LONG-TERM VOLATILITY DYNAMICS OF THE GERMAN STOCK MARKET : INSIGHTS FROM TWO DECADES OF DAILY RETURNS

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Abstract

This study provides an empirical analysis of the volatility dynamics of the Deutscher Aktienindex (DAX) stock index over a 20-year period based on daily observations, specifically from January 2, 2006, to March 20, 2026. Utilizing a dataset of 5,140 daily return points, the research explores the time-varying nature of market risk and the presence of volatility clustering. The primary objective is to identify a robust econometric framework capable of capturing the asymmetric response of volatility to market shocks, commonly known as the leverage effect.

To achieve this, the study evaluates several GARCH-family models specifications, including GARCH, EGARCH, GJR-GARCH, and APARCH models, paired with various error distributions such as Normal, Student-t, GED, and Skewed-t. Initial testing confirms that the return series is stationary, non-normally distributed, and characterized by significant

"fat tails". Based on the lowest Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the GJR-GARCH model with a Skewed-t distribution is identified as the most suitable model for the DAX index.
The results demonstrate high volatility persistence and provide strong evidence of the leverage effect, where negative market shocks impact volatility more significantly than positive ones. Diagnostic checks, including the Ljung-Box test, confirm that the model successfully captures the underlying volatility structure. These findings offer valuable insights for investors and policymakers regarding risk assessment and strategic decision-making in the German equity market.

Keywords: *DAX Index, GJR-GARCH model, Volatility Clustering, Leverage Effect, Skewed-t Distribution, Market Risk*

JEL Classification Codes: *C22, C52, G11, G17*

1.Introduction

When we look at European financial markets, the Deutscher Aktienindex (DAX) stands out as a heavyweight. It tracks the biggest and most liquid companies listed on the Frankfurt Stock Exchange. Historically, the index was heavily tilted toward traditional German industrial powerhouses, think massive automakers, financial giants, and chemical or pharmaceutical conglomerates. Over the last two decades, it has been a front-row spectator to some massive shifts in global finance. From the post-reunification boom of the 90s and the painful dot-com crash of the early 2000s, to the 2008 global financial crisis and the 2020 COVID-19 shock, the DAX has ridden every wave. More recently, between 2023 and 2025, we have seen it stage a remarkably strong rally.

Because the DAX is so central to European commerce, understanding its volatility isn't just an academic exercise, it is absolutely vital for managing risk. If you look at a graph of DAX closing prices over the last twenty years, you will see a story of constant motion. Prices are never static. Instead, they fluctuate constantly, reacting to both minor microeconomic shifts and massive global events.

What is particularly interesting is how this volatility behaves. If you plot the daily returns, you will notice volatility clustering. This means that turbulent, high-risk periods tend to group together, and quiet, calm periods do the same. To make sense of this, researchers rely on GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models. These statistical tools are specifically designed to analyze financial time series by using past errors and past variances to map out how risk changes over time.

While standard GARCH or ARCH models do a great job of tracking basic variance, they tend to treat good news and bad news equally. In the real world of trading, we know this isn't true. Markets tend to panic much faster when things go south. To capture this lopsided behavior, we lean on more advanced extensions like EGARCH and APARCH, which account for power transformations and asymmetric responses (Pourmansouri and Birau, 2024).

However, for this specific study, we use the GJR-GARCH model with a skewed-t distribution to analyze 5,140 daily data points from January 2, 2006, to March 20, 2026. This setup gives us the best of both worlds: the GJR-GARCH framework specifically hunts for the leverage effect (where negative news spikes volatility much harder than positive news), while the skewed-t distribution respects the heavy "fat tails" and skewness of real-world financial data. Ultimately, by stripping away normal assumptions and looking at the raw, asymmetric nature of market fear, this paper offers a realistic, reliable toolkit for modern risk assessment.

2.Literature Review

The specialized literature highlights numerous empirical studies on the behavior of stock markets, both developed (such as Spulbar et al., 2023a; Anand et al., 2023; Birau, 2014; Spulbar et al., 2023b) and developing (such as Birau and Trivedi, 2015; Meher et al., 2023; Hawaldar et al., 2020; Aman et al., 2024) markets. Spulbar et al. (2023a) have conducted an empirical research study based on a cluster of developed stock markets such Germany, but also Austria, France and Spain using a series of econometric models and statistical tests. In addition, another research study that investigated the behavior of the developed stock market in Germany is Spulbar et al. (2023b) which is focused on another cluster of stock markets such as UK, France, and German stock markets.

Celebi and Hönig (2019) developed a research study on the dynamics of the German stock market analyzing the influence of certain macroeconomic factors under extreme events, during the long term selected period from 1991 to 2018. Tilfani et al. (2020) provided a comprehensive comparative research between different European stock markets, including the German market by applying advanced econometric methodologies.

Rutkowska-Ziarko (2022) also investigated the behavior of the German stock market, i.e. Frankfurt Stock Exchange based on certain stock indices such as DAX, MDAX and SDAX indices. Moreover, another study provided by Niyitegeka and Habiyaremye (2024) estimated the financial linkage and interconnections between the BRICS stock markets and the German market for the selected time frame from 2005 to 2017.

3.Research Gap

While the academic world has no shortage of studies on stock market volatility, a closer look reveals a persistent gap in how we handle the "messiness" of real-world data. Most traditional models assume that market shocks are symmetrical, meaning they treat a sudden price jump and a sudden price crash as if they have the same impact on future risk. However, anyone who has watched the German market knows that investors react much more intensely to bad news than to good news. Many existing papers rely on standard GARCH models that overlook this "leverage effect," leading to risk assessments that are often too optimistic during times of crisis.

Furthermore, there is a recurring issue with how researchers treat the distribution of returns. A common shortcut is to assume that stock returns follow a "normal" bell curve. Our analysis of the DAX index shows this simply isn't true; the data is "heavy-tailed," meaning extreme market crashes happen far more often than basic models predict. While some studies have moved toward using Student-t distributions to fix this, they often ignore skewness, the fact that the "tails" of the distribution aren't even on both sides.

This study specifically addresses these oversights by combining the GJR-GARCH framework with a skewed-t distribution. By doing so, we bridge the gap between theoretical modeling and the actual, asymmetric behavior of the Frankfurt Stock Exchange over a massive 20-year window, from 2006 all the way to March 2026. This approach doesn't just add another paper to the pile; it provides a more rigorous and "honest" look at market risk that standard linear models frequently miss.

4.Research Methodology

The study is quantitative, exploratory, and applied in nature. Econometrics tools are utilized to capture price mobility. The study analyses conditional variance and provides an empirical estimation of volatility spillover.The study is based on daily return data to analyse price movements over time. There are total 5140 data points of Deutscher Aktienindex (DAX), German stock exchange ranging from January 02, 2006 to March 20, 2026. For better capture this behaviour, the GJR-GARCH model is used, as it estimates conditional variance and accounts for asymmetry in volatility, where negative shocks can affect volatility differently than positive shocks.

Significance of the Study

The study is relevant because it provides realistic understanding of how the volatility behaves in the financial markets. By applying GJR-GARCH model with skewed-t distribution. It captures the changing nature of volatility over time and shows that negative shocks have a stronger impact than positive shocks. The study offers more accurate insights into market risk and helps investors and policymakers for better decisions. It contributesto the existing literature by applying more advanced and reputable approach to analyse price volatility.

Limitations of the Study

Despite providing useful insights, the study has several limitations

i) Analysis is based on historical data, which may not fully captured future market behavior.
ii) Although GJR-GARCH model effective in capturing asymmetry, which may mot reflect real world complexities.
iii) Skewed-t distribution improves modelling of non-normality but it may not perfect for extreme market conditions or sudden shocks in future.
iv) Daily data may miss short-term fluctuations and important economic conditions.

5.Empirical analysis, estimation, and results

For better understanding price movement comprehensively it is essential to look at the price movement graph to visually understand the volatility.

The below is the graphical representation of the DAX index price over a period of approximately two decades. The graph of closing stock prices shows clear fluctuations over time, which indicates that the prices are not stable and tend to move up and down very frequently due to some minor and major microeconomic circumstances. There are period where high volatility can be observed, followed by calmer period can also be observed in the given time frame, this pattern suggests the presence of volatility clustering.

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Figure 1. Daily Closing Price of Index

Following the analysis of daily closing price index, the return series is transformed and their distribution is examined by using histogram for better understanding the characteristics of return.

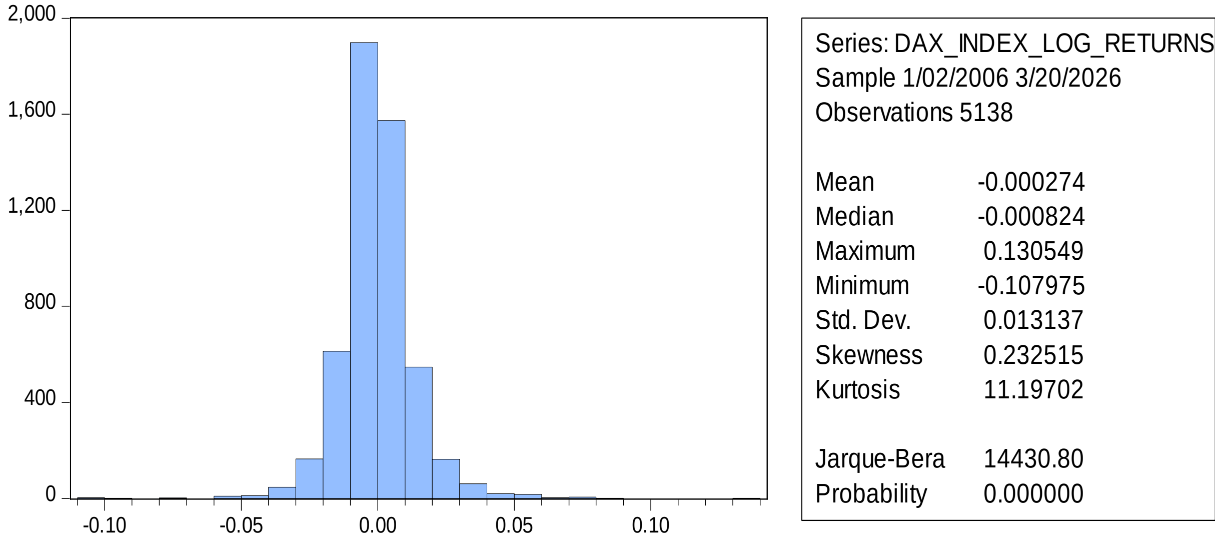

Figure 2. Test Distribution Analysis

The above histogram shows that most of the returns are concentrated approximately to zero that means most days the change in price is minimal. However, the extreme values indicating sudden increase or decrease in returns.

The average return is close to zero, which indicates that over time it gains and losses tend to balance out. Standard deviation indicates that there is volatility in returns that reflects market risk. Skewness is positive which suggests that there is slightly extreme positive returns compared with negative ones. Kurtosis value is high that shows the presence the of fat tails. Jarque-Bera statistics with probability value zero clearly indicates that the data is not normally distributed.

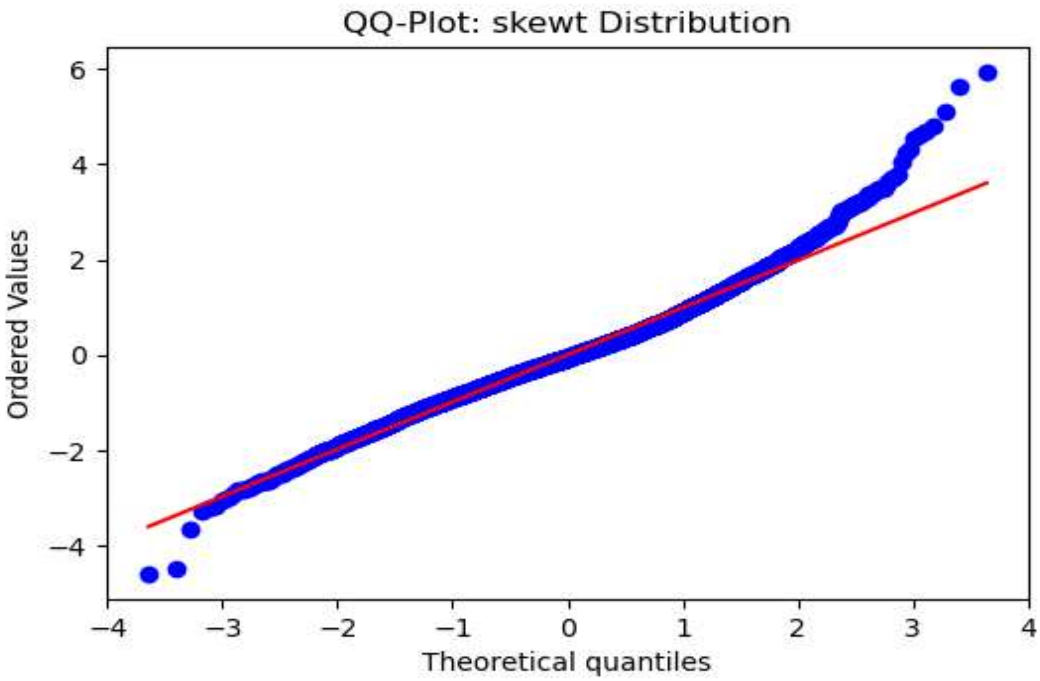

Figure 3. QQ-Plot: Skewt Distribution

The above graph shoes the data points does not follow the straight line perfectly, especially at the extreme ends. While at the middle portion of data is relatively close to the line, both the lower and upper tails is relatively close to the line. Strong deviation at the tails suggests the presence of fat tails, which indicated the extreme values occur more frequently than expected under normality. QQ plot clearly highlights non-normal behavior in return series.

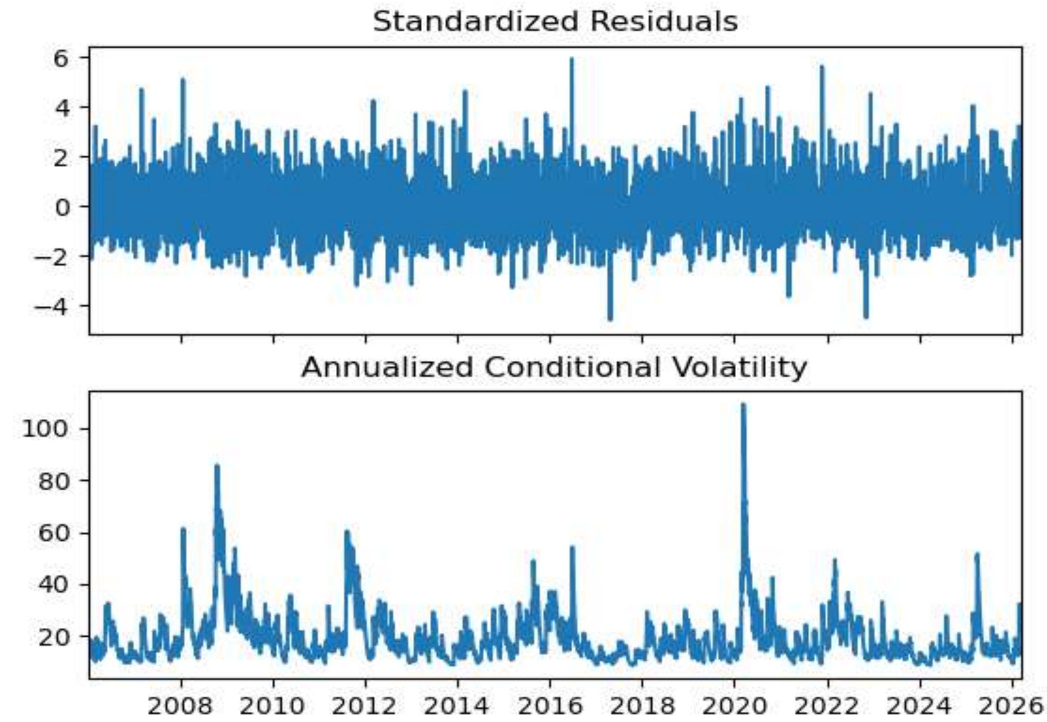

Figure 4. Standardized residual and annualized volatility

The above graph shows standardized residual fluctuate randomly around zero with any clear pattern, indicating the model has effectively captured the underlying structure of the data. The conditional volatility plot shows clear time-varying behavior, with the period of high volatility with low volatility. Sharp spikes highlight the impact of major market stocks and the persistence of volatility suggests these effects last over time. The result confirms that the model successfully captures the dynamic and changing nature of market volatility.

Metric	Value
ADF Statistic	-72.01479207
p-value	0
1% Crit	-3.431623617
5% Crit	-2.862102804

Table 1. Augmented Dickey-Fuller test

Augmented Dickey-Fuller test was employed to test the stationary,as p-value is less than 0.05 and t-statistics is less than 5% significant level, the data set is stationary. For better understanding of data some statistical parameters are discussed below.

Metric	Value
LM Stat	806.4316018

p-value	8.55E-167
F-Stat	95.4864097
F-p-value	8.51E-182

Table 2. ARCH LM Test

ARCH LM test is conducted to determine the presence of ARCH effect in the residual of the mean equation. Thisis important to confirm whether the data exhibits volatility clustering, LM test shows p-value of effectively zero which strongly states that there is no ARCH effect which justifies the subsequent application of GARCH models

Model	Distribution	AIC	BIC	Persistence	Log-Likelihood
GJR-GARCH	Skewd-t	15292.83601	15338.64695	1.0776	-7639.418007
GJR-GARCH	GED	15335.48645	15374.75296	1.0649	-7661.743224
GJR-GARCH	T	15337.44521	15376.71173	1.0745	-7662.722606
GARCH	Skewd-t	15472.00102	15511.26753	0.9873	-7730.00051
GARCH	GED	15478.82083	15511.54293	0.9822	-7734.410417
APARCH	Skewd-t	15469.66507	15515.47601	1	-7727.832536
APARCH	GED	15476.93765	15516.20416	0.9968	-7732.468825
EGARCH	Skewd-t	15479.55579	15518.8223	1.1951	-7733.777895
EGARCH	GED	15489.81744	15522.53954	1.1963	-7739.908721
GARCH	T	15501.5694	15534.2915	0.9882	-7745.784702
APARCH	T	15499.38904	15538.65556	1	-7743.694522
EGARCH	T	15510.04439	15542.76649	1.1999	-7750.022196
GJR-GARCH	Normal	15560.65627	15593.37837	1.0565	-7775.328136
GARCH	Normal	15744.06499	15770.24266	0.9767	-7868.032493
APARCH	Normal	15742.79705	15775.51915	0.9889	-7866.398527
EGARCH	Normal	15772.04484	15798.22251	1.1901	-7882.022419

Table3. Decision Table

The model involved in comparison of GARCH, GJR-GARCH, APARCH, and EGARCH specifications are paired with Skewed-t, GED, Student-t and normal error distributions.
By scrutinizing, GJR-GARCH model with Skewed-t distribution emerges as perfect model by achieving lowest AIC(15292.83), BIC(15338.65) and Log-Likelihood(7639.41).

Symbol	Coefficient	p-value
Mu (μ)	-0.03175557	0.018503087
Omega (ω)	0.033279434	8.39E-06
Alpha (α)	0.193263966	1.70E-12
Gamma (γ)	-0.193263966	1.49E-13
Beta (β)	0.884321154	0
Eta (η)	6.456141689	7.77E-30
Lambda (λ)	0.127746408	3.61E-13

Table 4. Parameter Estimates for the GJR-GARCH

The above table estimates the result for the GJR-GARCH model with skewed-t distribution provides insights into the volatility dynamics of series. The parameters are significant at 1% level, that recommend the robustness of the model. ARCH coefficient (α) and GARCH coefficient (β) suggests shocks to the system slowly over time. Asymmetry parameter (γ) is highly significant provides empirical evidences of leverage effect. These results demonstrate DAX index returns have high persistence, asymmetric, and heavy-tailed, skewed distribution.The estimation results for the GJR-GARCH model with a skewed-t distribution provide critical insights into the volatility dynamics of the DAX index, with all parameters showing statistical significance at the 1% level. The mean equation constant (μ) is -0.0317 with a p-value of 0.0185, indicating that the average daily log return is slightly negative. The constant variance (ω) of 0.0333 represents the baseline risk inherent in the market even in the absence of recent shocks. The ARCH coefficient (α) of 0.1932 confirms that recent market shocks have a significant immediate impact on current volatility, while the GARCH coefficient (β) of 0.8843 highlights high volatility persistence, meaning that once the market becomes turbulent, the risk remains elevated for a long period. A key finding is the highly significant asymmetry parameter (γ) of -0.1932, which provides strong empirical evidence of the leverage effect, where negative market shocks impact volatility more intensely than positive ones. Furthermore, the distributional parameters (η and λ) of 6.456 and 0.1277 respectively confirm that the DAX returns are characterized by a heavy-tailed, skewed distribution, justifying the use of a skewed-t framework over a standard normal one. These results collectively demonstrate that the DAX index is subject to extreme fluctuations and asymmetric risk, confirming the model's robustness in capturing complex market behaviors.

Lags	Lb stat	Lb p-value
10	12.17373	0.273598

Table 5. Ljung-Box Test for Standardized Residual Autocorrelation

The residual test is performed on standardized residual at 10 lags, test yield statistic of 12.17373 with p-value 0.273598. the p-value is greater than standard 0.05 threshold, it attempts to fail null hypothesis of autocorrelation. This suggests that GJR-GARCH model has successfully captured all patterns of volatility. Thus, it confirms that the model is specified and findings market risk and leverage effect are statistically reliable.

6.Conclusion

This quantitative and exploratory study analyzes the volatility dynamics of the DAX index using 5,140 daily data points from January 2, 2006, to March 20, 2026, to provide a realistic understanding of financial market behavior. By comparing multiple GARCH-family specifications, the research identifies the GJR-GARCH model with a skewed-t distribution as the superior framework for capturing the index's characteristic stationarity, heavy tails, and right-skewness. The empirical results demonstrate high volatility persistence and a significant leverage effect, where negative market shocks impact volatility more intensely than positive ones, a finding supported by significant ARCH (α), GARCH (β), and asymmetry (γ) parameters. Diagnostic validation via the Ljung-Box test confirms that the model successfully captures all patterns of volatility, making it a statistically reliable tool for risk assessment and strategic decision-making in the German equity market. Consequently, it is recommended that investors and policymakers move away from traditional normal distribution assumptions and utilize these asymmetric insights to better manage portfolio risk and time market interventions during downturns

This paper suggests that GJR-GARCH model with skewed-t distribution is the most suitable framework for capturing volatility of DAX index. Study successfully identified market returns characterized by significant stationarity, heavy-tailed, and right-skewness. Empirical results highlight leverage effect, where the market volatility reacts asymmetrically. Volatility is highly persistent that means shocks impart market over long period. And the model is statistically sound as observed in Ljung-Box test which confirms residuals. These insights provide investors and policymakers for risk assessment and decision-making in Germany equity market. Although the autocorrelation model is complex in itself but given the complex nature , there is remain scope of more detailed study through VaR, ML, Copula etc.

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