

# STRENGTH ANALYSIS METHODS OF CIRCULAR PULL BROACH COGS

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**Abstract:** A very big importance in a pull broach designing is represented by its mechanic computation, which trots out the pull broach resistance on various blank tooling, pull broach productivity and also the loadings which is subdued to and the stresses that appear during the chipping process. The pull broach geometric complexity leads to one difficulty concerning the resistance computing methods application (and implicitly, simplifying assumptions application). This present study presents a resistance computing of pull broach cogs, which dresses a circular hole trotting out more methods which can be used in this computing, and the teoretic aspects are then trotted out by an example of a numerical computation for a particular case.

**Keywords:** pull broaches, mechanical resistance, mechanical stress, finite-elements analysis, circular plate

## 1. Introduction

The pull broaches are tools of high-levelled productivity, which are used at chipping processing of circular holes, various inner channels, and also at simple or profiled plane outer surface processing. The pull-broaches are high-levelled, constructive and operational complexity tools which leads to a high-cost price. From this reason, it is only used in big production or in operations found in lots of adjusting strips (eg: for quoin channel tooling). An important chapter in pull broach designing is represented by its resistance computing. The high geometrical complexity of the pull broach leads to difficulties in picking the best computing method which has to ensure its resistance during the chipping operation, and to trot out the best phenomenon which takes place during the tooling process. The present study presents a strength computing of a circular pull broach cogs which processes a circular hole, possible stresses on which the pull broach is subddued during the tooling process, being studied. There are approached more mathematics computing simplifying methods and also finite elements analysis methods. The pull broaches are generally made of high steels. For this present study the high steel HS-18-01 STAS SR EN ISO 4957 is chosen as material. It is considered that the pull broach works on a draggind broached machine. In a general case, a circular pull broach which processes a circular hole has a form presented in figure 1.

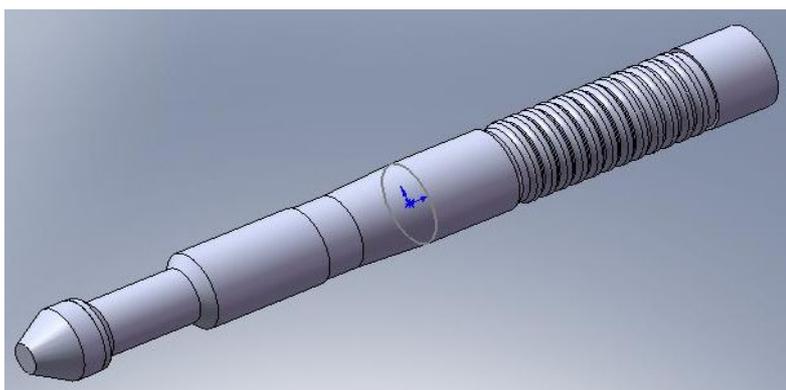


Fig.1. circular pull broach

## 2. The strength computing of the pull broach cogs

The circular pool borach is a tool which has, in general, its cutting part made of three parts: roughing, finishing and calibrating parts. The strength computing will be made for the first roughing cog that contacts the material and parts the addition processing, which is the most intensively loaded.

## 2.1 The cog is subdued to bending

Two cases are taken into consideration: variant 1- the cog is a bar with a fixed end; variant 2- the cog is a circular plate.

### 2.1.1 Variant 1

This method considers the pull broach cog as a bar with a fixed end and is actuated at the other end by the chipping force according to the figure 2, using the notations:  $h$ - the height of the cog,  $F$ - the chipping force,  $M_2$  and  $V_2$  reactions (the unknown introduced by constraint).

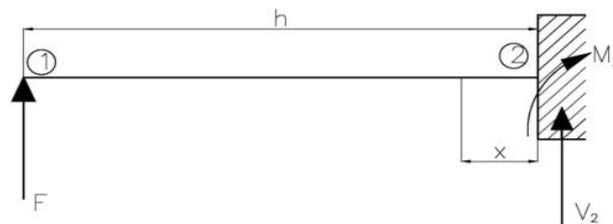


Fig. 2. bar with a fixed end

According to the upper schematization, the cog is subdued only to bending. The bending stress is determined by the relation (1):

$$\sigma_{il} = \frac{M_{imax}}{W}, M_{imax} = F \cdot h \quad (1)$$

where  $M_{imax}$  – maximum bending moment,  $W$ - axial strength modulus.

So that the pull broach would not be broken during the tooling process, the condition  $\sigma_{il} < \sigma_{ai}$  has to be fulfilled, where  $\sigma_{ai}$  is the stress admissible to the bending of the material of which is made the pull broach (in our case there is HS-18-01 STAS SR EN ISO 4957).

To demonstrate the upper announced theoretical relations, this method is used for a chosen particular case, in this way:

$F = 2400\text{N}$ ;  $h = 3,5 \text{ mm}$ ;  $f_1 = 3\text{mm}$ ;  $W = 4,5\text{mm}^3$ ,  $\sigma_{ai} = 1000 \text{ N/mm}^2$  (3), where  $f_1$  is the width of the cog.

The results are:  $M_{imax} = 8400 \text{ N}\cdot\text{mm}$ ,  $\sigma_{il} = 1867 \text{ N/mm}^2$  (4)

We observe that  $\sigma_{il} > \sigma_{ai}$  which means that the cog doesn't resist at the bending stress. In this case, so that the cog would resist, the next measures have to be taken: changing the chosen material with a more resistant one to bending; increasing the axial strength modulus which leads to a smaller bending stress; decreasing the height of the cog which leads to the decreasing of the bending moment.

### 2.1.2 Variant 2

In this case, the cog is also subdued to bending, but it is taken into consideration the fact that the cog is a plate, the action of the chipping force won't be in a point, but on the all exterior contour. Because of the geometrical complexity the cog has, it is considered that the pull broach cog has the same width on all its height (practically, the most inimical case is considered because the section of the cog base decreases). This is presented in figures 3,4 and 5. According to the considerations that were made, axial chipping force, actuates the cog just like in figure 5. To make this computing, the cog will be considered as a circular plate, constrained on the interior contour and actuated on the exterior contour. According to [5], the maximum stress that makes the bending is computed with the relation:

$$\sigma_{imax} = (2 \cdot F \cdot k) / (f_1)^2 \quad (5)$$

where  $k$  is a factor that heeds the  $a/b$  ratio with the values given in tabel 1.

Table 1

a/b	1,25	1,5	2
K	0,227	0,428	0,753

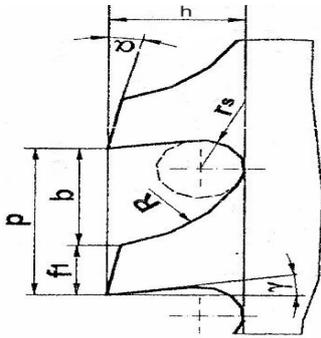


Fig.3. real pull broach cog[4]

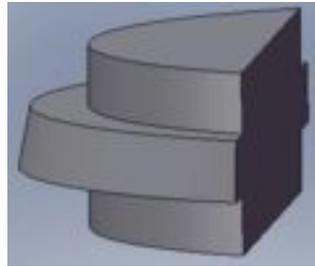


Fig.4-simplified pull broach cog:  
3D sectional representation

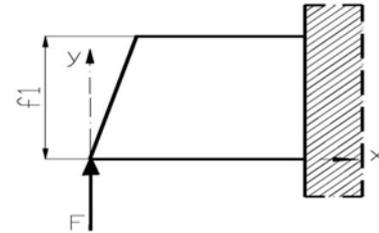


Fig.5.chipping force action

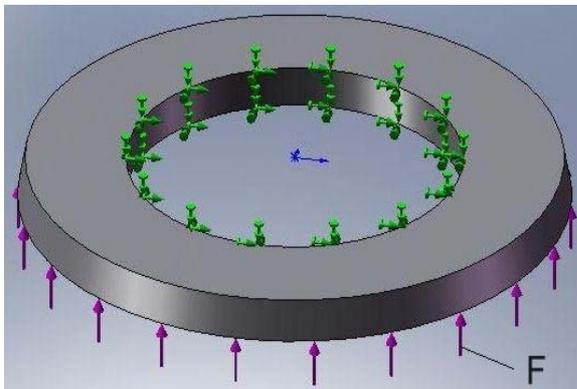


Fig.6. The cog is a circular plate constrained on the interior contour and actuated on the exterior contour

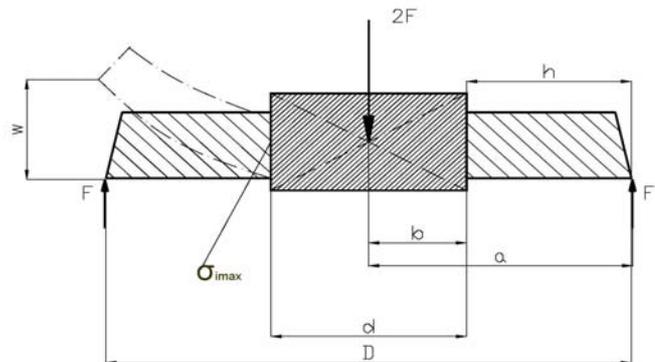


Fig.7. cog loading schematization [5]

If the a/b ratio values is found in tabel 1,  $\sigma_{imax}$  is directly computed and the next condition must be fulfilled:  $\sigma_{imax} < \sigma_{ai}$  (6), so that the cog resists bending. If the a/b ratio is not found in tabel 1, the interpolation is used to find out the value of k, following next steps: we create the function  $y(x) = mx^2 + nx + p$ , the values are consecutively given to x and three unknown equations system is made; the unknown 3 equations system is solved and then the values m,n,p are obtained resulting that all the parameters of the function y(x) are known;  $x=a/b$  is inserted in the function y(x) resulting  $k=y(a/b)$ ;  $\sigma_{imax}$  will result by computing the value of k; the strength condition (6) is checked. To demonstrate the upper theoretical relations, this method is used for a particular case, the same from 2.1.1 subitem:

$$F=2400N; h=3,5mm; f_1=3mm; \sigma_{ai}=1000N/mm^2; D=45mm; d=44,5mm \quad (7)$$

The steps showed at 2.1.2 subitem are followed and there are obtained next results (it is mentioned that is the case where k is not directly chosen from the tabel):

$$a=22,5 \text{ mm}; b=19,25mm; m=1,253; n=-3,311; p=2,437; y(x) = 1,235x^2 - 3,311x + 2,437; \quad (8)$$

$$x=a/b = 1,184; k=y(1,184) = 0,247; \sigma_{imax} = 131,8N/mm^2 < \sigma_{ai}.$$

The strength condition at the bending of the cog is checked by having :

$$\sigma_{imax} = 131,8N/mm^2 < \sigma_{ai} = 1000N/mm^2 .$$

## 2.2 The cog is subdued to bending impact

It is considered that the cog enters the material with impact. There are considered two cases: variant 3- the cog is a bar with a fixed end and actuated at the other end by chipping force ; variant 4- the cog is a circular plate actuated by the chipping force on all the exterior contour.

### 2.2.1 Variant 3

It is considered the same case as the one at the subitem 2.1.1. The static displacement is computed with the relation (9):

$$v = (F \cdot h^3) / (3 \cdot E \cdot I_z) \quad (9)$$

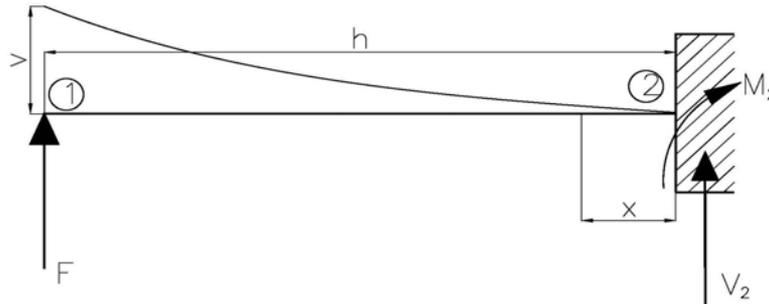


Fig.8.variant 3 computing scheme

where E- longitudinal elastic modulus,  $I_z$ - inertia moment.

Then, the impact intensifier  $\psi$  is computed with relation (10):

$$\psi = 1 + \sqrt{1 + \frac{2 \cdot h_c}{v}} \quad (10)$$

where  $h_c$ - the height between the force F and the cog until they make contact. If the chipping force is suddenly applied, then  $h_c=0$ . The maximum stress produced to impact will be:  $\sigma_{soc} = \sigma_{st} \cdot \psi$  (11), where  $\sigma_{st}$  – the maximum stress produced at the statical application of the chipping force (it is identical with the stress  $\sigma_{i1}$  produced at the bending case presented at the subitem 2.1.1). The condition  $\sigma_{soc} < \sigma_a$  (12) must be fulfilled, where  $\sigma_a$  is the impact admissible stress of the material of which the pull broach is made. The dynamic displacement is computed with the relation:

$$f = \psi \cdot v \quad (13)$$

To demonstrate the upper defined theoretical relations, this method is used for a particular case (the same treated as the one from the item 2.1) defined this way:  $F=2400\text{N}$ ;  $h=3,5\text{mm}$ ;  $f_1=3\text{mm}$ ;  $I_z=6,75\text{mm}^3$ ;  $\sigma_a=1000\text{N/mm}^2$ ;  $E=2,1 \cdot 10^6\text{N/mm}^2$ ;  $h_c=0$ . (14)

Numerically replacing, we obtain:

$$v = 2,42 \cdot 10^{-3}\text{mm}; \psi = 2; \sigma_{soc} = 3733\text{N/mm}^2; f = 4,84 \cdot 10^{-3}\text{mm} \quad (15)$$

It is noticed that  $\sigma_{soc} > \sigma_a$ , the condition (12) is not respected and the cog does not resist to impact.

### 2.2.2 Variant 4

It is used the scheme from figure 8. According to [5], the displacement is computed with the next relation:

$$w = k_1 \cdot \frac{F \cdot a^2}{E \cdot f_1^3} \quad (16)$$

where  $k_1$  is a factor that caters for a/b ratio with the values given in the table 2.

Table 2

a/b	1,25	1,5	2
k <sub>1</sub>	0,0051	0,0249	0,0877

If the a/b value is found in tabel 2, w is directly computed and then the impact intensifier,  $\psi$ , is computed with relation (10). The maximum stress produced at impact is computed with relation (11), the strength condition (12) is checked and the dynamic displacement is computed with relation (13). If the a/b value is not found in tabel 2, the interpolation is used to find out the value of k<sub>1</sub> following the methodology described at 2.1.2 subitem. To demonstrate the upper defined theoretical relations, this method is used for a particular case (the same treated at subitem 2.2.1). There are obtained the next results (it is mentioned that it is the case where k<sub>1</sub> is not directly chosen from the tabel 2):

$$a=22,5\text{mm}; b=19,25\text{mm}; m=0,062; n=-0,091; p=0,022; y(x)=0,062x^2-0,091x+0,022;$$

$$x=a/b=1,184; k_1=y(1,184)=1,175 \cdot 10^{-3}. \quad (17)$$

$$\text{The displacement is computed with relation (16): } w=2,517 \cdot 10^{-5}\text{mm}. \quad (18)$$

Then, the other elements are computed, obtaining:  $\Psi=2$ ;  $\sigma_{\text{soc}}=263,6 \text{ N/mm}^2$ ;  $f=5,035 \cdot 10^{-5}\text{mm}$ . It is noticed that  $\sigma_{\text{soc}} < \sigma_a$ , the strength condition is fulfilled and the cog resists at impact.

### 2.3 Finite element method

It is considered that the pull broach cog, which according to figure 10, is fixed on the interior contour and actuated on the exterior contour by the force F, the chopping force. It is used the same particular case defined at the previous items. After the finite element analysis, the stress map from figure 10 resulted. It is noticed that the maximum stress produce by force F is  $\sigma_{\text{max}}=34,14\text{N/mm}^2 < \sigma_{\text{ai}}=1000\text{N/mm}^2$ , so the cog resist to bending. The finite element analysis is made only for the static case.

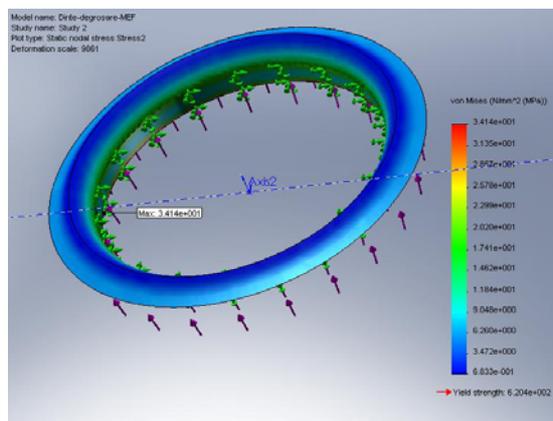


Fig.9. stress distribution in the cog.

### 3. Conclusions

We can extract the following conclusions:

- the using of the first method (the one from 2.1 subitem) leads to implausible results because it is noticed that the cog does not resist to the bending solicitation, the bending stress being too big than the one really produced; same conclusions can be made from the impact loading too.

- the results obtained with the second assumption are sustained by the finite element

analysis (in this cases, similar values are obtained), the bigger bending stress obtained with the second method resulted from the fact that the most inimical case was chosen (which is: the width of the cog is constant on all its height)

- the second variant gives satisfying results also for bending impact computing, a sollicitation to which the cog resists

- for a complete strength computing, it is recommended to use the methods presented at the subitems 2.1.2, 2.2.2 and 2.3.

### References

[1] **Case, J., Chilver, L., Ross, C.**, *Strength of Materials and Structures*, Fourth Edition, Published by Arnold, London, 1999

[2] **Ilincioiu, D.**, *Rezistența Materialelor*, Ediția a-2-a, Editura ROM TPT, Craiova, 2007

[3] **Nash, W.**, *Theory and Problems of Strength of Materials*, Fourth Edition, McGraw-Hill, 1998

[4] **Stoian, A.**, *Proiectarea Broșelor*, Editura Universitaria, Craiova, 2004

[5] **Timoshenko, S., Woionowsky-Krigev, S.**, *Teoria Plăcilor Plane și Curbe*, Editura Tehnică, București, 1968