manufacturing cost and waste cost. As we can notice, the main costs were established as being low. If they double, as in the second row of the table, waste costs would be higher, requiring a higher acceptance level.

3. Conclusions

The optimal determination of assembly tolerances depending on the parts manufacturing cost was made by developing Lagrange's multipliers method.

The advantages for developing this method are:

- it can easily handle the algebraic calculation method or the statistic calculation method.
- it allows alternative cost-tolerance methods.

Disadvantages:

- calculations apply only for the parts processed on classic machine tools;
- admitted limitations cannot be imposed to technological operations;
- we cannot promptly deal with the simultaneity problem by optimizing the interdependent design characteristics. The problems that have such characteristics can be optimized by using a non-linear technical program.

The total cost is adopted and includes a part of the assembly cost. The total cost is adjusted, in order for a part of it to include the cost of assembly waste. A very important thing is that the optimal acceptance ratio was calculated, that is the waste ratio results in the minimal real cost. It requires an iterative solution. The iterative solution applied to the acceptance ratio allows to find the value that minimizes the combined cost between manufacturing cost and waste cost. As we can notice, the main costs were established as being low. If they double, as in the second row of the table, waste costs would be higher, requiring a higher acceptance level.

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