A METHOD OF THE MINIMIZING OF THE TOTAL ACQUISITIONS COST WITH THE INCREASING VARIABLE DEMAND

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Abstract

Over time, mankind has tried to find different ways of costs reduction. This subject which we are facing more often nowadays, has been detailed studied, without reaching a general model, and also efficient, regarding the costs reduction. Costs reduction entails a number of benefits over the entity, the most important being: increase revenue and default to the profit, increase productivity, a higher level of services / products offered to clients, and last but not least, the risk mitigation of the economic deficit. Therefore, each entity search different modes to obtain most benefits, for the company to succeed in a competitive market. This article supports the companies, trying to make known a new way of minimizing the total cost of acquisitions, by presenting some hypotheses about the increasing variable demand, proving them, and development of formulas for reducing the costs. The hypotheses presented in the model described below, can be maximally exploited to obtain new models of reducing the total cost, according to the modes of the purchase of entities which approach it.

Key words: *inventories, stock management, increasing variable demand, total acquisition cost, minimizing costs*

JEL Clasification: *B41, C02*

1. Introduction

Inventories are a very important part within a company and therefore, these must be managed with great prudence. The optimum stocks management shall ensure the consumption continuity. [1] In stock management systems, we need the calculation of the stock value for any product, at any particular moment in time. [2]

Throughout time, the main objective in the production field was the minimizing of the costs. If should be taken considered this objective and the example the two models presented by Professor Eugen Țigănescu, together with the lecturer Dorin Mitrut, in their study „Operational research foundation" [3] (Willson model and Willson model with stockouts), will be elaborate a new model to minimize the total acquisition cost. The models used in the works mentioned above include five hypotheses that are demonstrated subsequent by the author, to establish a prototype of the costs reduction model.

The hypothesis proposed by the author refer to: constant demand in time, the supply at equal time intervals, equal amounts of supply, supplying when the stock becomes 0 and the last hypothesis refers to instant supply.

2. The presentation of the method of minimizing the total acquisition cost

Starting from these hypothesis, but using a more common framework, ie variable demand in time, will be present below, a new model of inventories management, which aims to minimize the total acquisition cost (CT). In order to achieve this objective, are known five hypotheses, which will be demonstrated subsequent:

The first hypothesis of the model is variable demand in time, ie unequally demand on the equal interval of time. From this hypothesis, results the following relation:

$$\frac{n}{\tau} > \frac{N}{T} \text{ (1) = demand per unit of time} \Rightarrow \text{s(t) = increasing variable,}$$

where: n = quantity ordered and brought on each supply

τ = interval between two successive supplies

N = total demand on the period T
T = total period of time which the storage is studied
s(t) = stock levels from the warehouse at time t

$$\frac{n}{\tau} > \frac{N}{T} \Leftrightarrow \frac{N}{T} < \frac{n}{\tau} \Leftrightarrow N \times \tau < n \times T / \div n \Leftrightarrow \frac{N \times t}{n} < T / \div t \Leftrightarrow \frac{N}{n} < \frac{T}{t} \quad (2)$$

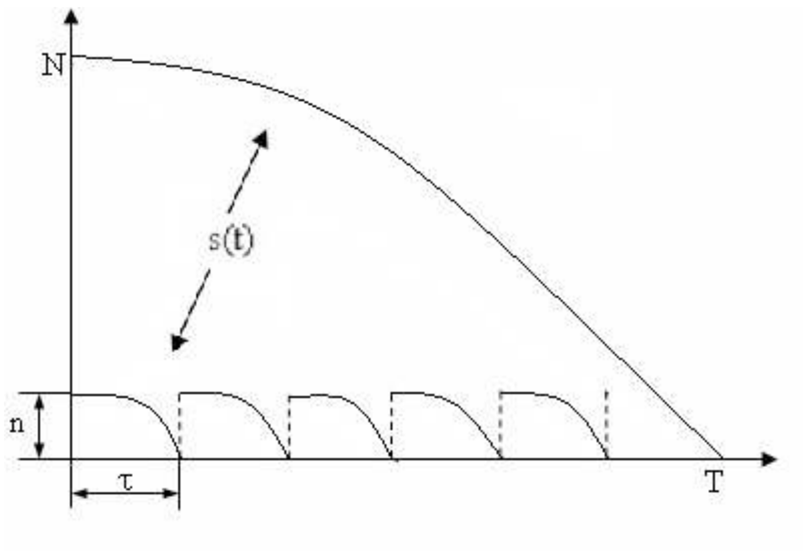
2. The second hypothesis assumes a fixed period of supply, ie the supply at equal intervals of time. Therefore, the manufacturer buys raw materials on the same time each month, which means that the interval between two successive supplies (τ) is the same between any two commands.

3. The third hypothesis assumes equal amounts of supply. This means that the entity purchase the same amount of raw materials, at every supply, ie n is the same for all orders.

4. The fourth hypothesis implies that the purchase is made in the moment when the stock becomes 0, there are not accepted time intervals having the stock 0. If the manufacturer grants the supplier of raw materials full trust, this can provide when the stock is 0, but no matter how much trust he has, there is a risk of blocking the production, because the goods will not be delivered in time. However, considering the idealistic case, the relationship results form this hypothesis $\Rightarrow$ $s(t) \geq 0$ for anyone would be t.

5. The fifth and final hypothesis of the model indicates that the purchase is made instantaneously, the duration between the moment of placing the order and the entry of goods into the warehouse being 0. From here, it results the fact that, at the end of the interval between two successive supplies, s(t) will have a jump from 0 to n.

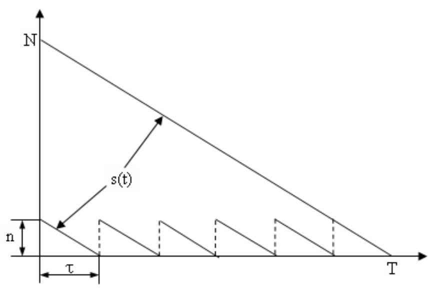
The situations resulted from this model, will be better analysed on the basis of the graphic representation of the evolution of the stocks, as follows:



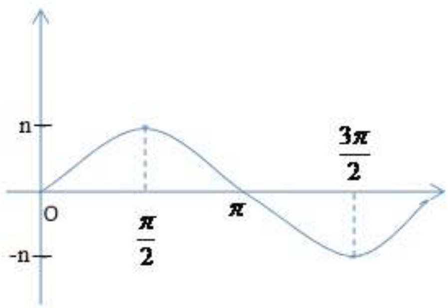
Graphic no.1 - The evolution of the stock over time intervals, in the case of a increasing variable demand

From the graphic no. 1, shown above, results are the following: The first graphic representation (the one above) assume the fact that all the necessary quantity will have brought at the beginning of period, and the second graphic representation (the bottom one) assume the fact that would bring n units in the interval between two successive purchase in τ units of time (will be brought n units of the interval τ in the same τ units of time).

Starting from the graphic representation of the constant demand in the time (the graphic no. 2), elaborated by authors Eugen Ţigănescu and Dorin Mitruţ, in the study „ Operational research foundation" (mentioned above) and combined with graphic representation of the sinus function (graphic no. 3), will be analysed the following issues:



Graphic no. 2 – The evolution of the stock over time intervals, in case of a constant demand [3]



Graphic no. 3 – the graphic n x sin *u*

The interval $[\frac{\pi}{2};\pi]$, from the graphic n x sin u (graphic no. 3), may be associated with the time interval „τ" from the graphic no. 2, and the curve from $\frac{\pi}{2}$ to π from the graphic n x sin u, it may be associated the curves in the stocks evolution chart, on the time intervals in case of increasing variable demand (graphic no. 1).

Considering the fact that the evolution is periodical, by τ period, will be calculated the total acquisition cost considering the fact that during a time period there is one single launch, ie a cost (c_1) and the storage expenditure during a period τ, and by the fact that the stock varies sinusoidally from n la 0, the cost of storage (c_s) is calculated:

$$c_s \times \sin u \times \tau \ (3),\ u\in(0;\frac{\pi}{2}], \text{ where u = a some constant.}$$

It is worth mentioning that generally determination the cost of storage is conducted by the formula: $c_S \times \int_0^T \sin u \times s(t)dt = c_S \times \sin u \int_0^T s(t)dt$. (4)

- the number of periods will be determined thereby: $\frac{N}{n} < \frac{T}{\tau}$

- the total acquisition cost will be determined: $C_T = (c_1 + c_s \times \sin u \times \tau) \times \frac{N}{n}$ (5)

With the relations outlined above, the minimum of the function will be determined:

$C_T(n,\tau) = (c_1 + c_S \times \sin u \times \tau) \times \frac{N}{n}$ (6), where n and τ are strictly positive, n ∈ (0;N], τ ∈ (0;T] and u ∈ $(\frac{\pi}{2};\pi]$, together verifying the relationship $\frac{N}{n} < \frac{T}{\tau}$.

From the relationship $\frac{N}{n} < \frac{T}{\tau} \Rightarrow \tau \times N = n \times T \Rightarrow \tau = n \times \frac{T}{N}$, (7) then:

$$C_T(n) = (c_1 + c_s \times \sin u \times n \times \frac{T}{N}) \times \frac{N}{n} = \frac{c_1 \times N}{n} + c_s \times \sin u \times n \times \frac{T}{N} \times \frac{N}{n} =$$

$$\frac{c_1 \times N}{n} + c_s \times n \times \sin u \times T \ (8)$$

From the above formulas, the total cost (C_T) was separated in the total expenditure with the launches and total expenditure with the storage, where it can be seen that the first are decreasing in n and the other are increasing sinusoidally. This means that if the whole amount will be brought in one installment, the storage costs will be very high and if it is brought very often a little quantity, launch costs will be very high. A solution for this problem would be to determine the total cost derived according to n placed in the interval (0,N], thereby:

$$C'_T = \frac{c_l \times N}{n} + c_s \times n \times \sin u \times T \ (9)$$

$\frac{c_l \times N}{n} + c_s \times n \times \sin u \times T = 0$; (10) according derivative rules:

$\left(\frac{1}{n}\right)' = -\frac{1}{n^2}$; $(n^2)' = 2n$; $n' = 1$; $(3n)' = 3$, (11) obtain:

$-\frac{c_1 \times N}{n^2} + c_s \times T \times \sin u = 0 \Rightarrow -\frac{c_1 \times N}{n^2} + \frac{c_s \times T \times \sin u}{1} = 0 \Rightarrow \frac{c_s \times T \times \sin u}{1} = \frac{c_1 \times N}{n^2} \Rightarrow$

$c_s \times T \times \sin u \times n^2 = c_1 \times N \times 1 \Rightarrow n^2 = \frac{c_1 \times N}{c_s \times T \times \sin u} \Rightarrow n = \pm\sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}}$, (12) which has zeros:

$$n_{1,2} = \pm\sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}} \Rightarrow n_1 = -\sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}} \notin (0, N]; \quad (13)$$

$$n_2 = \sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}} \in (0, N] \Leftrightarrow \sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}} \le N \quad (14) \quad /^2 \Rightarrow$$

$$\frac{c_1 \times N}{c_s \times T \times \sin u} \le N^2 \qquad /\div N \neq 0 \Rightarrow$$

$$\frac{c_1}{c_s \times T \times \sin u} \le N \qquad /\times T \times \sin u \Rightarrow$$

$$\frac{c_1}{c_s} \le N \times T \times \sin u \quad (15)$$

As a conclusion of this model there are we have two situations:

- if $\frac{c_l}{c_S} \ge N \times T \times \sin u$ (16), ie if the cost of launch is more than $N \times T \times \sin u$ times higher than the cost of storage, the variation formula will be:

$$C_T(n) = \frac{c_1 \times N}{n} + c_s \times n \times \sin u \times T \Rightarrow$$

$$C_T(N) = \frac{c_1 \times N}{N} + c_s \times N \times \sin u \times T \Rightarrow$$

$$C_T(N) = c_1 + c_s \times N \times T \times \sin u \quad (17)$$

Table no. 1 **The variation of n from 0 to N (case 1)**

n	0 N
$C_T'(n)$	- - - - - - - - -
$C_T(n)$	↘ $c_1 + c_S \times N \times T \times \sin u$

and so will be a single purchase at beginning of period T in that will bring the entire quantity N, total cost being $c_l + c_S \times N \times T \times \sin u$.

- if $\frac{c_l}{c_S} < N \times T \times \sin u$ (18) obtain:

$$C_T(n) = \frac{c_1 \times N}{n} + c_s \times n \times \sin u \times T \Rightarrow$$

$$C_T(n) = \frac{c_1 \times N}{\sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}}} + c_s \times T \times \sin u \times \sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}} \Rightarrow$$

$$C_T(n) = \sqrt{\frac{(c_1 \times N)^2}{\frac{c_1 \times N}{c_s \times T \times \sin u}}} + \sqrt{(c_s \times T \times \sin u)^2 \times \frac{c_1 \times N}{c_s \times T \times \sin u}} \Rightarrow$$

$$C_T(n) = \sqrt{\frac{(c_1 \times N)^2}{1} \times \frac{c_s \times T \times \sin u}{c_1 \times N}} + \sqrt{(c_s \times T \times \sin u)^2 \times \frac{c_1 \times N}{c_s \times T \times \sin u}} \Rightarrow$$

$$C_T(n) = \sqrt{c_1 \times N \times c_s \times T \times \sin u} + \sqrt{c_s \times T \times \sin u \times c_1 \times N} \Rightarrow$$

$$C_T(n) = 2\sqrt{c_1 \times N \times c_s \times T \times \sin u} \quad (19)$$

Table no. 2 **The variation of n from 0 to N (case 2)**

n	0	$\sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}}$	N
$C_T'(n)$	- - - - - -	0	+ + + + + +
$C_T(n)$	↘	$2\sqrt{c_1 \times N \times c_s \times T \times \sin u}$	↗

In conclusion will be made: $\frac{N}{n}$ purchasings at intervals of $t_{opt} = \sqrt{\frac{c_l \times T}{c_S \times N \times \sin u}}$ (20) that will be brought in a quantity $n_{opt} = \sqrt{\frac{c_l \times N}{c_S \times T \times \sin u}}$ (21), variant which will purchase with the lowest possible total cost:

$C_T = 2\sqrt{c_1 \times N \times c_s \times T \times \sin u}$ (22), where:

$$\frac{N}{n} = \frac{N}{\sqrt{\frac{c_1 \times N}{c_s \times T \times \sin u}}} = \sqrt{\frac{N^2}{\frac{c_1 \times N}{c_s \times T \times \sin u}}} = \sqrt{\frac{N^2}{1} \times \frac{c_s \times T \times \sin u}{c_1 \times N}} = \sqrt{\frac{N \times c_s \times T \times \sin u}{c_l}} \quad (23)$$

When the only accepted solution for this situation are n and t integers, computed like this:

$n_1 = \left[\sqrt{\frac{c_l \times N}{c_S \times T \times \sin u}}\right]$ and $n_2 = \left[\sqrt{\frac{c_l \times N}{c_S \times T \times \sin u}}\right] + 1$ (24)

$t_1 = \left[\sqrt{\frac{c_l \times T}{c_S \times N \times \sin u}}\right]$ and $t_2 = \left[\sqrt{\frac{c_l \times T}{c_S \times N \times \sin u}}\right] + 1$ (25)

whichever is the cheapest solution. ([x] = the whole of x).

3. Conclusions

The five hypotheses presented in the previous model, can be affirmative or negative, so can be seen from two points of view, as follows:
The hypothesis no. 1:

The constant demand in time, which means the equal demand on the equal intervals of time $\Rightarrow \frac{n}{\tau}=\frac{N}{T}$ = the demand per unit of time $\Rightarrow$ s(t)= limited

The variable demand in time, which assumes unequal demands on unequal time intervals $\Rightarrow \frac{n}{\tau}\neq\frac{N}{T}\Rightarrow$ s(t)= variable

The hypothesis no. 2:

The fixed period of purchase, which means that the purchase will be made at equal time intervals $\Rightarrow\tau$ (the interval between two successive purchase) It is the same between any two commands

The variable period of purchase, which means that the purchase will be made at unequal time intervals $\Rightarrow\tau$ (the interval between two successive purchase) It isn't the same between any two commands

The hypothesis no. 3:

The equal amounts of supply, this hypothesis assumes the fact that every time the company purchase the same quantity of goods $\Rightarrow$ n (the quantity ordered and brought to each supply) it's the same for all orders

The variable amounts of supply, this hypothesis assumes the fact that the entity does not purchase every time the same quantity of goods $\Rightarrow$ n (the quantity ordered and brought to each supply) it isn't the same for all orders

The hypothesis no. 4:

Supplying is made in the moment when the stock is 0, there are not accepted intervals of time for the stock to be 0 $\Rightarrow$ s(t) $\geq$ 0 for any t

Supplying isn't made in the moment when the stock is 0, there are accepted intervals of time for the stock to be 0, and the demand isn't satisfied $\Rightarrow$ s(t) > 0 for any t (s(t) it is strictly greater than 0, which means that when supplying with good there is a stock > 0).

The hypothesis no. 5:

The supply is made instantly, ie the period between the time of order placement and the entry of goods in warehouse is zero $\Rightarrow$ at the end of a period τ, s(t) it has a jump from 0 to n

The supply isn't made instantly, ie the period between the time of order placement and the entry of goods in warehouse is different zero $\Rightarrow$ at the end of a period τ there is something in the warehouse, so s(t) it has a jump from m (ie from how much goods are in the warehouse) to m+n (n representing the quantity that will supply).

Considering these existing options (in number of 10) and the fact that can be taken into account by 5 hypothesis, the entity can create its own stocks management model. Through this information, we can calculate the number of models that can be configured using a mathematical formula to combinations, so:

$$C_n^k=\frac{n!}{k!(n-k)!}, \text{ where } n\geq k\geq 0;\ k,n\in N$$

Through applying this formula we obtain:

$$C_{10}^5=\frac{10!}{5!(10-5)!}=\frac{5!\times 6\times 7\times 8\times 9\times 10}{5!\times 1\times 2\times 3\times 4\times 5}=\frac{6\times 7\times 2\times 3}{1}=252 \text{ models}$$

Applying mathematical formula to combinations in this context, it allows identifying existing number of potential models for minimizing the cost of supply, although not all of them can be done in everyday life.

The hypothesis refers to "supplying is made in the moment when the stock is 0, not being accepted the periods in which the stock to be 0" or "the supply is made instantly, ie the period between the time of order placement and the entry of goods in warehouse is zero" they can be hardly encountered nowadays.

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