CALCULATION ALGORITHM FOR STUDYING EFFORTS IN THE 4R SPHERICAL QUADRILATERAL MECHANISMS BECAUSE OF TECHNICAL DEVIATIONS

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Abstract: *Due to technical deviations, in the elements of the 4R spatial spherical mechanism appear efforts that additionally loads the mechanism, efforts that can be determined with the calculation algorithm that will be presented in this paper.*

Keywords: elastic calculation, cardan transmission .

1. THEORETICAL ASPECTS

On the basis of the mathematical model presented in paper [7]., and using the expression of technical deviations in pluckerian coordinates, a calculation algorithm is elaborated for determining the efforts from the 4R mechanism.

The 4R spherical spatial mechanism (see Figure 1.), with the axes OA, OB, OC, OD concurrent in O, and with $\tilde{\Delta}_A, \tilde{\Delta}_B, \tilde{\Delta}_C$ the technical deviations of elements AB, BC, CD.

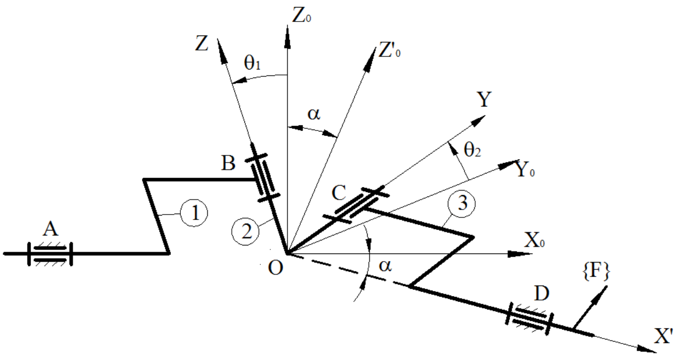

Figure 1.

Taking into account the relations between efforts and displacements [9] ,[11]., and notating with $\{R_A\}, \{R_D\}$ the efforts from the both ends of the mechanism expressed in pluckerian coordinates, the stability equation can be written like:

$$\{R_A\} + \{R_D\} = \{0\} \tag{1}$$

if noted with:

$$\{\Delta_{AD}\} = \{\Delta_{AB}\} + \{\Delta_{BC}\} + \{\Delta_{CD}\}, \{\tilde{\Delta}\} = \{\tilde{\Delta}_{AB}\} + \{\tilde{\Delta}_{BC}\} + \{\tilde{\Delta}_{CD}\} \tag{2}$$

the total relative displacements, caused by external loads, with technical deviations, and with $[H_{AD}]$, $[K_{AD}]$ the flexibility matrixes. The rigidity can be written with the relation:

$$\{R_A\} = [K_{AD}](\{\Delta_{AD}\} + \{\tilde{\Delta}_{AD}\}) \tag{3}$$

from where

$$\{\Delta_{AD}\} = \{\tilde{\Delta}_{AD}\} + [H_{AD}]\{R_A\} \tag{4}$$

If the A joint is considered blocked, the following formal diagram is obtained. (see Figure 2.)

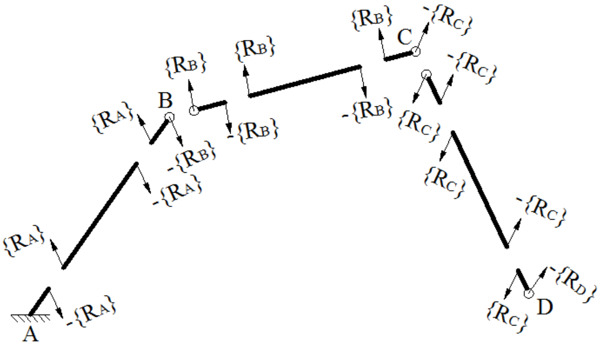

Figure 2.

From the stability condition of the cinematic couples it results:

$$\{R_B\} = \{R_C\} = \{R_D\} \tag{5}$$

By writing the relation (4) for all isolated elements the equations are obtained:

$$\{\Delta_{AB}\} = \{\tilde{\Delta}_{AB}\} + [H_{AB}]\{R_A\} \; ; \; \{\Delta_{BC}\} = \{\tilde{\Delta}_{BC}\} + [H_{BC}]\{R_A\} \; ; \; \{\Delta_{CD}\} = \{\tilde{\Delta}_{CD}\} + [H_{CD}]\{R_A\} \tag{6}$$

By summing the relations (6) and using the notations

$$[U] = [[U_B], [U_C], [U_D]] \; ; \; \{\xi\} = [\xi_B, \xi_C, \xi_D]^T \; ; \; [\tilde{U}] = [[\tilde{U}_B], [\tilde{U}_C], [\tilde{U}_D]]^T \tag{7}$$

equation [7] is obtained.

$$\{R_A\} = [K_{AD}]([U]\{\xi\} - \{\tilde{\Delta}\}) \tag{8}$$

And by taking into account the condition[12].

$$[\tilde{U}]\{R_A\} = \{0\} \tag{9}$$

the unknown is deducted $\{\xi\}$

$$\{\xi\} = -([\tilde{U}][K_{AD}][U])^{-1}[\tilde{U}][K_{AD}]\{\tilde{\Delta}\} \tag{10}$$

If the matrixes, $[K_{AD}]$, $[H_{AD}]$ are expressed under the form:

$$[K_{AD}] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \; ; \; [H_{AD}] = \begin{bmatrix} [H_{11}] & [H_{12}] \\ [H_{21}] & [H_{22}] \end{bmatrix} \tag{11}$$

and observing that the matrixes $[U]$, $[\tilde{U}]$ are formed like

$$[U] = \begin{bmatrix} [I] \\ [0] \end{bmatrix} \; ; \; [\tilde{U}] = [[0]\;[I]]^T \tag{12}$$

and by using the notations

$$\{\tilde{\Delta}\} = [\tilde{\theta}_x, \tilde{\theta}_y, \tilde{\theta}_z, \tilde{\delta}_x, \tilde{\delta}_y, \tilde{\delta}_z]^T \; ; \; \{\tilde{\theta}\} = [\tilde{\theta}_x, \tilde{\theta}_y, \tilde{\theta}_z]^T \; ; \; \{\tilde{\delta}\} = [\tilde{\delta}_x, \tilde{\delta}_y, \tilde{\delta}_z]^T \tag{13}$$

$$\{R_A\} = [F_{AX}^0, F_{AY}^0, F_{AZ}^0, M_{AX}^0, M_{AY}^0, M_{AZ}^0]^T ; \; \{R_A^0\} = [F_{AX}^0, F_{AY}^0, F_{AZ}^0]^T ;$$

$$\{M_A^0\} = [M_{AX}^0, M_{AY}^0, M_{AZ}^0]^T \tag{14}$$

there are obtained [5],[6]. The results

$$\{\xi\} = -[I]^{-1}(\{\tilde{\theta}\} + [K_{21}]^{-1}[K_{22}]\{\tilde{\delta}\}) \; ; \; \{M_A^0\} = \{0\} \tag{15}$$

$$\{R_A^0\} = \left([K_{12}] - [K_{11}][K_{21}]^{-1}[K_{22}]\right)\{\delta\} \tag{16}$$

The representations (15), (16) are in the general reference system. In the local systems also appear the moments:

$$\begin{Bmatrix} R_A^l \\ M_A^l \end{Bmatrix} = \begin{bmatrix} [R]^T & [0] \\ [R]^T[G]^T & [R]^T \end{bmatrix} \begin{Bmatrix} R_A^0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} [R]^T\{R_A^0\} \\ [R]^T[G]^T\{R_A^0\} \end{Bmatrix} \tag{17}$$

2. CALCULATING THE EFFORTS

The deviations are first locally defined and then in one of the three reference systems OX_0Y^*Z, OX^*YZ, $OX^*YZ_0^*$ [10]. that do not depend on θ_1,θ_2 and then in the system $OX_0Y_0Z_0$ and under this form by summing:

$$\{\Delta\} = \{\tilde{\Delta}_{AB}\} + \{\tilde{\Delta}_{BC}\} + \{\tilde{\Delta}_{CD}\} \tag{18}$$

the relations (15), (16) are involved

The calculation of $\{\tilde{\Delta}_{AB}\}$

(see Figure 3.)

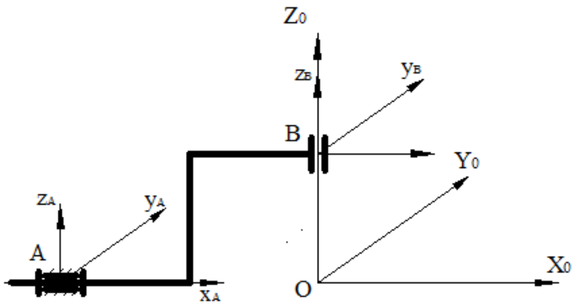

Figure 3.

$$\{\Delta_A\} = [\theta_{Ax}^l, \theta_{Ay}^l, \theta_{Az}^l, \delta_{Ax}^l, \delta_{Ay}^l, \delta_{Az}^l]^T \tag{19}$$

$$X_A = -OA, Y_A = 0,\ Z_A = 0; \tag{20}$$

$$[G_A] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -X_A \\ 0 & X_A & 0 \end{bmatrix} \tag{21}$$

$$\{\Delta_A\} = \begin{bmatrix} [I] & [0] \\ [G_A] & [I] \end{bmatrix} \{\Delta_A^l\} \tag{22}$$

$$\{\tilde{\Delta}_A\} = [T_{AB}]\{\Delta_A\} \tag{23}$$

$$[T_{AB}] = \begin{bmatrix} [R_{AB}] & [0] \\ [0] & [R_{AB}] \end{bmatrix}; \quad [R_{AB}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_1 & -s\theta_1 \\ 0 & s\theta_1 & c\theta_1 \end{bmatrix} \tag{24}$$

$$\{\Delta_B\} = \{\theta^l_{Bx}, \theta^l_{By}, \theta^l_{Bz}, \delta^l_{Bx}, \delta^l_{By}, \delta^l_{Bz}\}^T \tag{25}$$

$$X_B = 0\,, Y_B = 0\,,\ Z_B = OB\,; \tag{26}$$

$$[G_B] = \begin{bmatrix} 0 & 0 & Y_B \\ 0 & 0 & 0 \\ -Y_B & 0 & 0 \end{bmatrix}; \tag{27}$$

$$\{\Delta_B\} = \begin{bmatrix} [I] & [0] \\ [G_B] & [I] \end{bmatrix} \{\Delta^l_B\}; \tag{28}$$

$$\{\tilde{\Delta}_B\} = [T_{AB}]\{\Delta_B\}\,;\ \{\tilde{\Delta}_{AB}\} = \{\tilde{\Delta}_A\} - \{\tilde{\Delta}_B\} \tag{29}$$

The calculation of $\{\tilde{\Delta}_{BC}\}$

(see Figure 4.)

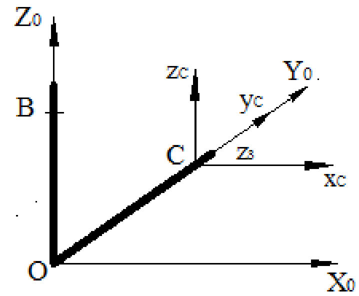

Figure 4.

$$\{\Delta_C\} = \{\theta^l_{Cx}, \theta^l_{Cy}, \theta^l_{Cz}, \delta^l_{Cx}, \delta^l_{Cy}, \delta^l_{Cz}\}^T \tag{30}$$

$$X_C = 0\,, Y_C = OC\,,\ Z_C = 0\,; \tag{31}$$

$$[G_C] = \begin{bmatrix} 0 & 0 & -Y_C \\ 0 & 0 & 0 \\ Y_C & 0 & 0 \end{bmatrix} \tag{32}$$

$$\{\Delta_C\} = \begin{bmatrix} [I] & [0] \\ [G_C] & [I] \end{bmatrix} \{\Delta^l_C\} \tag{33}$$

$$\{\tilde{\Delta}_C\} = [T_{BC}]\{\Delta_C\}\,;\ \{\tilde{\Delta}_{BC}\} = -\{\tilde{\Delta}_C\} \tag{34}$$

$$[T_{BC}] = \begin{bmatrix} [R_{BC}] & [0] \\ [0] & [R_{BC}] \end{bmatrix};\ [R_{BC}] = \begin{bmatrix} c\theta_1 c\theta_2 + s\theta_1 s\theta_2 c\alpha & s\alpha s\theta_2 & 0 \\ -c\theta_1 s\theta_2 s\alpha & c\theta_2 & -s\theta_1 \\ -s\theta_1 s\theta_2 s\alpha & c\alpha s\theta_2 & c\theta_1 \end{bmatrix} \tag{35}$$

The calculation of $\{\tilde{\Delta}_{CD}\}$

(see Figure 5.)

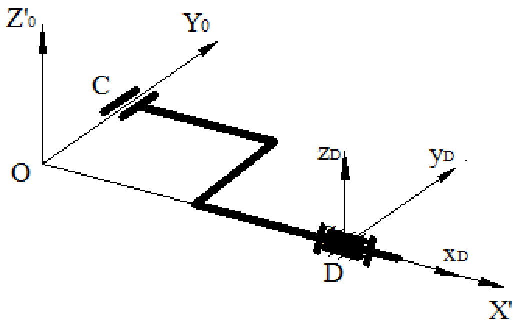

Figure 5.

$$\{\Delta_D\} = \left[\theta^l_{Dx}, \theta^l_{Dy}, \theta^l_{Dz}, \delta^l_{Dx}, \delta^l_{Dy}, \delta^l_{Dz}\right]^T \tag{36}$$

$$X_D = OD, Y_D = 0,\ Z_D = 0 \tag{37}$$

$$[G_D] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -X_D \\ 0 & X_D & 0 \end{bmatrix} \tag{38}$$

$$\{\Delta_D\} = \begin{bmatrix} [I] & [0] \\ [G_C] & [I] \end{bmatrix} \{\Delta^l_D\} \tag{39}$$

$$\{\tilde{\Delta}_D\} = [T_{CD}]\{\Delta_D\}\ ;\ \{\tilde{\Delta}_{CD}\} = -\{\tilde{\Delta}_D\} \tag{40}$$

$$[T_{CD}] = \begin{bmatrix} [R_{CD}] & [0] \\ [0] & [R_{CD}] \end{bmatrix}\ ;\ [R_{CD}] = \begin{bmatrix} c\alpha & s\alpha s\theta_2 & s\alpha c\theta_2 \\ 0 & c\theta_2 & -s\theta_2 \\ -s\alpha & c\alpha s\theta_2 & c\alpha c\theta_2 \end{bmatrix} \tag{41}$$

3.CALCULATION ALGORITHM

1. For a given θ_1 and chosen $\{\Delta_A\}, \{\Delta_B\}, \{\Delta_C\}, \{\Delta_D\}$ it is determined $\{\tilde{\Delta}_{AB}\}, \{\tilde{\Delta}_{BC}\}, \{\tilde{\Delta}_{CD}\}$, with the relations (19)- (41) and $\{\tilde{\theta}\}, \{\tilde{\delta}\}$.

$$\{\tilde{\Delta}\} = \{\tilde{\Delta}_{AB}\} + \{\tilde{\Delta}_{BC}\} + \{\tilde{\Delta}_{CD}\} = \begin{bmatrix} \{\tilde{\theta}\} \\ \{\tilde{\delta}\} \end{bmatrix} \tag{42}$$

2. The $[V]$ matrix is calculated with the relation

$$[V] = \begin{bmatrix} 0 & s\theta_1 s\alpha & c\alpha \\ -s\theta_1 & c\theta_2 & 0 \\ c\theta_1 & s\theta_2 c\alpha & -s\alpha \end{bmatrix} \tag{43}$$

3. The rigidity matrixes $[K_{11}], [K_{12}], [K_{21}], [K_{22}]$ are calculated from the equation

$$\left[K_{AD}\right]=\begin{bmatrix}[K_{11}] & [K_{12}]\\ [K_{21}] & [K_{22}]\end{bmatrix} \tag{44}$$

4. $\{\xi^0\}, \{F_A^0\}$ are calculated with the relations (15), (16)
5. The reactions are calculated in the own reference system knowing that:

$$\{R_B^0\} \quad \{R_C^0\} \quad \{R_D^0\} \quad \{R_A^0\} \tag{45}$$

6. The graphs $\xi_B, \xi_C, \xi_D, R_{Ax}, R_{Ay}, R_{Az}, M_{Ax}, M_{Ay}, M_{Az},$ $R_{Bx}, R_{By}, R_{Bz}, M_{Bx}, M_{By}, M_{Bz},$ $R_{Cx}, R_{Cy}, R_{Cz}, M_{Cx}, M_{Cy}, M_{Cz},$ $R_{Dx}, R_{Dy}, R_{Dz}, M_{Dx}, M_{Dy}, M_{Dz}$

4. CONCLUSIONS

The calculation algorithm established after the mathematical model presented in paper [7]., makes possible the study of technological deviations influence over the 4R spherical quadrilateral mechanism.

Using the relative displacement method and the elastic calculation made possible the elaboration of such algorithm that by numerical simulation can highlight the influence of different technical deviations that appear in the mechanism elements.

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