ON GRAVITY CENTER OF A HOMOGENEOUS FLAT PLATE

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Abstract: *The paper presents aspects on justification of the graphic method applied in determining the gravity center of a homogeneous flat plate of a given configuration.*

Keywords: Homogeneous plate, center of gravity;

1. INTRODUCTION

The existence of the field of gravity of the Earth leads to an interaction at distance between a body and the Earth, namely the force of gravitational attraction. Gravitational attraction is the classic case of parallel forces. These forces can be reduced to a resultant applied in the center of parallel forces, which in this case becomes the center of gravity. The phrase center of gravity is often interchangeable with the phrase center of masses, but they are different physical concepts. the two centers overlap in a uniform gravitational field.

The center of mass is a concept of a more general character than the center of gravity, being independently defined in relation to the latter. In the absence of the gravitational field, the center of gravity loses its meaning, while the center of mass continues to exist. Likewise, in a non-uniform gravitational field, the two points (centers) take different places. To be noted that in the context of the current technical problems, one can consider that the position of the two centers coincide.

The position of the center of gravity of a body is determined in a simpler way when the latter is homogeneous and has elements of symmetry.

The determination of the center of gravity of a compound body, can be made with the help of partial centers of gravity, the body having a shape that allows it being divided in several component parts, for each part the mass (weight) and the position of the center of gravity are known.

The center of gravity of a homogeneous body and of regular shape depends on the geometrical elements of the body, not on the nature of the material of which it is made, and can be decomposed in component parts, of regular geometric shape as well. If these bodies have a plan, an axis or a center of symmetry, then the center of gravity is found in the plane, on the axis or in the respective center of symmetry.

The center of gravity of simple homogeneous bodies of regular shape (for instance, square, rectangle, triangle) can be determined graphically as well (geometrically).

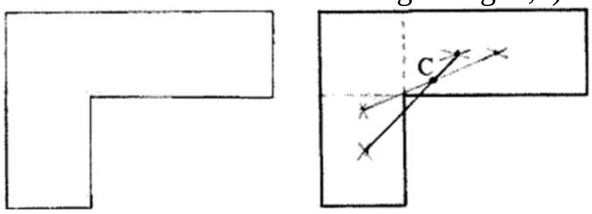
Aspects will be further presented on the graphic method applied in the determination of the center of gravity of a homogeneous flat plate of given configuration and the justification of this method.

2. GRAPHIC DETERMINATION OF THE CENTER OF MASS OF THE PLATE

Using a ruler and a sharp pencil, the center of gravity of the plate in Fig. 1. a, needs to be determined. The plate has right angles, it is homogeneous, and has constant thickness. It is required to present how to proceed, by motivating physically the method, explaining the procedure applied and showing how the center of gravity of the plate has been located.

The resolution is shown below.

The plate in the shape of the letter *L* is made up by joining two rectangular plates, of unequal length. Their centers of gravity are at the intersection of the diagonals (in the very middle of the respective rectangles). The center of the plate in the shape of the letter *L* is definitely on the line that joins the centers of the two rectangular plates. The arms of the letter *L* are not equal, however. Therefore, the choice of the two joined rectangular plates can be done in to separate ways (see the two dotted lines in the drawing in Fig. 1,b).



a) b)

Fig. 1. The plate and the intersection of the lines that join the centers of gravity in the two decompositions

Point *C* (of the intersection of the two straight lines obtained by joining the centers of gravity of the selected plates in the two ways) is the center of gravity of the *L* shaped plate.

3. JUSTIFICATION OF THE APPLIED GRAPHIC METHOD

We shall consider the first mode of selecting the component plates as in Fig. 2 with centers of gravity, the rectangular plate 1 of m1 mass and the center of gravity in point 1 C′ , and the rectangular plate 2 of m2 mass and the center of gravity in point 2 C′, respectively.

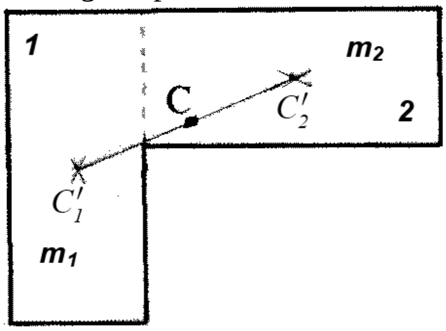


Fig. 2. The first mode of decomposition

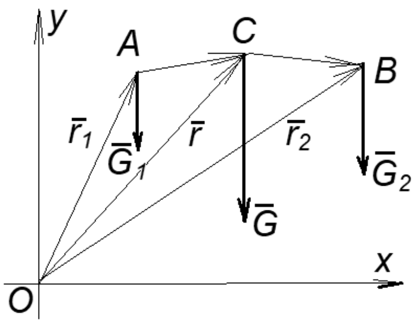


Fig. 3. The scheme for the first mode

Let us take in plane *xOy* (Fig. 3) point *A* of mass m1 and position vector $\overline{r}_2 = x_2\overline{i} + y_2\overline{j}$, and point *B*, respectively, of mass m2 and position vector $\overline{r}_2 = x_2\overline{i} + y_2\overline{j}$ the center of mass (weight) of the two points of the coordinates given by the equations:

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \quad y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}, \tag{1}$$

We further check whether vectors $\overline{AB}$ and $\overline{AC}$ are colinear ($C \in \overline{AB}$) and whether the center of gravity *C* is between points *A* and *B*. We thus have:

$$\overline{OA}+\overline{AC}=\overline{OC} \Rightarrow \overline{AC}=\overline{OC}-\overline{OA}$$
$$\overline{OA}+\overline{AB}=\overline{OB} \Rightarrow \overline{AB}=\overline{OB}-\overline{OA} \quad , \qquad (2)$$

where:

$$\overline{OA}=\overline{r_1}=x_1\overline{i}+y_1\overline{j},\ \overline{OB}=\overline{r_2}=x_2\overline{i}+y_2\overline{j},\ \overline{OC}=\overline{r}=x\overline{i}+y\overline{j}, \qquad (3)$$

Whence:

$$\overline{AB}=\left(x_2-x_1\right)\overline{i}+\left(y_2-y_1\right)\overline{j}$$

$$\overline{AC}=\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}-x_1\right)\overline{i}+\left(\frac{m_1y_1+m_2y_2}{m_1+m_2}-y_1\right)\overline{j}=$$

$$=\left(\frac{\cancel{m_1x_1}+m_2x_2-\cancel{m_1x_1}-m_2x_1}{m_1+m_2}\right)\overline{i}+\left(\frac{\cancel{m_1y_1}+m_2y_2-\cancel{m_1y_1}+m_2y_1}{m_1+m_2}\right)\overline{j}= \quad , \qquad (4)$$

$$=\frac{m_2}{m_1+m_2}\left[\left(x_2-x_1\right)\overline{i}+\left(y_2-y_1\right)\overline{j}\right]=\frac{m_2}{m_1+m_2}\overline{AB}$$

Thus:

$$\overline{AC}=\lambda_1\overline{AB},\quad \left(\lambda_1=\frac{m_2}{m_1+m_2}\lhd 1\right), \qquad (5)$$

In conclusion vectors $\overline{AB}$ and $\overline{AC}$ are colinear ($\overline{AC}=\lambda_1\overline{AB}$ and the vectors having the same origin) and the center of gravity C is between points A and B, $\lambda_1 < 1$.

We shall further consider that the second mode of selection of the component plates, as in Fig. 4, with centers of gravity, rectangular plate $1'$ of mass m_1-m and the center of gravity in point C_1'' , and respectively, the rectangular plate $2'$ of mass m_2+m and the center of gravity in point C_2''. We have $m<m_1$ and $m<m_2$.

For the second mode, let us have in plane xOy (Fig. 5) point A' of mass m_1-m and the position vector $\overline{r_1'}=x_1'\overline{i}+y_1'\overline{j}$, and, respectively, point B' of mass m_2+m and the position vector $\overline{r_2'}=x_2'\overline{i}+y_2'\overline{j}$ the center of mass (weight) of the two points of the given coordinates given by the equations:

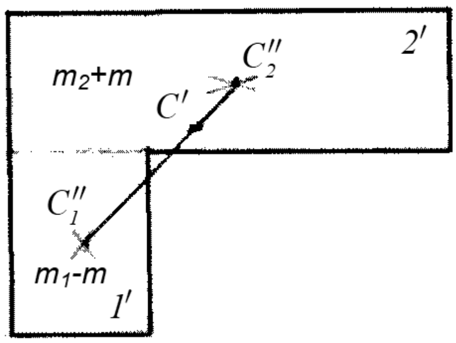


Fig. 4. The second mode of decomposition

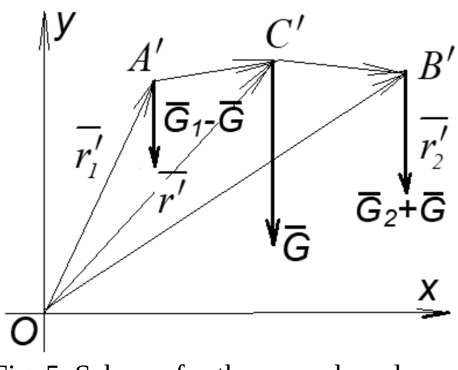


Fig. 5. Scheme for the second mode

The center of gravity C' should coincide with the center of gravity C ($C' \equiv C$) since the same plate cannot have two centers of gravity.

We further verify whether vectors $\overline{A'B'}$ și $\overline{A'C'}$ ($\overline{A'C'} \equiv \overline{A'C}$) are colinear ($C' \equiv C \in \overline{A'B'}$) and whether the center of gravity C' (C) is between points A' and B'.

We thus have:

$$\begin{aligned} &\overline{OA'} + \overline{A'C} = \overline{OC} \;\; \Rightarrow \;\; \overline{A'C} = \overline{OC} - \overline{OA'} \\ &\overline{OA'} + \overline{A'B'} = \overline{OB'} \;\; \Rightarrow \;\; \overline{A'B'} = \overline{OB'} - \overline{OA'} \end{aligned}, \tag{7}$$

where:

$$\begin{aligned} &\overline{OA'} = \overline{r_1'} = \overline{x_1'\overline{i} + y_1'\overline{j}} \\ &\overline{OB'} = \overline{r_2'} = x_2'\overline{i} + y_2'\overline{j} \\ &\overline{OC'} \equiv \overline{OC} = \overline{r} = x\overline{i} + y\overline{j} \end{aligned} , \tag{8}$$

Whence:

$$\overline{A'B'} = \left(x_2' - x_1'\right)\overline{i} + \left(y_2' - y_1'\right)\overline{j}\,, \tag{9}$$

and

$$\overline{A'C} = \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - x_1'\right)\overline{i} + \left(\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} - y_1'\right)\overline{j}\,, \tag{10}$$

But x ′=x and y ′=y , it resuls:

$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{x_1' m_1 + x_2' m_2 + m\left(x_2' - x_1'\right)}{m_1 + m_2} \tag{11}$$

and

$$\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{y_1' m_1 + y_2' m_2 + m\left(y_2' - y_1'\right)}{m_1 + m_2} \tag{12}$$

We can write taking into consideration equations (11) and (12) that:

$$\begin{aligned} \overline{A'C} &= \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - x_1'\right)\overline{i} + \left(\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} - y_1'\right)\overline{j} = \\ &= \left[\frac{x_1' m_1 + x_2' m_2 + m\left(x_2' - x_1'\right)}{m_1 + m_2} - x_1'\right]\overline{i} + \left[\frac{y_1' m_1 + y_2' m_2 + m\left(y_2' - y_1'\right)}{m_1 + m_2} - y_1'\right]\overline{j} = \end{aligned}$$

$$
\begin{aligned}
&= \left[\frac{x_1' m_1 + x_2' m_2 + m\left(x_2' - x_1'\right) - x_1'\left(m_1 + m_2\right)}{m_1 + m_2}\right]\overline{i} + \\
&+ \left[\frac{y_1' m_1 + y_2' m_2 + m\left(y_2' - y_1'\right) - y_1'\left(m_1 + m_2\right)}{m_1 + m_2}\right]\overline{j} = \\
&= \frac{1}{m_1 + m_2}\left[\left(m_2 + m\right)\left(x_2' - x_1'\right)\overline{i} + \left(m_2 + m\right)\left(y_2' - y_1'\right)\overline{j}\right] = \\
&= \frac{m_2 + m}{m_1 + m_2}\overline{A'B'}
\end{aligned} \quad , \qquad (13)
$$

$$
\overline{A'C} = \lambda_2 \overline{A'B'}, \quad \left(\lambda_2 = \frac{m_2 + m}{m_1 + m_2} \lhd 1\right), \qquad (14)
$$

In conclusion $\overline{A'B'}$ and $\overline{A'C}$ are colinear ($\overline{A'C} = \lambda_2 \overline{A'B'}$ and vectors having the same origin) and the center of gravity $C' \equiv C$ is between A' and B', $\lambda_2 < 1$.

Thus in the end the center of gravity of the plate is at the intersection of the lines going through points C_1', C_2' (line $\overline{C_1'C_2'}$) and, respectively, C_1'', C_2'' (line $\overline{C_1''C_2''}$).

That is $C \in \overline{C_1'C_2'} \cap C_1''C''$.

Collinearity is in geometry the property of a number greater than two points to belong to the same line. Several non-collinear points are points that cannot belong to the same line. it can be demonstrated using vectors and complex numbers, as with coplanarity. Out of the specific methods to demonstrate collinearity used in geometry, we mention the following methods. Demonstration of collinearity of points using the applications of complex numbers in geometry. If points A, B, C have affixes zA, zB, zC, respectively, A, B, C are collinear if, and only if
$\left(z_B - z_A\right) / \left(z_C - z_A\right) \in \mathbf{R}^*$

Demonstration of collinearity by identification of a line that includes the respective points. To show that points *A*, *B*, *C* are collinear a line is identified to which they would belong.

$$
\frac{y_3 - y_1}{y_2 - y_1} = \frac{x_3 - x_1}{x_2 - x_1}, \qquad (15)
$$

The condition of collinearity of the three points can also be written in the form of a determinant:

The condition of collinearity of three points $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ is obtained if we put the condition that point $C(x_3,y_3)$ should verify the equation of line *AB*, that is:

$$
\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}, \qquad (16)
$$

To be mentioned that the fact he above mentioned vectors are collinear can be demonstrated also by calculating the vector product.

Thus we have:

$$\overline{AC}\times\overline{AB}=\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ \frac{m_2}{m_1+m_2}\left(x_2-x_1\right) & \frac{m_2}{m_1+m_2}\left(y_2-y_1\right) & 0\\ x_2-x_1 & y_2-y_1 & 0\end{vmatrix}=, \quad (17)$$

$$=\frac{m_2}{m_1+m_2}\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ x_2-x_1 & y_2-y_1 & 0\\ x_2-x_1 & y_2-y_1 & 0\end{vmatrix}=0$$

$$\overline{A'C}\times\overline{A'B'}=\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ \frac{m_2+m}{m_1+m_2}\left(x_2'-x_1'\right) & \frac{m_2+m}{m_1+m_2}\left(y_2'-y_1'\right) & 0\\ x_2'-x_1' & y_2'-y_1' & 0\end{vmatrix}=, \quad (18)$$

$$=\frac{m_2+m}{m_1+m_2}\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ x_2'-x_1' & y_2'-y_1' & 0\\ x_2'-x_1' & y_2'-y_1' & 0\end{vmatrix}=0$$

4. CONCLUSIONS

In the paper a graphic method has been analytically justified with the help of vector calculation presented to determine the center of gravity of a flat homogeneous plate of geometrical configuration given in the shape of an *L*.

5. REFERENCES

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