THE ACTION OF PERIODIC VARIABLE FORCES IN TIME ON AN ELASTIC MEDIUM

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ABSTRACT: *This paper solves the problems of the dynamics of thin-walled bar frames of open sections excited externally parametrically, decomposing the functions of the section displacements in time, after the bending functions in the main planes of inertia and simple twisting. It is then shown that the dynamic instability domains will move to the high frequency area, as in the absence of elastic medium.*

KEY WORDS: forces, displacement, equation, vibration

1. Introduction

Considering a thin-walled frame of open sections supported on a Winckler-type elastic medium, externally driven by time-varying forces, which produce axial forces in the frame bars, also time-varying, the differential equations of parametric forced vibrations are [4]:

$$C\frac{d^2T}{dt^2}+\left[D+F-\alpha\ a-\beta\phi(t)B\right]T=0 \qquad (1)$$

T being an infinite vector, $T^T=\{T_1,T_2,T_3,........\}$, having as components the vectors of time functions, corresponding to the three displacements of the cross section of the bar (two linear displacements in the main directions of inertia and an angular displacement of rotation around the bending-twisting center), $T_K^T=\{T_k^u(t)T_k^v(t)T_k^\varphi(t)\}$. *C, D, F, A ,B* are infinite square hyper matrices containing the geometric and elastic characteristics of the bar and the support medium and α and β are two parameters.

This system of equations is used to determine the proper vibrations of thin-walled bar frames of open sections and to study dynamic stability, in other words to determine the main areas of dynamic instability.

2. Limits of dynamic instability of fields

Equations (1) from above constitute a system of second order differential equations, linear, homogeneous and with variable coefficients.

The stability of the solutions of such equations, being a known problem in the theory of equations with periodic coefficients [1,2], we will admit for Φ(t)=cosθt, the simple harmonic form, representing the most frequent and fundamental practical case, given that periodic excitations most of the time they can be decomposed into simple harmonics.

In this case the equations (1) are written:

$$C\frac{d^2T}{dt^2}+\left[D+F-\alpha A-\beta\cos\theta B\right]T=0 \tag{2}$$

Since at the limits of the fields of dynamic instability the system of differential equations (2) must admit periodic solutions with the excitation period and with the double of this period, solutions for the form are sought for it:

$$T=\sum_{k=2,4,6,\ldots}^{\infty}\left(a_k\sin\frac{k\theta t}{2}+b_k\cos\frac{k\theta t}{2}\right) \tag{3}$$

$$T=\frac{1}{2}b_0+\sum_{k=2,4,6,\ldots}^{\infty}\left(a_k\sin\frac{k\theta t}{2}+b_k\cos k\theta t\right) \tag{4}$$

With the help of solutions (3) the critical pulsations θ result at the limits of the instability domains 1,3,5 ....:

$$\begin{vmatrix} D-F-\alpha A\pm\frac{1}{2}\beta B-\frac{1}{4}\theta^2C & -\frac{1}{2}\beta B & 0 \\ -\frac{1}{2}\beta B & D-F-\alpha A-\frac{9}{4}\theta^2C & -\frac{1}{2}\beta B \\ 0 & -\frac{1}{2}\beta B & D-F-\alpha A-\frac{25}{4}\theta^2C \end{vmatrix}=0, \tag{5}$$

and with the help of solutions (4) we obtain the equations for determining the critical pulsations at the limits of the instability domains 2,4,6 ....:

$$\begin{vmatrix} D-F-\alpha A & -\frac{1}{2}\beta B & 0 \\ -\frac{1}{2}\beta B & D-F-\alpha A-4\theta^2C & -\frac{1}{2}\beta B \\ 0 & -\frac{1}{2}\beta B & D-F-\alpha A-16\theta^2C \end{vmatrix}=0 \tag{6}$$

$$\begin{vmatrix} D-F-\alpha A & -\frac{1}{2}\beta B & 0 & 0 \\ -\frac{1}{2}\beta B & D-F-\alpha A-\theta^2C & -\frac{1}{2}\beta B & 0 \\ 0 & -\frac{1}{2}\beta B & D-F-\alpha A-4\theta^2C & -\frac{1}{2}\beta B \\ 0 & 0 & -\frac{1}{2}\beta B & D-F-\alpha A-16\theta^2C \end{vmatrix}=0 \tag{7}$$

In practical computational cases, the determinants of equations (5), (6) and (7) are obviously limited to n-order determinants and thus the respective equations will be algebraic equations of 3n degree determining approximately the critical pulsations of the excitation at the limits of the first 3n areas of dynamic instability.

When the amplitude of the variable component of the load is very small ($\beta \to 0$), the equations of the critical pulsations at the limits of the dynamic instability domains become:

$$\left| D + F - \alpha A - \frac{1}{4} k^2 \theta^2 \right| = 0; \quad k=1,2,3,.... \tag{8}$$

The equations of the free vibrations pulsations of the frame supported on elastic medium and loaded by the static components of the excitation of parameter α are:

$$\left| D + F - \alpha A - \omega^2 C \right| = 0 \tag{9}$$

Comparing (8) and (9) it follows:

$$t = \frac{2\omega}{k}; \quad k=1,\ 2,3,....... \tag{10}$$

that is, the critical pulsation of the load is an integer submultiple of twice the pulsations of the free vibrations of the frame as in the case of the lack of static load and elastic support.

The boundaries of the first domain of dynamic instability, the main domain, is the widest and most important from a practical point of view, are obtained with sufficient precision considering only the first element of the determinant in equations (5):

$$\left| D + F - \alpha A \pm \frac{1}{2} \beta B - \frac{1}{4} \theta^2 C \right| = 0 \tag{11}$$

Equations (11) can be written:

$$|R| = 0, \tag{12}$$

R being the cell matrix having as element the matrices of the third order, square:

$$R = \begin{Bmatrix} r_{ik}^{uu} & 0 & r_{ik}^{u\varphi} \\ 0 & r_{ik}^{vv} & r_{ik}^{v\varphi} \\ r_{ik}^{\varphi u} & r_{ik}^{\varphi v} & r_{ik}^{\varphi\varphi} \end{Bmatrix} \tag{13}$$

which elements are:

$$r_{ik}^{uu} = d_{ik}^{uu} + h_{ik}^{uu} - \alpha a_{ik}^{uu} \pm \frac{1}{2} \beta b_{ik}^{uu} - \frac{1}{4} \theta^2 c_{ik}^{uu};$$

$$r_{ik}^{vv} = d_{ik}^{vv} + h_{ik}^{vv} - \alpha a_{ik}^{vv} \pm \frac{1}{2} \beta b_{ik}^{vv} - \frac{1}{4} \theta^2 c_{ik}^{vv};$$

$$r_{ik}^{\varphi\varphi} = d_{ik}^{\varphi\varphi} + h_{ik}^{\varphi\varphi} - \alpha a_{ik}^{\varphi\varphi} \pm \beta b_{ik}^{\varphi\varphi} - \frac{1}{4} \theta^2 c_{ik}^{\varphi\varphi}; \tag{14}$$

$$r_{ik}^{u\varphi} = h_{ik}^{u\varphi} - \alpha a_{ik}^{u\varphi} \pm \beta b_{ik}^{u\varphi} - \frac{1}{4} \theta^2 c_{ik}^{u\varphi};$$

$$r_{ik}^{v\varphi} = h_{ik}^{v\varphi} - \alpha a_{ik}^{v\varphi} \pm \beta b_{ik}^{v\varphi} - \frac{1}{4}\theta^2 c_{ik}^{v\varphi};$$

The elements of the matrices (13) can be easily calculated if in the differential equations (1), the time functions T (t), are given by the decompositions in series of functions of the displacements of the cross sections of the frame bars, according to the functions of the displacements of the frame bars in the basis of the displacement method under the action of unit displacements acting on the basic system.

The r_{ik} coefficients that contain the pulsations of their own vibrations are called coefficients of influence or dynamic reactions. These coefficients are calculated for two types of bars: the double recessed bar and the double articulated bar, but in the case of the thin-walled bar, the displacement of the support section will be considered one of the displacements [5,7].

The calculation of influence coefficients and of dynamic reactions for these two types of bars implies several integrations [6].

These elements, also called dynamic reactions, can be written with the help of the dynamic reaction tables of the two types of bars intervening in the method of displacements and located on elastic medium. The structure of the elements of the coefficients of dynamic reactions is quite complex:

$$d_{ik}^{uu} = \sum EI_y \int_0^1 \frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$d_{ik}^{\varphi\varphi} = \sum EI_\omega \frac{dZ_i^{2\varphi}}{dz^2}\frac{dZ_k^{2\varphi}}{dz^2}dz + \sum GI_i \int_0^i \frac{dZ_i^\varphi}{dz}\frac{dZ_k^\varphi}{dz}dz;$$

$$c_{ik}^{uu} = \sum m \int_0^i Z_i^u(z) Z_k^u(z) dz + \sum m r_y^2 \int_0^i \frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$f_{ik}^{\varphi\varphi} = \sum \int_0^i \left[k_x (y_0 - h_y)^2 + k_y (x_0 - h_x)^2 + k_\varphi \cdot \varphi\right] Z_i^\varphi(z) Z_k^\varphi(z) dz; \qquad (15)$$

$$a_{ik}^{uu} = \sum \int_0^i N_0(z) \frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$a_{ik}^{\varphi\varphi} = \sum (e_y \beta_1 + e_x \beta_2 + r_0^2) \int_0^i N_0(z) \frac{dZ_i^\varphi(z)}{dz}\frac{dZ_k^\varphi(z)}{dz}dz;$$

$$b_{ik}^{vv} = \sum \int_0^i N_i(t) \frac{dZ_i^v(z)}{dz}\frac{dZ_k^v(z)}{dz}dz;$$

$$b_{ik}^{v\varphi} = -\sum (x_0 - e_0) \int_0^i N_i(z) \frac{dZ_i^v(z)}{dz} \frac{dZ_k^{\varphi}(z)}{dz} dz;$$

$$c_{ik}^{vv} = \sum m \int_0^i Z_i^v(z) Z_k^v(z) + \sum m r_x^2 \int_0^i \frac{dZ_i^v(z)}{dz} \frac{dZ_k^{\varphi}(z)}{dz} dz;$$

The reaction coefficients contain the geometric and elastic characteristics of the bar as well as the parameters of the excitatory load and the elastic characteristics of the support medium [3].

Under these conditions the equation of pulsations of free vibrations of the elastic support frame is:

$$\left| D + F - \alpha A \pm \frac{1}{2} \beta B - \omega^2 C \right| = 0 \tag{16}$$

CONCLUSIONS

This paper presents a system of equations used for determining the main areas of dynamic instability of frames with thin walls of open sections.

In view of this system of equations it is easily found that the presence of the elastic medium leaves unchanged the width of the dynamic instability domains, realizing only a translation of them towards the higher frequency domain.

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