ASPECTS OF MATHEMATICAL MODELING AND INTERPRETATION OF A MANUFACTURING SYSTEM

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Abstract: *In the paper developing we started from a model that allows a detailed decoding of causal relationships and getting the laws that determine the evolution of the phenomenon.*

The model chosen for the study is a discrete event system applicable to optimize the transport system used in pottery. In order to simulate the manufacturing process we chose Matlab package that contains pntool library, by which can be realized modeling of analyzed graphs. Since the timings of manufacture are very high and the process simulation is conducted with difficulty, we divided the graph according to the transport system.

Keywords: Petri networks, system with discrete events, optimization, transport system

1.INTRODUCTION

The informatics models are designed to model the flows of information, to process and to store information with the task of enabling the analysis, designing and optimization of these. According to the literature, the traditional patterns are linear or nonlinear, deterministic or continuous, stochastic or deterministic [1] [2] [3]. The models that help to improve communication, ease the learning and allow the study of performance of the system are divided into three categories:

- physical model, including technological equipment to transport and to ensure the transfer of information ,
- the functional model describing the model of the system functioning,
- the organizational model, which describes the system in terms of inter-dependency that exist between operators.

In manufacturing systems are conducted synchronous, parallel and concurrent events.

Parallel events can be simultaneous processing of some pieces on various machine tools, carried out simultaneously with the transport of semi-products, with the execution of some loading/unloading operations. The main modeling techniques used are:

- techniques based on systems of vectors assembly,
- techniques based on different types of graphs,
- techniques based on petri network,
- techniques based on model queues .

For simulating the transport process are necessary data on the structure of the route and the time spent by a piece in the manufacturing system, as well as using of a particularized simulation model derived from the general simulation models.

2.THE GENERAL MODEL

Contributions concerning the timing of manufacturing are based on the following assumptions:

- The transition sequence of the products on more machine – tools, *n*, given by the group technology, any product P_i must be processed on machines in a defined order, $M_1, M_2, M_3, ..., M_p$.
- If the machine M_i is occupied by a product P_{j1}, it can not process in the same time the product P_{j2}.
- The machine M_{i+1} can't begin processing of the product P_{ji} until it was process on the machine M_i. The processing times of products P_j on the machines M_i are given by the matrix: $\left(t_{ij}\right); \ i = 1,2,...,p; \ \ j = 1,2,...,n.$

Since the first machine can work without a break, meaning it does not take into account the auxiliaries times established by legislation and that are considered as having constant values, the problem is reduced to minimizing the total time of inactivity for the rest of the machine – tools, i.e. to minimize the total time of processing on *p* machines of *n* products.

In order to determine the total time of inactivity [1] of the machines, we start on the following considerations:

- it is denoted by *T* the total time from the beginning of the first product passing on the machine M_1 and the end of the last product passing on the machine M_p,
- it establishes an arbitrary order for passing of the products on every machines,
- it considers X_{jq} the waiting time between the end of the product P_{jq-1} passing on the machine M_i and the beginning of the product P_{jq} passing on the machine M_i.

On this basis, it obtains the solution of the calculation of the total time: $T = \sum_{r=1}^{n} A_{pjr} + \sum_{r=1}^{n} X_{jr}$ where $A_{prj} = t_{jr}$ is the time of processing off the product P_{jr} on the machine M_p. Taking into account that the times values A_{pjr} are constants, for minimizing the time *T* it can do the minimization of the function: $\sum_{r=1}^{n} X_{jr}$.

It is considered that the processing time of the product P_{j1} on the machine M_1 is A_{1j1}. On the machine M_2 the waiting time is given by the relation: $X_{j1}^{(2)} = A_{j1}$

On the machines $M_3, ..., M_p$ the waiting times are set customizing the indicated relations from **Error! Reference source not found.**:

$$X_{j1}^{(3)} = A_{j1}^{(1)} + A_{j1}^{(2)}$$

$$\dots$$

$$X_{j1}^{(p)} = A_{j1}^{(1)} + A_{j1}^{(2)} + A_{j1}^{(3)} + ... + A_{j1}^{(p-1)}$$

The waiting time $X_{j1}^{(2)}$, on the machine M_2, is calculated by the relation:

$$X_{j1}^{(2)} = A_{j1}^{(1)} + A_{j2}^{(1)} - A_{j1}^{(2)} - X_{j1}^{(2)}$$

$$\text{if } A_{j1}^{(1)} + A_{j2}^{(2)} \geq A_{j1}^{(2)} + X_{j1}^{(2)} \text{ or } X_{j1}^{(2)} = 0$$

$$\text{if } A_{j1}^{(1)} + A_{j2}^{(1)} < A_{j1}^{(2)} + X_{j1}^{(2)}.$$

For the machine M_3:

$$X_{j2}^{(3)} = A_{j1}^{(1)} + A_{j2}^{(1)} + A_{j2}^{(2)} - A_{j1}^{(3)} - X_{j1}^{(3)}$$

$$\text{if } A_{j1}^{(1)} + A_{j2}^{(1)} + A_{j2}^{(2)} \geq A_{j1}^{(3)} + X_{j1}^{(3)}, \text{ or } X_{j2}^{(3)} = 0$$

$$\text{if } A_{j1}^{(1)} + A_{j2}^{(1)} + A_{j2}^{(2)} < A_{j1}^{(3)} + X_{j1}^{(3)}.$$

For the machine M_p, the waiting time $X_{j2}^{(p)}$ is calculated based on the relation [1]:

$$X_{j2}^{(p)} = A_{j1}^{(1)} + A_{j2}^{(1)} + \sum_{i=1}^{p-1} A_{j2}^{(i)} - A_{j1}^{(p)} - X_{j1}^{(p)}$$

$$\text{if } A_{j1}^{(1)} + A_{j2}^{(1)} + \sum_{i=1}^{p-1} A_{j2}^{(i)} \geq A_{j1}^{(p)} + X_{j1}^{(p)}, \text{ or } X_{j2}^{(p)} = 0$$

$$\text{if } A_{j1}^{(1)} + A_{j2}^{(1)} + \sum_{i=1}^{p-1} A_{j2}^{(i)} < A_{j1}^{(p)} + X_{j1}^{(p)} \; .$$

The above relations can also write in the form [1]:

$$\begin{cases} X_{j2}^{(2)} = \max(A_{j1}^{(1)} + A_{j2}^{(1)} - A_{j1}^{(2)} - X_{j1}^{(2)}, 0) \\ X_{j2}^{(3)} = \max(A_{j1}^{(1)} + A_{j2}^{(1)} + A_{j2}^{(2)} - A_{j1}^{(3)} - X_{j1}^{(3)}, 0) \\ \ldots \\ X_{j2}^{(p)} = \max(A_{j1}^{(1)} + \sum_{i=1}^{p-1} A_{j2}^{(i)} - A_{j1}^{(p)} - X_{j1}^{(p)}, 0) \end{cases}$$

$$\begin{cases} X_{j3}^{(2)} = max(\sum_{k=1}^{3} A_{jk}^{(1)} - \sum_{k=1}^{2} A_{jk}^{(2)} - \sum_{k=1}^{2} X_{jk}^{(2)}, 0) \\ X_{j3}^{(3)} = max(\sum_{k=1}^{2} A_{jk}^{(1)} + \sum_{i=1}^{3} A_{j3}^{(i)} - \sum_{k=1}^{2} A_{jk}^{(3)} - \sum_{k=1}^{2} X_{jk}^{(3)}, 0) \\ \ldots \\ X_{j3}^{(p)} = max(\sum_{k=1}^{2} A_{jk}^{(1)} + \sum_{i=1}^{p} A_{j3}^{(i)} - \sum_{k=1}^{2} A_{jk}^{(p)} - \sum_{k=1}^{2} X_{jk}^{(p)}, 0) \end{cases}$$

$$\begin{cases} X_{jn}^{(2)} = max(\sum_{k=1}^{n-1} A_{jk}^{(1)} - \sum_{k=1}^{n-2} A_{jk}^{(2)} - \sum_{k=1}^{n-1} X_{jk}^{(2)}, 0) \\ X_{jn}^{(3)} = max(A_{jn}^{(2)} + \sum_{k=1}^{n} A_{jk}^{(1)} - \sum_{k=1}^{n-1} A_{jk}^{(3)} - \sum_{k=1}^{n-1} X_{jk}^{(3)}, 0) \\ \ldots \\ X_{jn}^{(p)} = max(\sum_{k=1}^{n-1} A_{jk}^{(1)} + \sum_{i=1}^{p} A_{jn}^{(i)} - \sum_{k=1}^{n-1} A_{jk}^{(p)} - \sum_{k=1}^{n-1} X_{jk}^{(p)}, 0) \end{cases}$$

The total sum of inactivity times on the machine M_2, denoted by $X^{(2)}$ is [1]:

$$X^{(2)} = max(A_{j1}^{(1)}, A_{j1}^{(1)} + A_{j2}^{(1)} - A_{j1}^{(2)} - X_{j1}^{(2)}, \sum_{k=1}^{3} A_{jk}^{(1)} - \sum_{k=1}^{2} A_{jk}^{(2)} - \sum_{k=1}^{2} X_{jk}^{(2)}, \ldots,$$

$$\sum_{k=1}^{n-1} A_{jk}^{(1)} - \sum_{k=1}^{n-1} A_{jk}^{(2)} - \sum_{k=1}^{n-1} X_{jk}^{(2)}, 0)$$

On the machine M_3, the total time of inactivity is given by:

$$X^{(3)} = max(A_{j1}^{(1)} + A_{j1}^{(2)}, A_{j1}^{(1)} + A_{j2}^{(1)} + A_{j1}^{(2)} - A_{j1}^{(3)} - X_{j1}^{(3)}, \sum_{k=1}^{2} A_{jk}^{(1)} + \sum_{i=1}^{3} A_{j3}^{(i)} -$$

$$- \sum_{k=1}^{2} A_{jk}^{(3)} - \sum_{k=1}^{2} X_{jk}^{(3)}, \ldots, A_{jn}^{(2)} + \sum_{k=1}^{n} A_{jk}^{(1)} - \sum_{k=1}^{n-1} A_{jk}^{(3)} - \sum_{k=1}^{n-1} X_{jk}^{(3)}, 0)$$

For the machine M_p [1]:

$$X^{(p)} = max(A_{j1}^{(1)} + A_{j1}^{(2)} + \ldots + A_{j1}^{(p-1)}, A_{j1}^{(1)} + \sum_{i=1}^{p-1} A_{j2}^{(i)} - A_{j1}^{(p)} - X_{j1}^{(p)},$$

$$, \sum_{k=1}^{2} A_{jk}^{(1)} + \sum_{i=1}^{p} A_{j3}^{(i)} - \sum_{k=1}^{2} A_{jk}^{(2)} - \sum_{k=1}^{2} X_{jk}^{(2)}, \ldots, \sum_{k=1}^{n-1} A_{jk}^{(1)} + \sum_{i=1}^{p} A_{jk}^{(i)} - \sum_{k=1}^{n-1} A_{jk}^{(p)} - \sum_{k=1}^{n-1} X_{jk}^{(p)}, 0)$$

Summing the last relations we obtain the total time of inactivity for the entire technological cycle, for the processing of all parts. It obtains the following relation [1]:

$$\sum_{r=1}^{n} X_{jr} = \sum_{i=1}^{p} \sum_{r=1}^{n} X_{jr}^{(i)}$$

This relationship expresses the waiting times in the form of expressions that depend on machines working times. This relation is minimum when the processing order S is given by the relation: $(P_{j1}, P_{j2}, \ldots, P_{jn})$, is optimal.

If is denoted by $D_n^{(1)}(S)$ the total time of inactivity on products processing in S order, on the machine M_i, then it obtains [1]:

$$D_n^{(2)}(S) = \max_{1\le\alpha\le n}\left(\sum_{r=1}^{n} A_{jr}^{(1)} - \sum_{r=1}^{n-1} A_{jr}^{(2)}\right)$$

$$D_n^{(3)}(S) = \max_{1\le\alpha\le n}\left(\sum_{r=1}^{n}(A_{jr}^{(1)} + A_{jk}^{(2)}) - \sum_{r=1}^{n-1}(A_{jr}^{(2)} + A_{jr}^{(3)})\right)$$

$$\ldots .$$

$$D_n^{(p)}(S) = \max_{1\le\alpha\le n}\left(\sum_{r=1}^{n}\sum_{i=1}^{p-1} A_{jr}^{(i)} - \sum_{r=1}^{n-1}\sum_{i=1}^{p} A_{jr}^{(i)}\right)$$

If it is denoted:

$$L_\alpha^{(2)} = \sum_{r=1}^{n} A_{jr}^{(1)} - \sum_{r=1}^{n-1} A_{jr}^{(2)}$$

$$L_\alpha^{(3)} = \sum_{r=1}^{n}(A_{jr}^{(1)} + A_{jr}^{(2)}) - \sum_{r=1}^{n-1}(A_{jr}^{(2)} + A_{jr}^{(3)})$$

$$\ldots$$

$$L_\alpha^{(p)} = \sum_{r=1}^{n}\sum_{i=1}^{p-1} A_{jr}^{(i)} - \sum_{r=1}^{n-1}\sum_{i=1}^{p} A_{jr}^{(i)}$$

In this case the total time of inactivity on processing can also express thus:

$$D_n^{(2)}(S) = \max_{1\le\alpha\le n} L_\alpha^{(2)}$$

$$D_n^{(3)}(S) = \max_{1\le\alpha\le n} L_\alpha^{(3)}$$

$$\ldots$$

$$D_n^{(p)}(S) = \max_{1\le\alpha\le n} L_\alpha^{(p)}$$

If it is supposed that is processing “n” types of products on two machines and it chooses a random order of the products:

$$S_1 = P_{j1}, P_{j2}, \ldots, P_{jk-1}, P_{jk}, P_{jk+1}, \ldots, P_{jn}$$

3.THE STUDY OF THE MODEL

The model of the transport system can be assimilated with a discrete event [2] [3] [4]. These systems form a class of nonlinear dynamic systems that are using their mathematical tools other than differential equations used in the theory and practice of automatically adjust. The systems are used to describe the mathematical model, but also the process to be analyzed.

Description of the system is based on a set that includes events that are arrivals and exits. The system status is given by the total number of elements that are at a given time in the system, ie the space is a set.

It is considered the flow line, fig.1, fig. 2, which is provided with the asynchronous transfer system, some stations have in the structure storage areas, while others have not. The storage spaces are determined by the processing time to the next stage. The semi-products are placed in the working station where they are processed and transported to the next working stations.

It is denoted by T_t the theoretical time of a processing cycle. T_t represents the necessary time for processing and transporting of a certain semi-products set, from the first working station to the next one. T_t is different from the processing time [3] [4].

Due to the malfunctions that may occur, the average time of production T_p is higher than the time T_t. The time T_d is considered as being the time of diagnosis and performing necessary repairs. The time T_d may also have more components T_{dk} depending on the reasons for stopping. The frequency of the flow line stopping due to the defects k is denoted by F_k.

If there exists at least one reason that can stop the flow line, the average time of production is determined by the relation:

$$T_p = T_t + \sum_{k=1}^{h} F_k \cdot T_{dk}$$

where *h* is the maximum number of possibilities of defects that the line can have.

It is considered the flexible manufacturing system, consisting of the following resources, placed according to the representation from fig.1 and fig. 2:

- Processing centers, M1- Burning, M2- Form of payment, M3- Frosting;
- Conveyors, truck;
- Intermediary storages, D1;
- robot, R1;

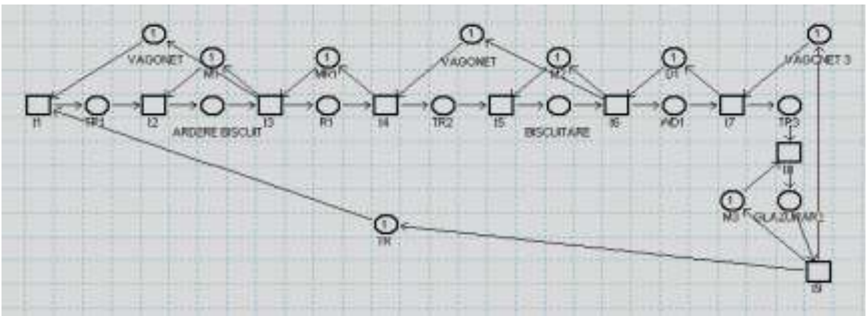

Fig.1: Petri Network

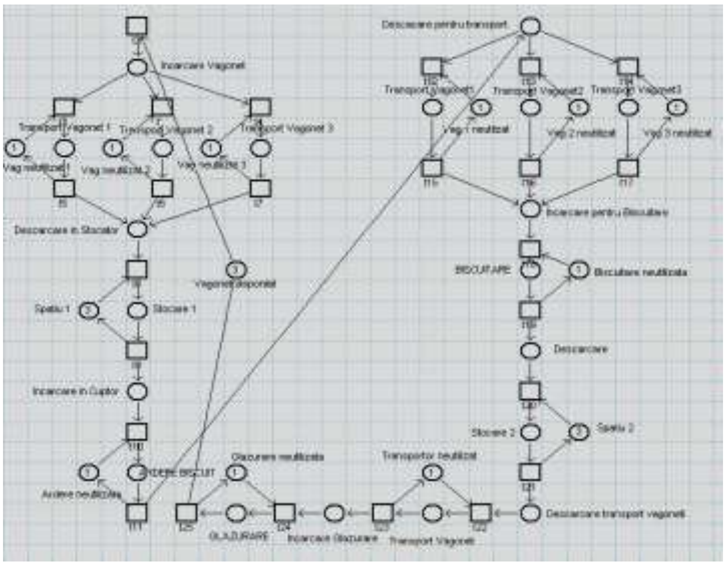

Fig.2 Petri network with auxiliary times

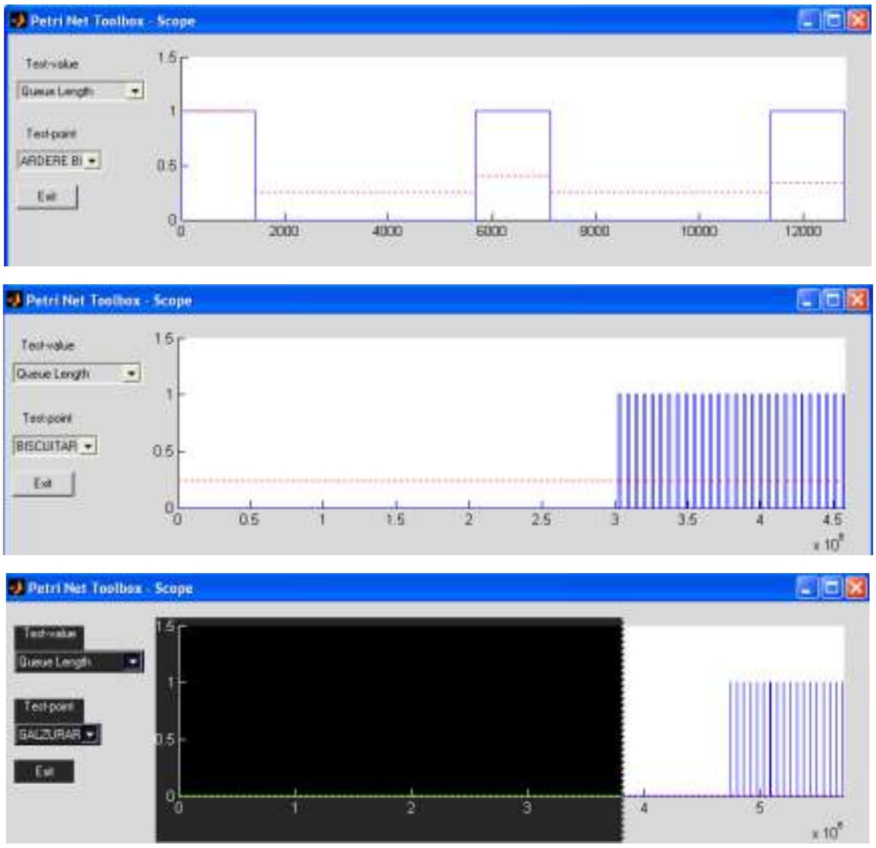

Fig.3: The evolution of indicators Queue Length corresponding to the analyzed positions (continuous line – current value; dashed line – global value resulting from mediation on the simulation interval)

The max-plus model are highlighted grid positions assigned time duration and transitions are used to designate the name of the state variables.

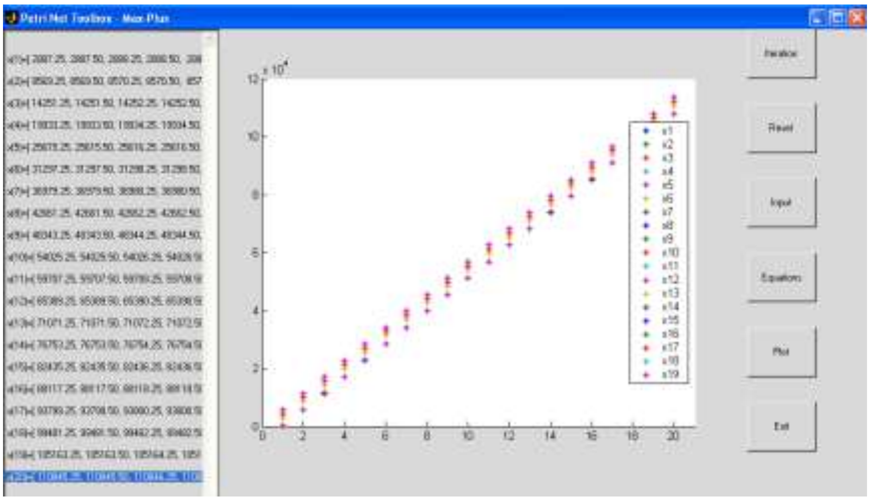

Fig.4: Graphical representation corresponding to the section II displayed in the window opened by Petri Net Toolbox for max-plus analyze

4.CONCLUSION

It was developed the transport system that established the dimensioning, internal structure and the corresponding couplings. In addition to the workstations in the system are included control stations and corresponding logistics sub-systems.

Applying product manufacturing times and transportation times obtained by measuring the spot, it is obtained graphical representations that show the average times of transport activity, but also the evolution of the average time of processing related to the transport activity time, using as parameters sets of finite products.

The theoretical results that are obtained can be implemented directly in the manufacturing process. By optimizing the proposed transport system is obtained a reduction of the waiting time, transportation time, queue length reduction and easier coordination of combustion furnaces.

A contribution is referred to the study of simulation solutions proposed. First the method was developed for a flexible system. It resulted from the performed researches the possibility of using it also in the case of classical manufacturing systems considering the analyzed aspects in this paper.

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