# STRUCTURAL APPROACH TO THE MATHEMATICAL DESCRIPTION AND COMPUTER VISUALIZATION OF PLANE KINEMATIC CURVES FOR THE DISPLAY OF GEARS

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***Abstract:*** *The structural approach stated in this paper allows to simulate the different plane kinematic curves without their concrete analytic equations. The summarized unified mapping system for rack gearing is used. The examples of plane kinematic curves received by the structural method on computer are adduced.*
**Keywords:** gear meshes, multiparametric mappings, kinematic curves, structural method, unified system.

## 1. Introduction

A kinematic curve may be obtained by the one-parametric mapping as a continuous trajectory of composite movement. As a prototype, we have adopted a point and also have expressed parameters of movement mapping operators as one independent parameter. The foundations of this approach in the application to gearing are performed at the Kharkov Polytechnic Institute under the leadership of Professor B. A. Perepelitsa [1], as well as jointly with the Kiev Polytechnic Institute (Corresponding Member of NAS of Ukraine Professor P. R. Rodin) and the Institute of Superhard Materials of the NAS Ukraine (leading research officer A. V. Krivosheya) [2]. They are used for the development of toothed objects of increased complexity and functionality [3], including conical two-parameter transmissions [4] and reducers of increased accuracy and compactness [5], and they are developing in accordance with these challenges of the modern technological order of economically advanced countries.

In this article it's realized the structural approach to the simulation of plane kinematic curves that operates only by structures without deriving the concrete analytic equations. It uses design formed in cooperation for the development of generalized mappings unified structure for gearing [6], and also the support by databases of methods and analysis by own [3, 7], as well as from Tula [8] and Petersburg [9] scientific schools.

## 2. Analytical research

The initial position of coordinate systems, operators and centrodes of rotations are showed on the Fig. 1. The centrodes with radius $r_{w1}, r_{w2}, r_{w3}$ are placed in the systems 1, 2, 3 (let's call them "the first, the second, and the third coordinate system"). That scheme is conventionally conforms to a three-members rack gearing.

Let's impose, the following restrictions on the domain of existence of curves under investigation: the point trajectory is placed in one plane (plane curves); number of pairs of elemental movement is equal to number of members in the summarized structure, that is to 3;

elemental movements are even; the initial point is on the first centrode.

In this task the point A (prototype) experience the following one-parametric changes (see Fig. 1):

1) rotation $\bar{\varphi}_1$ in the system 1 about the axis $z_1$ (angle of rotation $\varphi_1$ and angular velocity $\omega_1$); parallel translation $\bar{l}_1$ along the axis $y_1$ (length of translation $l_1$ and linear velocity $v_1$); transition $\bar{c}_{12}$ from the system 1 to the system 2 ($c_{12}$ – centre-to-centre distance);

2) rotation $\bar{\varphi}_2$ in the system 2 about the axis $z_2$ (angle of rotation $\varphi_2$ and angular velocity $\omega_2$; parallel translation $\bar{l}_2$ along the axis $y_2$ (length of translation $l_2$ and linear velocity $v_2$); transition $\bar{c}_{23}$ from the system 2 to the system 3 ($c_{23}$ – centre-to-centre distance);

3) rotation $\bar{\varphi}_3$ in the system 3 about the axis $z_3$ (angle of rotation $\varphi_3$ and angular velocity $\omega_3$); parallel translation $\bar{l}_3$ along the axis $y_3$ (length of translation $l_3$ and linear velocity $v_2$).

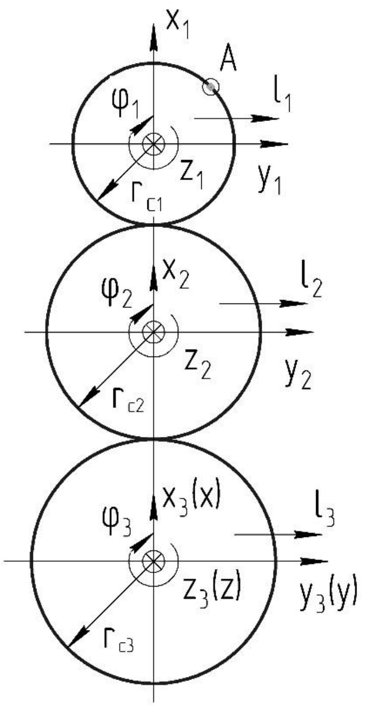



***Fig. 1.*** *The scheme of a three-members rack gearing*

The mapping operators $\bar{\varphi}_1, \bar{l}_1$, $\bar{\varphi}_2, \bar{l}_2$, $\bar{\varphi}_3, \bar{l}_3$ act simultaneously, their affine parameters are functionally connected to the independent parameter *t*:

$$
\begin{aligned}
&\varphi_1 = \omega_1 t; \qquad l_1 = v_1 t;\\
&\varphi_2 = \omega_2 t; \qquad l_2 = v_2 t;\\
&\varphi_3 = \omega_3 t; \qquad l_3 = v_3 t;\\
&i_{21} = \frac{\omega_2}{\omega_1};\ i_{21} = \frac{\omega_2}{v_1};\ i_{21} = \frac{v_2}{\omega_1};\\
&i_{32} = \frac{\omega_3}{\omega_2};\ i_{32} = \frac{\omega_3}{v_2};\ i_{32} = \frac{v_3}{\omega_2},
\end{aligned}
\tag{1}
$$

where *i* - velocities ratio.

Consequently, this ratio is multi-parametric in relationship to affine operators and one-parametric to the independent parameter *t*.

In this mapping the operators $\bar{\varphi}_1, \bar{l}_1, \bar{c}_{12}, \bar{\varphi}_{2,}, \bar{l}_2, \bar{c}_{23}, \bar{\varphi}_3, \bar{l}_3$ are active. The others operators, appearing in the summarized unified mapping structure, don't work:

$$
\begin{array}{llll}
\tau_1 = 0; & \bar{\tau}_1 = 1; & \tau_2 = 0; & \bar{\tau}_2 = 1;\\
\xi_{12} = 0; & \bar{\xi}_{12} = 1; & \xi_{23} = 0; & \bar{\xi}_{23} = 1;\\
\nu_{1\,add} = 0; & \bar{\nu}_{1\,add} = 1; & \nu_{2\,add} = 0; & \bar{\nu}_{2\,add} = 1;\\
\nu_{12} = 0; & \bar{\nu}_{12} = 1; & \nu_{23} = 0; & \bar{\nu}_{23} = 1;\\
g_{12} = 1; & \bar{g}_1 = 1; & g_2 = 1; & \bar{g}_2 = 1;\\
l_{12} = 0; & \bar{l}_1 = 0; & l_2 = 0; & \bar{l}_2 = 0;\\
\varepsilon_2 = 0; & \bar{\varepsilon}_2 = 1; & \varepsilon_3 = 0; & \bar{\varepsilon}_3 = 1;
\end{array}
\tag{2}
$$

$$\tau_{1\,add}=0; \quad \bar{\tau}_{1\,add}=1; \quad \tau_{2\,add}=0; \quad \bar{\tau}_{2\,add}=1;$$
$$\tau_3=0; \quad \bar{\tau}_3=1.$$

Substituting ones and zeros into the unified structure for the active operators (2), we can obtain a particular structure for concrete mapping:

$$\bar{R}=\bar{\varphi}_3\bar{\varphi}_2\bar{\varphi}_1\bar{R}_1+\bar{\varphi}_3\bar{\varphi}_2(\bar{l}_1+\bar{r}_{w1}+\bar{r}_{w2})+\bar{\varphi}_3(\bar{l}_2+\bar{r}_{w2}+\bar{r}_{w3})+\bar{l}_3. \quad (3)$$

In the task

$$\bar{r}_{w1}+\bar{r}_{w2}=\bar{c}_{12}; \; \bar{r}_{w2}+\bar{r}_{w3}=\bar{c}_{23}, \quad (4)$$

where $r_{w1}=r_{c1};\; r_{w2}=r_{c2};\; r_{w3}=r_{c3}$ – centrodes radius (see Fig. 1).

Substituting (3) into (4), we can obtain

$$\bar{R}=\bar{\varphi}_3\bar{\varphi}_2\bar{\varphi}_1\bar{R}_1+\bar{\varphi}_3\bar{\varphi}_2(\bar{l}_1+\bar{c}_{12})+\bar{\varphi}_3(\bar{l}_2+\bar{c}_{23})+\bar{l}_3. \quad (5)$$

The derived particular structure (5) includes 3 pair of elemental movements (rotation and parallel translation). This conforms to 3 members in the unified structure (n=3). For two pairs of elemental movements $\varphi_3=0 \;(\bar{\varphi}_3=1),\; l_3=0 \;(\bar{l}_3=0),\; c_{23}=0 \;(\bar{c}_{23}=0)$, that‘s why for n=2

$$\bar{R}=\bar{\varphi}_2\bar{\varphi}_1\bar{R}_1+\bar{\varphi}_2(\bar{l}_1+\bar{c}_{12})+\bar{l}_2. \quad (6)$$

For one pair of elemental movement $\varphi_2=0 \;(\bar{\varphi}_2=1)$, $l_2=0 \;(\bar{l}_2=0)$, $c_{12}=0$ $(\bar{c}_{12}=0)$, that’s why for n=l

$$\bar{R}=\bar{\varphi}_1\bar{R}_1+\bar{l}_1. \quad (7)$$

The matrices of active operators in the homogeneous coordinate system appear as shown in [9]:

$$m_{\varphi\, l}=\begin{pmatrix} cosf & \text{-}sinf & 0 & 0\\ sinf & cosf & 0 & l\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}=\begin{pmatrix} cos\omega t & \text{-}sin\omega t & 0 & 0\\ sin\omega t & cos\omega t & 0 & vt\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix};\; m_{oc}=\begin{pmatrix} 1 & 0 & 0 & c\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (8)$$

Using these matrices, we can obtain the particular matrix mapping equation for the investigated case (n=3) in the homogeneous coordinate system:

$$m_{oR}=m_{\varphi_3 l_3}m_{or_{c3}}m_{or_{c2}}m_{\varphi_2 l_2}m_{or_{c2}}m_{or_{c1}}m_{\varphi_1 l_1}m_{oR_1} \quad (9)$$

or

$$m_{oR}=m_{\varphi_3 l_3}m_{oc_{23}}m_{\varphi_2 l_2}m_{oc_{12}}m_{\varphi_1 l_1}m_{oR_1}. \quad (10)$$

For n=2: $\quad m_{oR}=m_{\varphi_2 l_2}m_{oc_{12}}m_{\varphi_1 l_1}m_{oR_1}. \quad (11)$

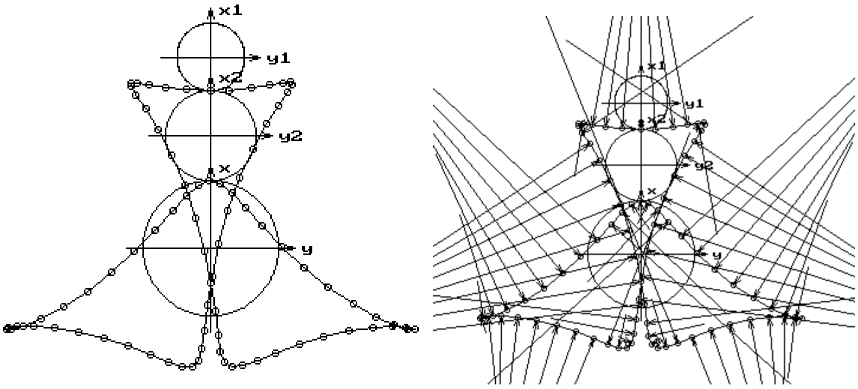
For n=1: $\quad m_{oR}=m_{\varphi_1 l_1}m_{oR_1}. \quad (12)$

The equations (5) and (10), being obtained from the summarized unified structure, describe the domain on which plane kinematic curves are defined. That domain is bounded by the constraints above. This is a summarized structural mathematical model of all the curves that enter this domain. Radius-vector $\bar{R}_1$ specifies in the equations the point A coordinates and the radius-vector $\bar{R}$ (to be determined) defines the coordinates of points of a plane as images in the system *XYZ* (system 3 as a final one is substituted by *XYZ*).

Utilizing summarized unified structure as well as the mapping method allows to synthesize different plane curves by the structural method without derivation of their concrete analytic equations. We utilize only the structures and the particular values of parameters as input information in a specially designed program in Turbo Pascal.

The result of the program implementation are arrays of points of simulated kinematic curves and geometric properties at these points (vectors of tangents, second derivatives, center lines and curvature vectors), with display them on the computer screen and to enter the received information in external files.

Let's give the examples of plane kinematic curves obtained by the computer using the structural method and the developed software (Fig. 2, Fig. 3).




a b

***Fig. 2.*** *The curve formed by rotations in a three-link gearing with the ratio* $\omega_1 : \omega_2 : \omega_3 = 4 : 3 : (-2)$ *at* $\omega_1 = 1$ *rad/s* (a) *and curvature vectors at its points* (b)

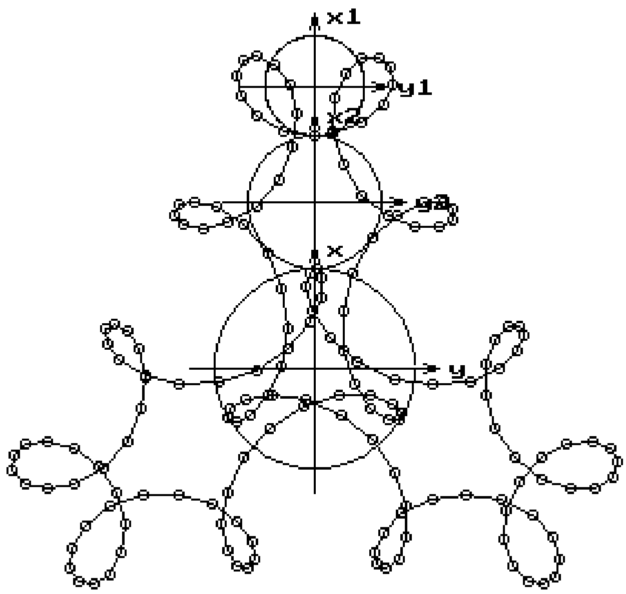



***Fig. 3.*** *The curve formed by rotations in a three-link gearing with the ratio* $\omega_1 : \omega_2 : \omega_3 = 4 : 3 : (-2)$ *at* $\omega_1 = 3$ *rad/s*

The curve showed on the Fig. 2a is derived by the three rotations $\overline{\varphi}_1, \overline{\varphi}_2, \overline{\varphi}_3$ while having the exterior tangency of centrodes. The structure of this curve is $\overline{\varphi}_1, \overline{c}_{12}, \overline{\varphi}_2, \overline{c}_{23}, \overline{\varphi}_3$. The form of the curve is symmetric because of conforming to two completed cycles of the independent parameter changes. On the Fig. 2b it's showed the vectors of curvature radius in curve points. The Fig. 3 depicts the same curve but with other angular velocities ω ratios. The remaining parameter values in the computer demonstrations of the structural method given here are as follows: $r_1$ = 15 mm, $r_2$ = 20 mm, $r_3$ = 30 mm, $x_1$ = –15 mm, $y_1$ = 0 mm.

## 3. Conclusion

The presented structural approach to the generalized multiparametric mathematical mapping of space in application to gearing allows to describe the kinematic curves and its geometric properties by the arrange of points without analytical equations for this curves.

The approach is tested in the computer programming environment of Turbo Pascal in the area of determining for plane kinematic curves with constraints by setting of systems of specific conditions.

The development is open to applications for further analysis and synthesis of improved and new gear transmissions and mechanisms.

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