IDEALUL VAG, INTUITIONIST AL BF-ALGEBRA EVALUAT IN CADRUL UNUI INTERVAL

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Rezumat
Notiunea de multime vaga, intuitionista, evaluata intr-un interval, a fost introdusa pentru prima data de Atanassov si Gargov [3] in 1989 ca o generalizare atat a multimilor vagi evaluate intr-un interval dar si a celor intuitioniste. In [7] si [8] Satyanarayana si altii au aplicat conceptul de multimi vagi, intuitioniste, evaluate intr-un interval, pentru BF- subalgebra si idealuri vagi ale BF-algebra. In aceasta lucrare, noi introducem notiunea de idealuri duale, intuitioniste, vagi, evaluate intr-un interval si verificam anumite proprietati interesante.
Cuvinte cheie: BF-algebra, multimi vagi intuitioniste, idealuri vagi, intuitioniste i-v

1 Introducere si preliminarii
Notiunea de multime vaga, intuitionista, evaluata intr-un interval a fost introdusa, pentru prima data de Zadeh [10] ca o extindere la multimile vagi. Multimile vagi, evaluate intr-un interval sunt niste multimi vagi a caror functie de apartenenta este multi-evaluata si dintr-un interval pe o scara a apartenentei. Ideea ofera cea mai simpla metoda de a capta imprecizia procentului de apartenenta la o multime vaga. Pe de alta parte, Atanassov [1] a introdus notiunea de multimi vagi, intuitioniste, ca o extensie la multimile vagi pentru care nu numai un procent al parteneriatului este oferit, dar este implicat si un procent de non-apartenenta. Atanassove si Gargov [3] au introdus notiunea de multimi vagi evaluate intr-un

ON INTERVAL-VALUED INTUITIONISTIC FUZZY DUAL IDEALS OF BF-ALGEBRAS

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Abstract: The notion of interval-valued intuitionistic fuzzy sets was first introduced by Atanassov and Gargov [3] in 1989 as a generalization of both interval-valued fuzzy sets and intuitionistic fuzzy sets. In [7] and [8] Satyanarayana with others applied the concept of interval-valued intuitionistic fuzzy sets to BF-subalgebras and fuzzy ideals of BF-algebras. In this paper we introduce the notion of interval-valued intuitionistic fuzzy dual ideals of BF-algebras and investigate some interesting properties.

Keywords: BF-algebras, intuitionistic fuzzy sets, i-v intuitionistic fuzzy ideals.

1. Introduction and Preliminaries
The notion of interval-valued fuzzy sets was first introduced by Zadeh [10] as an extension of fuzzy sets. An interval-valued fuzzy sets is a fuzzy set whose membership function is many-valued and from an interval in the membership scale. This idea gives the simplest method to capture the imprecision of the membership grade for a fuzzy set. On the other hand, Atanassov [1] introduced the nation of intuitionistic fuzzy sets as an extension of fuzzy set which not only a membership degree is given, but also a non-membership degree is involved. Atanassove and Gargov [3] introduced the notion of interval-valued intuitionistic fuzzy sets which is a generalization of both intuitionistic fuzzy sets and interval-valued fuzzy sets. In [7] Satyanarayana with others applied the concept of interval-valued

interval care reprezinta o generalizare atat a multimilor vagi intuitioniste dar si a multimilor vagi evaluate intr-un interval. In [7] Satyanarayana si altii au aplicat conceptul de idealuri ale multimilor vagi intuitioniste, evaluate intr-un interval ale BF-algebra. In aceasta lucrare introducem notiunea de idealuri duale ale multimilor vagi evaluate intr-un interval pentru BF-algebra si verificam anumite proprietati interesante.

Prin BF-algebra, intelegem o algebra care respecta axiomele:

(1). $x * x = 0$,

(2). $x * 0 = x$,

(3). $0 * (x * y) = y * x$, pentru toate $x, y \in X$

De-a lungul acestei lucrari, X este o BF-algebra.

Daca exista un element 1 al X care respecta $x \le 1$, pentru toate $x \in X$, atunci elementul 1 este numit unitatea lui X. O BF-algebra cu unitate este numita delimitata. Intr-o algebra delimitata BF, demonstram $1 * x$ prin Nx pe scurt. O algebra-BF, X, este numita involuntar daca NNx=x, pentru toate $x \in X$.

Definitie 1.1 O submultime nevida D intr-o BF-algebra X este numita un ideal dual al X daca respecta:

(D_1) $1 \in D$,

(D_2) $N(Nx * Ny) \in D$ si $y \in D$ implica $x \in D$, pentru oricare $x, y \in X$..

Acum analizam cateva concept vagi, logice. O multime vida in X este o functie $\mu : X \to [0,1]$. Pentru multimile vide μ si λ a lui X si $s, t \in [0,1]$, multimile $U(\mu; t) = \{x \in X : \mu(x) \ge t\}$ se numesc bucati superioare la nivel t din μ si $L(\mu; t) = \{x \in X : \lambda(x) \le s\}$ se numeste bucata inferioara a λ.

Multimea vida μ in X se numeste sub-agebra duala, vaga a lui X, in cazul in care $\mu(N(Nx * Ny)) \ge \min\{\mu(x), \mu(y)\}$, pentru toate $x, y \in X$.

intuitionistic fuzzy ideals of BF-algebras. In this paper we introduce the notion of interval-valued intuitionistic fuzzy dual ideals of BF-algebras and investigate some interesting properties.

By a BF-algebra we mean an algebra satisfying the axioms:

(1). $x * x = 0$,

(2). $x * 0 = x$,

(3). $0 * (x * y) = y * x$, for all $x, y \in X$

Throughout this paper, X is a BF-algebra.

If there is an element 1 of X satisfying $x \le 1$, for all $x \in X$, then the element 1 is called unit of X. A BF-algebra with unit is called bounded. In a bounded BF-algebra, we denote $1 * x$ by Nx for brief. A bounded BF-algebra X called involutary if NNx=x, for all $x \in X$.

Definition 1.1 A nonempty subset D in a BF-algebra X is said to be a dual ideal of X if it satisfies:

(D_1) $1 \in D$,

(D_2) $N(Nx * Ny) \in D$ and $y \in D$ imply $x \in D$, for any $x, y \in X$..

We now review some fuzzy logic concepts. A fuzzy set in X is a function $\mu : X \to [0,1]$. For fuzzy sets μ and λ of X and $s, t \in [0,1]$, the sets $U(\mu; t) = \{x \in X : \mu(x) \ge t\}$ is called upper t-level cut of μ and $L(\mu; t) = \{x \in X : \lambda(x) \le s\}$ is called lower s-level cut of λ.

The fuzzy set μ in X is called a fuzzy dual sub algebra of X, if $\mu(N(Nx * Ny)) \ge \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

An intuitionistic fuzzy set (shortly IFS) in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively

O multime vida, intuitionista (numita pe scurt IFS) intr-o multime nevida X este un obiect care are forma $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$, unde functia $\mu_A : X \to [0,1]$ si $\lambda_A : X \to [0,1]$ denota apartenenta (si anume $\mu_A(x)$) si lipsa apartenentei (si anume $\lambda_A(x)$) pentru fiecare element $x \in X$ la multimea A si anume aceea ca $0 \le \mu_A(x) + \lambda_A(x) \le 1$ pentru toate $x \in X$. De dragul simplicitatii, utilizam simbolul

$A = (X, \mu_A, \lambda_A)$ sau $A = (\mu_A, \lambda_A)$.

Prin numar de interval D la $[0,1]$, mentionam un interval $[a^-, a^+]$, unde $0 \le a^- \le a^+ \le 1$. Multimea tuturor subintervalelor inchise ale $[0,1]$ este denumita prin $D[0,1]$

Pentru numere ale intervalului $D_1 = [a_1^-, b_1^+]$, $D_2 = [a_2^-, b_2^+]$.

Definim

- $D_1 \cap D_2 = \min(D_1, D_2) = \min([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$
- $D_1 \cup D_2 = \max(D_1, D_2) = \max([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$

$$D_1 + D_2 = [a_1^- + a_2^- - a_1^-.a_2^-, b_1^+ + b_2^+ - b_1^+.b_2^+]$$

si punem

- $D_1 \le D_2 \Leftrightarrow a_1^- \le a_2^-$ si $b_1^+ \le b_2^+$
- $D_1 = D_2 \Leftrightarrow a_1^- = a_2^-$ si $b_1^+ = b_2^+$,
- $D_1 < D_2 \Leftrightarrow D_1 \le D_2$ si $D_1 \ne D_2$
- $mD = m[a_1^-, b_1^+] = [ma_1^-, mb_1^+]$, unde $0 \le m \le 1$.

In mod evident $(D[0,1], \le, \vee, \wedge)$ formeaza o lattice completa cu $[0,0]$ ca cel

such that $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in X$. For the sake of simplicity we use the symbol form $A = (X, \mu_A, \lambda_A)$ or $A = (\mu_A, \lambda_A)$.

By interval number D on $[0,1]$, we mean an interval $[a^-, a^+]$, where $0 \le a^- \le a^+ \le 1$. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$

For interval numbers $D_1 = [a_1^-, b_1^+]$, $D_2 = [a_2^-, b_2^+]$. We define

- $D_1 \cap D_2 = \min(D_1, D_2) = \min([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$
- $D_1 \cup D_2 = \max(D_1, D_2) = \max([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$

$$D_1 + D_2 = [a_1^- + a_2^- - a_1^-.a_2^-, b_1^+ + b_2^+ - b_1^+.b_2^+]$$

and put

- $D_1 \le D_2 \Leftrightarrow a_1^- \le a_2^-$ and $b_1^+ \le b_2^+$
- $D_1 = D_2 \Leftrightarrow a_1^- = a_2^-$ and $b_1^+ = b_2^+$,
- $D_1 < D_2 \Leftrightarrow D_1 \le D_2$ and $D_1 \ne D_2$
- $mD = m[a_1^-, b_1^+] = [ma_1^-, mb_1^+]$, where $0 \le m \le 1$.

Obviously $(D[0,1], \le, \vee, \wedge)$ form a complete lattice with $[0,0]$ as its least element and $[1,1]$ as its greatest element.

Let L be a given nonempty set. An interval-valued fuzzy set (briefly, i-v fuzzy set) B on L is defined by $B = \{(x, [\mu_B^-(x), \mu_B^+(x)]) : x \in L\}$, where $\mu_B^-(x)$ and $\mu_B^+(x)$ are fuzzy sets of L such that $\mu_B^-(x) \le \mu_B^+(x)$ for all $x \in L$. Let $\tilde{\mu}_B(x) = [\mu_b^-(x), \mu_B^+(x)]$, then mai mic element al sau si [1,1] ca cel mai mare element al sau.

Fie L o multime nevida. O multime vaga, evaluata intr-un interval (briefly, i-v fuzzy set) B la L este definite ca $B = \{(x, [\mu_B^-(x), \mu_B^+(x)]) : x \in L\}$, unde $\mu_B^-(x)$ si $\mu_B^+(x)$ sunt multimi vagi ale L precum $\mu_B^-(x) \le \mu_B^+(x)$ pentru toate $x \in L$. Fie $\tilde{\mu}_B(x) = [\mu_b^-(x), \mu_B^+(x)]$, atunci $B = \{(x, \tilde{\mu}_B(x)) : x \in L\}$ unde $\tilde{\mu}_B : L \to D[0,1]$

O reprezentare $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ $: L \to D[0,1] \times D[0,1]$ se numeste multime vida, intuitionista, evaluata intr-un interval (i-v IF multime, pe scurt) la L daca $0 \le \mu_A^+(x) + \lambda_A^+(x) \le 1$ si $0 \le \mu_A^-(x) + \lambda_A^-(x) \le 1$ pentru toate $x \in L$ (si anume, $A^+ = (X, \mu_A^+, \lambda_A^+)$ si $A^- = (X, \mu_A^-, \lambda_A^-)$ sunt multimi vagi, intuitioniste), acolo unde reprezentarea $\tilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)] : L \to D[0,1]$ si $\tilde{\lambda}_A(x) = [\lambda_A^-(x), \lambda_A^+(x)] :$ $L \to D[0,1]$ denota gradul de apartenenta (si anume $\tilde{\mu}_A(x)$) si non-apartenenta (si anume $\tilde{\lambda}_A(x)$) pentru fiecare element $x \in L$ la A.

2 IDEAL DUAL INTUITIONIST VAG, EVALUAT INTR-UN INTERVAL AL ALGEBREI BF

Definitie 2.1: Un IFS $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ evaluat in cadrul unui interval este numit ideal dual inuitionist, vag, evaluat intr-un interval (pe scurt ideal dual i-v IF) al algebrei BF- X in cazul in care respecta urmatoatele diferente

(i-v IF1) $\tilde{\mu}_A(1) \ge \tilde{\mu}_A(x)$ si $\tilde{\lambda}_A(1) \le \tilde{\lambda}_A(x)$

(i-v IF2)

$B = \{(x, \tilde{\mu}_B(x)) : x \in L\}$ where $\tilde{\mu}_B : L \to D[0,1]$

A mapping $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ $: L \to D[0,1] \times D[0,1]$ is called an interval-valued intuitionistic fuzzy set (i-v IF set, in short) in L if $0 \le \mu_A^+(x) + \lambda_A^+(x) \le 1$ and $0 \le \mu_A^-(x) + \lambda_A^-(x) \le 1$ for all $x \in L$ (that is, $A^+ = (X, \mu_A^+, \lambda_A^+)$ and $A^- = (X, \mu_A^-, \lambda_A^-)$ are intuitionistic fuzzy sets), where the mappings $\tilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)] : L \to D[0,1]$ and $\tilde{\lambda}_A(x) = [\lambda_A^-(x), \lambda_A^+(x)] :$ $L \to D[0,1]$ denote the degree of membership (namely $\tilde{\mu}_A(x)$) and degree of non-membership (namely $\tilde{\lambda}_A(x)$) of each element $x \in L$ to A respectively.

2. INTERVAL-VALUED INTUITIONISTIC FUZZY DUAL IDEAL OF BF-ALGEBRAS

Definition 2.1: An interval-valued IFS $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is called interval-valued intuitionistic fuzzy dual ideal (shortly i-v IF dual ideal) of BF-algebra X if satisfies the following inequality

(i-v IF1) $\tilde{\mu}_A(1) \ge \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(1) \le \tilde{\lambda}_A(x)$

(i-v IF2) $\tilde{\mu}_A(x) \ge \min\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

(i-v IF2) $\tilde{\lambda}_A(x) \le \max\{\tilde{\lambda}_A(N(Nx * Ny)), \tilde{\lambda}_A(y)\}$, for all $x, y, z \in X$.

Example 2.2 Consider a BF-algebra $X = \{0, 1, 2, 3\}$ with following table

$\tilde{\mu}_A(x) \geq \min\{\tilde{\mu}_A(N(Nx*Ny)), \tilde{\mu}_A(y)\}$

(i-v IF2)

$\tilde{\lambda}_A(x) \leq \max\{\tilde{\lambda}_A(N(Nx*Ny)), \tilde{\lambda}_A(y)\}$,

pentru toate $x, y, z \in X$.

Exemplu 2.2 Consideram o BF-algebra $X = \{0, 1, 2, 3\}$ cu urmatorul tabel

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Fie A o multime vaga evaluate in cadrul unui interval prin $\tilde{\mu}_A(0) = \tilde{\mu}_A(1) = [0.6, 0.7]$ si $\tilde{\mu}_A(2) = \tilde{\mu}_A(3) = [0.2, 0.3]$, $\tilde{\lambda}_A(0) = \tilde{\lambda}_A(1) = [0.1, 0.2]$, $\tilde{\lambda}_A(2) = \tilde{\lambda}_A(3) = [0.5, 0.7]$. Este usor de verificat daca A este un ideal dual inuitionist, vag, evaluat intr-un interval al lui X.

Teorema 2.3: Fie $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ un IFS i-v, atunci A este un i-v IF ideal dual al $X \Leftrightarrow A^- = (X, \mu_A^-, \lambda_A^-)$ si $A^+ = (X, \mu_A^+, \lambda_A^+)$ este un ideal dual IF al X.

Demonstratie: Sa presupunem ca $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ este un ideal dual i-v IF al lui X.

Din moment ce

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let A be an interval valued fuzzy set in X defined by $\tilde{\mu}_A(0) = \tilde{\mu}_A(1) = [0.6, 0.7]$ and $\tilde{\mu}_A(2) = \tilde{\mu}_A(3) = [0.2, 0.3]$, $\tilde{\lambda}_A(0) = \tilde{\lambda}_A(1) = [0.1, 0.2]$, $\tilde{\lambda}_A(2) = \tilde{\lambda}_A(3) = [0.5, 0.7]$. It is easy to verify that A is an interval valued intuitionistic fuzzy dual ideal of X.

Theorem 2.3: Let $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ be an i-v IFS then A is an i-v IF dual ideal of $X \Leftrightarrow A^- = (X, \mu_A^-, \lambda_A^-)$ and $A^+ = (X, \mu_A^+, \lambda_A^+)$ are IF dual ideal of X.

Proof: Assume $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v IF dual ideal of X.

Since $\tilde{\mu}_A(1) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(1) \leq \tilde{\lambda}_A(x)$, for all $x \in X$

$\Leftrightarrow$ $[\mu_A^-(1), \mu_A^+(1)] \geq [\mu_A^-(x), \mu_A^+(x)]$ and $[\lambda_A^-(1), \lambda_A^+(1)] \leq [\lambda_A^-(x), \lambda_A^+(x)]$

$\Leftrightarrow \mu_A^-(1) \geq \mu_A^-(x),\ \lambda_A^-(1) \leq \lambda_A^-(x)$ and $\mu_A^+(1) \geq \mu_A^+(x),\ \lambda_A^+(1) \leq \lambda_A^+(x)$, for all $x \in X$

Since

$\tilde{\mu}_A(x) \geq \min\{\tilde{\mu}_A(N(Nx*Ny)), \tilde{\mu}_A(y)\}$ pentru toate $x \in X$

$$= \min\left\{\left[\mu_A^-(N(Nx*Ny)), \mu_A^+(N(Nx*Ny))\right], \left[\mu_A^-(y), \mu_A^+(y)\right]\right\}$$

$$= \left[\min\{\mu_A^-(N(Nx*Ny)), \mu_A^-(y)\}, \min\{\mu_A^+(N(Nx*Ny)), \mu_A^+(y)\}\right]$$

$$\Leftrightarrow [\mu_A^-(x), \mu_A^+(x)] \geq \left[\min\{\mu_A^-(N(Nx*Ny)), \mu_A^-(y)\}, \min\{\mu_A^+(N(Nx*Ny)), \mu_A^+(y)\}\right]$$

$$\Leftrightarrow \mu_A^-(x) \geq \min\{\mu_A^-(N(Nx*Ny)), \mu_A^-(y)\} \text{ and } \mu_A^+(x) \geq \min\{\mu_A^+(N(Nx*Ny)), \mu_A^+(y)\},$$

pentru toate $x \in X$

Din moment ce

$\tilde{\lambda}_A(x) \leq \max\{\tilde{\lambda}_A(N(Nx*Ny)), \tilde{\lambda}_A(y)\}$

$$= \max\left\{\left[\lambda_A^-(N(Nx*Ny)), \lambda_A^+(N(Nx*Ny))\right], \left[\lambda_A^-(y), \lambda_A^+(y)\right]\right\}$$

for all $x, y \in X$.

Similarly

$$= \left[\max\{\lambda_A^-(N(Nx * Ny)), \lambda_A^-(y)\}, \max\{\lambda_A^+(N(Nx * Ny)), \lambda_A^-(y)\}\right],$$

pentru toate $x, y \in X$.

In mod similar

$\tilde{\lambda}_A(x) \le \max\{\tilde{\lambda}_A(N(Nx * Ny)), \tilde{\lambda}_A(y)\}$

$= \max\{[\lambda_A^-(N(Nx * Ny)), \lambda_A^+(N(Nx * Ny))], [\lambda_A^-(y), \lambda_A^+(y)]\}$

$= [\max\{\lambda_A^-(N(Nx * Ny)), \lambda_A^-(y)\}, \max\{\lambda_A^+(N(Nx * Ny)), \lambda_A^-(y)\}]$

, pentru toate $x \in X$

Astfel,

$[\lambda_A^-(x), \lambda_A^+(x)] \le [\max\{\lambda_A^-(N(Nx * Ny)), \lambda_A^-(y)\}, \max\{\lambda_A^+(N(Nx * Ny)), \lambda_A^-(y)\}]$

$\Rightarrow \lambda_A^-(x) \le \max\{\lambda_A^-(N(Nx * Ny)), \lambda_A^-(y)\}$

and $\lambda_A^+(x) \le \max\{\lambda_A^+(N(Nx * Ny)), \lambda_A^-(y)\}$,

pentru toate $x, y \in X$.

Astfel $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ este un ideal dual i-v IF $\Leftrightarrow A^- = (X, \mu_A^-, \lambda_A^-)$ si $A^+ = (X, \mu_A^+, \lambda_A^+)$ este un ideal dual IF.

Teorema 2.4: Let $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ este o multime i-v IF a lui X.

Atunci $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ este un ideal dual i-v IF al lui X daca si numai daca $\Box A = (X, (\tilde{\lambda}_A)^c, \tilde{\lambda}_A)$ este un ideal dual i-v IF al lui X.

$\Leftrightarrow \Box A^- = (X, (\mu_A^-)^c, \lambda_A^-)$ si $\Box A^+ = (X, (\lambda_A^+)^c, \lambda_A^+)$ sunt idealuri duale IF ale lui X.

Teorema 2.5: Fie $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ o multime i-v IF a lui X. Atunci $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ este un ideal dual i-v IF al lui X. $\Leftrightarrow \Diamond A = (X, \tilde{\mu}_A, (\tilde{\mu}_A)^c)$ este un ideal dual i-v IF al lui X

$\Leftrightarrow \Diamond A^- = (X, \mu_A^-, \mu_A^-)$ si $\Diamond A^+ = (X, \mu_A^+, (\mu_A^+)^c)$ sunt idealuri duale IF ale lui X.

Teorema 2.6: Fie A_1 si A_2 sunt idealuri duale i-v IF ale lui X. Atunci $A_1 \cap A_2$ este un ideal dual i-v IF- al lui X.

In consecinta 2.7: Fie $\{A_i \setminus i \in A\}$ o

for all $x \in X$

Therefore,

$$[\lambda_A^-(x), \lambda_A^+(x)] \le \begin{bmatrix} \max\{\lambda_A^-(N(Nx * Ny)), \lambda_A^-(y)\}, \\ \max\{\lambda_A^+(N(Nx * Ny)), \lambda_A^-(y)\} \end{bmatrix}$$

$\Rightarrow \lambda_A^-(x) \le \max\{\lambda_A^-(N(Nx * Ny)), \lambda_A^-(y)\}$

and $\lambda_A^+(x) \le \max\{\lambda_A^+(N(Nx * Ny)), \lambda_A^-(y)\}$,

for all $x, y \in X$. Hence $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v IF dual ideal $\Leftrightarrow A^- = (X, \mu_A^-, \lambda_A^-)$ and $A^+ = (X, \mu_A^+, \lambda_A^+)$ are IF dual ideal.

Theorem 2.4: Let $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ be an i-v IF set of X. Then $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v IF dual ideal of X if and only if $\Box A = (X, (\tilde{\lambda}_A)^c, \tilde{\lambda}_A)$ is an i-v IF dual ideal of X.

$\Leftrightarrow \Box A^- = (X, (\mu_A^-)^c, \lambda_A^-)$ and $\Box A^+ = (X, (\lambda_A^+)^c, \lambda_A^+)$ are IF dual ideals of X.

Theorem 2.5: Let $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ be an i-v IF set of X. Then $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v IF dual ideal of X. $\Leftrightarrow \Diamond A = (X, \tilde{\mu}_A, (\tilde{\mu}_A)^c)$ is an i-v IF dual ideal of X

$\Leftrightarrow \Diamond A^- = (X, \mu_A^-, \mu_A^-)$ and $\Diamond A^+ = (X, \mu_A^+, (\mu_A^+)^c)$ are IF dual ideals of X.

Theorem 2.6: Let A_1 and A_2 are i-v IF dual ideals of X. Then $A_1 \cap A_2$ is also an i-v IF-dual ideal of X.

Corollary 2.7: Let $\{A_i \setminus i \in A\}$ be a family of i-v IF dual ideal of X. Then $\bigcap_{i \in I} A_i$ is also an i-v IF dual ideal of X.

Theorem 2.8: If $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v IF-dual ideal of X, then the sets

$X_{\tilde{\mu}_A} = \{x \in X / \tilde{\mu}_A(x) = \tilde{\mu}_A(1)\}$ and familie a idealului dual i-v IF al lui X. Atunci $\bigcap_{i\in I} A_i$ este un ideal dual i-v IF al lui X.

Teorema 2.8: Daca $A=(X,\tilde{\mu}_A,\tilde{\lambda}_A)$ este un ideal dual i-v IF- a lui X, atunci multimile $X_{\tilde{\mu}_A}=\{x\in X/\tilde{\mu}_A(x)=\tilde{\mu}_A(1)\}$ si $X_{\tilde{\lambda}_A}=\{x\in X/\tilde{\lambda}_A(x)=\tilde{\lambda}_A(1)\}$ sunt idealuri duale ale lui X.

Dovada: Din moment ce $\tilde{\mu}_A(1)=\tilde{\mu}_A(1)$ and $\tilde{\lambda}_A(1)=\tilde{\lambda}_A(1)$ $\Rightarrow 1\in X_{\tilde{\mu}_A}$ and $1\in X_{\tilde{\mu}_A}$

In cazul in care $N(Nx*Ny),y\in X_{\tilde{\mu}_A}$ $\Rightarrow$ $\tilde{\mu}_A(N(Nx*Ny))=\tilde{\mu}_A(1)$, $\tilde{\mu}_A(y)=\tilde{\mu}_A(1)$ si astfel

$\tilde{\mu}_A(x)\ge\min\{\tilde{\mu}_A(N(Nx*Ny)),\tilde{\mu}_A(y)\}=\min\{\tilde{\mu}_A(1),\tilde{\mu}_A(1)\}=\tilde{\mu}_A(1)$

implica $\tilde{\mu}_A(x)\ge\tilde{\mu}_A(1)$ dar $\tilde{\mu}_A(x)\le\tilde{\mu}_A(1)$ implica $\tilde{\mu}_A(x)=\tilde{\mu}_A(1)$ implica $x\in X_{\tilde{\mu}_A}$ si anume

$N(Nx*Ny),y\in X_{\tilde{\mu}_A}\Rightarrow x\in X_{\tilde{\mu}_A}$.

Astfel, $X_{\tilde{\mu}_A}$ este un ideal dual al lui X. In mod asemanator $X_{\tilde{\lambda}_A}$ este un ideal dual a lui X.

Definitie 2.9: Fie $A=(X,\tilde{\mu}_A,\tilde{\lambda}_A)$ un i-v IFS in X. Pentru $[s_1, s_2],[t_1, t_2]\in D[0, 1]$ atunci $\bigcup(\tilde{\mu}_A;[s_1,s_2])=\{x\in X/\tilde{\mu}_A(x)\ge[s_1,s_2]\}$ se numeste nivel superior i-v al lui $\tilde{\mu}_A$ si set $L(\tilde{\lambda}_A;[t_1,t_2])=\{x\in X/\tilde{\lambda}_A(x)\le[t_1,t_2]\}$ se numeste parte inferioara i-v nivel $\tilde{\lambda}_A$.

Nota: (i) $\bigcup(\tilde{\mu}_A;[s_1,s_2])=\bigcup(\mu_A^-;s_1)\bigcap\bigcup(\mu_A^+;s_2)$

(ii) $L(\tilde{\lambda}_A;[t_1,t_2])=L(\lambda_A^-;t_1)\bigcap L(\lambda_A^+;t_2)$.

Teorema 2.10: Fie $A=(X,\tilde{\mu}_A,\tilde{\lambda}_A)$ atunci i-

$X_{\tilde{\lambda}_A}=\{x\in X/\tilde{\lambda}_A(x)=\tilde{\lambda}_A(1)\}$ are dual ideals of X.

Proof: Since $\tilde{\mu}_A(1)=\tilde{\mu}_A(1)$ and $\tilde{\lambda}_A(1)=\tilde{\lambda}_A(1)$ $\Rightarrow 1\in X_{\tilde{\mu}_A}$ and $1\in X_{\tilde{\mu}_A}$

If $N(Nx*Ny),y\in X_{\tilde{\mu}_A}$ $\Rightarrow$ $\tilde{\mu}_A(N(Nx*Ny))=\tilde{\mu}_A(1)$, $\tilde{\mu}_A(y)=\tilde{\mu}_A(1)$ and so

$\tilde{\mu}_A(x)\ge\min\{\tilde{\mu}_A(N(Nx*Ny)),\tilde{\mu}_A(y)\}=$

$=\min\{\tilde{\mu}_A(1),\tilde{\mu}_A(1)\}=\tilde{\mu}_A(1)$

implies $\tilde{\mu}_A(x)\ge\tilde{\mu}_A(1)$ but $\tilde{\mu}_A(x)\le\tilde{\mu}_A(1)$ implies $\tilde{\mu}_A(x)=\tilde{\mu}_A(1)$ implies $x\in X_{\tilde{\mu}_A}$ that is $N(Nx*Ny),y\in X_{\tilde{\mu}_A}\Rightarrow x\in X_{\tilde{\mu}_A}$.

Therefore, $X_{\tilde{\mu}_A}$ is an dual ideal of X. Similarly $X_{\tilde{\lambda}_A}$ is an dual ideal of X.

Definition 2.9: Let $A=(X,\tilde{\mu}_A,\tilde{\lambda}_A)$ be an i-v IFS in X. For $[s_1, s_2],[t_1, t_2]\in D[0, 1]$ then the set $\bigcup(\tilde{\mu}_A;[s_1,s_2])=\{x\in X/\tilde{\mu}_A(x)\ge[s_1,s_2]\}$ is called i-v upper level cut of $\tilde{\mu}_A$ and set $L(\tilde{\lambda}_A;[t_1,t_2])=\{x\in X/\tilde{\lambda}_A(x)\le[t_1,t_2]\}$ is called i-v lower level cut of $\tilde{\lambda}_A$.

Note:

(i) $\bigcup(\tilde{\mu}_A;[s_1,s_2])=\bigcup(\mu_A^-;s_1)\bigcap\bigcup(\mu_A^+;s_2)$

(ii) $L(\tilde{\lambda}_A;[t_1,t_2])=L(\lambda_A^-;t_1)\bigcap L(\lambda_A^+;t_2)$.

Theorem 2.10: Let $A=(X,\tilde{\mu}_A,\tilde{\lambda}_A)$ be an i-v IF set of X, Then the following conditions are equivalent.

(v) A is an i-v IF dual ideal of X.

(vi) $A^-=(X,\mu_A^-,\lambda_A^-)$ and $A^+=(X,\mu_A^+,\lambda_A^+)$ are IF dual ideals of X.

(vii) The non-empty sets

v IF o multime a lui X, atunci urmatoarele conditii sunt echivalente.

(i) A este un ideal dual i-v IF al lui X.

(ii) $A^- = (X, \mu_A^-, \lambda_A^-)$ si $A^+ = (X, \mu_A^+, \lambda_A^+)$ sunt idealuri duale IF ale lui X.

(iii) Multimile nevide $\bigcup(\mu_A^-; s_1), L(\lambda_A^-; t_1)$ si $\bigcup(\mu_A^+; s_2), L(\lambda_A^+; t_2)$

(iv) Multimile nevide $\bigcup(\tilde{\mu}_A; [s_1, s_2])$ si $L(\tilde{\lambda}_A; [t_1, t_2])$ sunt idealuri duale i-v.

Teorema 2.11: Daca $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ este un ideal dual i-v IF al lui X, atunci submultimile seturilor $\bigcup(\tilde{\mu}_A; \tilde{s})$ si $L(\tilde{\lambda}_A; \tilde{s})$ sunt idealuri duale i-v ale lui X pentri fiecare $\tilde{s} \in Im(\tilde{\mu}_A) \cap Im(\tilde{\lambda}_A) \subseteq D[0,1]$, unde $Im(\tilde{\mu}_A)$ si $Im(\tilde{\lambda}_A)$ sunt multimi de valori ale $\tilde{\mu}_A$ si respectiv $\tilde{\lambda}_A$.

Teorema 2.12: Daca pentru toate multimile nevide i-v $\bigcup(\tilde{\mu}_A; \tilde{s})$ si $L(\tilde{\lambda}_A; \tilde{s})$ ale multimii vagi, intuitioniste i-v $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ sunt idealuri duale ale lui X, atunci A este i-v un ideal dual, vag, intuitionist al lui X.

Definitie 2.13: Fie $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ o multime vaga, intuitionista, estimate in cadrul unui interval X si $\tilde{s}, \tilde{t} \in D[0,1]$ astfel incat $\tilde{s} + \tilde{t} \le [1,1]$. Atunci multimea $X_A^{\tilde{s}, \tilde{t}} = \{x \in X \setminus \tilde{s} \le \tilde{\mu}_A(x), \tilde{\lambda}_A(x) \le \tilde{t}\}$ se numeste submultime $(\tilde{s}, \tilde{t})$ la nivel A. Multimea tuturor $(\tilde{s}, \tilde{t}) \in Im(\tilde{\mu}_A) \times Im(\tilde{\lambda}_A)$ cum ar fi $\tilde{s} + \tilde{t} \le [1,1]$ se numeste imaginea lui $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$. In mod

$\bigcup(\mu_A^-; s_1), L(\lambda_A^-; t_1)$ and $\bigcup(\mu_A^+; s_2), L(\lambda_A^+; t_2)$

(viii) The non-empty sets $\bigcup(\tilde{\mu}_A; [s_1, s_2])$ and $L(\tilde{\lambda}_A; [t_1, t_2])$ are i-v dual ideals..

Proposition 2.11: If $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is i-v IF dual ideal of X, then the level subsets $\bigcup(\tilde{\mu}_A; \tilde{s})$ and $L(\tilde{\lambda}_A; \tilde{s})$ are i-v dual ideals of X for every $\tilde{s} \in Im(\tilde{\mu}_A) \cap Im(\tilde{\lambda}_A) \subseteq D[0,1]$, where $Im(\tilde{\mu}_A)$ and $Im(\tilde{\lambda}_A)$ are sets of values of $\tilde{\mu}_A$ and $\tilde{\lambda}_A$, respectively.

Proposition 2.12: If for all non-empty i-v level subsets $\bigcup(\tilde{\mu}_A; \tilde{s})$ and $L(\tilde{\lambda}_A; \tilde{s})$ of an i-v intuitionistic fuzzy set $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ are dual ideals of X, then A is i-v intuitionistic fuzzy dual ideal of X.

Definition 2.13: Let $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an interval-valued intuitionistic fuzzy set on X and $\tilde{s}, \tilde{t} \in D[0,1]$ such that $\tilde{s} + \tilde{t} \le [1,1]$. Then the set $X_A^{\tilde{s}, \tilde{t}} = \{x \in X \setminus \tilde{s} \le \tilde{\mu}_A(x), \tilde{\lambda}_A(x) \le \tilde{t}\}$ is called an $(\tilde{s}, \tilde{t})$ level subset of A. The set of all $(\tilde{s}, \tilde{t}) \in Im(\tilde{\mu}_A) \times Im(\tilde{\lambda}_A)$ such that $\tilde{s} + \tilde{t} \le [1,1]$ is called the image of $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$. Obviously, $X_A^{(\tilde{s}, \tilde{t})} = \cup(\tilde{\mu}_A, \tilde{s}) \cap L(\tilde{\lambda}_A, \tilde{t})$.

Theorem 2.14: An interval-valued intuitionistic fuzzy set $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ of X is an i-v intuitionistic fuzzy dual ideal of X if and only if $X_A^{(\tilde{s}, \tilde{t})}$ is an dual ideal of X, for every $(\tilde{s}, \tilde{t}) \in Im(\tilde{\mu}_A) \times Im(\tilde{\lambda}_A)$ with $\tilde{s} + \tilde{t} \le [1,1]$.

Proof: We only need to prove the necessity, because the sufficiency is trivial. Assume evident, $X_A^{(\tilde{s},\tilde{t})} = \cup(\tilde{\mu}_A, \tilde{s}) \cap L(\tilde{\lambda}_A, \tilde{t})$.

Teorema 2.14: O multime vaga, intuitionista, evaluate in cadrul unui interval $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ al lui X este un ideal dual al unei multimi vagi, intuitioniste i-v a lui X daca si numai daca $X_A^{(\tilde{s},\tilde{t})}$ este un ideal dual a lui X, pentru fiecare $(\tilde{s},\tilde{t}) \in Im(\tilde{\mu}_A) \times Im(\tilde{\lambda}_A)$ cu $\tilde{s} + \tilde{t} \le [1,1]$.

Demonstratie: Trebuie sa demonstram doar nevoia, deoarece capacitatea este neinsemnata. Sa presupunem ca $X_A^{(\tilde{s},\tilde{t})}$ este un ideal dual al lui X si $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ o multime vafa, intuitionista, evaluate in cadrul unui interval pentru X. Este usor sa vedem daca $\tilde{\mu}_A(1) \ge \tilde{\mu}_A(x)$ si $\tilde{\lambda}_A(1) \le \tilde{\lambda}_A(x)$. Luand in calcul $N(Nx * Ny), y \in X$ cum ar fi $A(N(Nx * Ny)) = (\tilde{s},\tilde{t})$ si $A(y) = (\tilde{s}_1,\tilde{t}_1)$ i.e $\tilde{\mu}_A(N(Nx * Ny)) = \tilde{s}$, $\tilde{\lambda}_A(N(Nx * Ny)) = \tilde{t}$, $\tilde{\mu}_A(y) = \tilde{s}_1$ si $\tilde{\lambda}_A(y) = \tilde{t}_1$. Fara a pierde generalitatile, putem presupune ca $(\tilde{s},\tilde{t}) \le (\tilde{s}_1,\tilde{t}_1)$, i.e $\tilde{s} \le \tilde{s}_1$ si $\tilde{t}_1 \le \tilde{t}$. Atunci $X_A^{(\tilde{s}_1,\tilde{t}_1)} \subseteq X_A^{(\tilde{s}_1,\tilde{t}_1)}$ inseamna $x, y \in X_A^{(\tilde{s},\tilde{t})}$. Acest lucru inseamna ca $N(Nx * Ny), y \in X_A^{(\tilde{s},\tilde{t})}$ din moment ce $X_A^{(\tilde{s},\tilde{t})}$ este un ideal dual al lui X. Astfel, deduce ca

$\tilde{\mu}_A(x) \ge \tilde{s} = \min\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

si

$\tilde{\lambda}_A(x) \le \tilde{t} = \max\{\tilde{\lambda}_A(N(Nx * Ny)), \tilde{\lambda}_A(y)\}$.

Acest lucru demonstreaza ca $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ este o multime vaga, duala, intuitionista i-v BF a lui X.

that $X_A^{(\tilde{s},\tilde{t})}$ is an dual ideal of X and $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ an interval-valued intuitionistic fuzzy set on X. It is easy to see that $\tilde{\mu}_A(1) \ge \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(1) \le \tilde{\lambda}_A(x)$. Consider $N(Nx * Ny), y \in X$ such that $A(N(Nx * Ny)) = (\tilde{s},\tilde{t})$ and $A(y) = (\tilde{s}_1,\tilde{t}_1)$ i.e $\tilde{\mu}_A(N(Nx * Ny)) = \tilde{s}$, $\tilde{\lambda}_A(N(Nx * Ny)) = \tilde{t}$, $\tilde{\mu}_A(y) = \tilde{s}_1$ and $\tilde{\lambda}_A(y) = \tilde{t}_1$. Without loss of generality, we may assume that $(\tilde{s},\tilde{t}) \le (\tilde{s}_1,\tilde{t}_1)$, i.e $\tilde{s} \le \tilde{s}_1$ and $\tilde{t}_1 \le \tilde{t}$. Then $X_A^{(\tilde{s}_1,\tilde{t}_1)} \subseteq X_A^{(\tilde{s}_1,\tilde{t}_1)}$ implies $x, y \in X_A^{(\tilde{s},\tilde{t})}$. This implies that $N(Nx * Ny), y \in X_A^{(\tilde{s},\tilde{t})}$ since $X_A^{(\tilde{s},\tilde{t})}$ is an dual ideal of X. Hence we deduce that

$\tilde{\mu}_A(x) \ge \tilde{s} = \min\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

and

$\tilde{\lambda}_A(x) \le \tilde{t} = \max\{\tilde{\lambda}_A(N(Nx * Ny)), \tilde{\lambda}_A(y)\}$.

This shows that $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v intuitionistic fuzzy dual BF ideal of X.

Theorem 2.15: Let $A = (\tilde{\alpha}_A, \tilde{\lambda}_A)$ and $B = (\tilde{\beta}_B, \tilde{\upsilon}_B)$ be i-v intuitionistic fuzzy dual ideals of X. Then the generalized Cartesian product $A \times B = (\tilde{\alpha}_A \wedge \tilde{\beta}_B, \tilde{\lambda}_A \vee \tilde{\upsilon}_B)$, where the functions $\tilde{\alpha}_A \wedge \tilde{\beta}_B : G \to D[0, 1]$ and $\tilde{\lambda}_A \vee \tilde{\upsilon}_B : G \to D[0,1]$ defined by $(\tilde{\alpha}_A \wedge \tilde{\beta}_B)(x) = \min\{\tilde{\alpha}_A(x), \tilde{\beta}_B(x)\}$ and $(\tilde{\lambda}_A \vee \tilde{\upsilon}_B)(x) = \max\{\tilde{\lambda}_A(x), \tilde{\upsilon}_B(x)\})$, for all $x, y \in X$ is an i-v intuitionistic fuzzy dual ideal of X.

Proof: For every $x \in X$, we have

Teorema 2.15: Fie $A=(\tilde{\alpha}_A,\tilde{\lambda}_A)$ si $B=(\tilde{\beta}_B,\tilde{\upsilon}_B)$ idealuri vagi, intuitioniste i-v ale lui X. Atunci produsul cartezian, generalizat, $A\times B=(\tilde{\alpha}_A\wedge\tilde{\beta}_B,\tilde{\lambda}_A\vee\tilde{\upsilon}_B)$, unde functiile $\tilde{\alpha}_A\wedge\tilde{\beta}_B : G\to D[0,\ 1]$ si $\tilde{\lambda}_A\vee\tilde{\upsilon}_B : G\to D[0,1]$ definite prin

$(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(x)=\min\{\tilde{\alpha}_A(x),\tilde{\beta}_B(x)\}$ si

$(\tilde{\lambda}_A\vee\tilde{\upsilon}_B)(x)=\max\{\tilde{\lambda}_A(x),\tilde{\upsilon}_B(x)\})$,

pentru toate $x,y\in X$ este un ideal dual, vag, intuitionist i-v al lui X.

Demonstratie: Pentru fiecare $x\in X$, avem

$$(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(1)=\min\{\tilde{\alpha}_A(1),\tilde{\beta}_B(1)\}$$
$$\geq\min\{\tilde{\alpha}_A(x),\tilde{\beta}_B(x)\}=(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(x),$$
$$(\tilde{\lambda}_A\vee\tilde{\upsilon}_B)(1)=\max\{\tilde{\lambda}_A(1),\tilde{\upsilon}_B(1)\}$$
$$\leq\max\{\tilde{\lambda}_A(x),\tilde{\upsilon}_B(x)\}=(\tilde{\lambda}_A\vee\tilde{\upsilon}_B)(x).$$

Si pentru oricare $x,y\in X$, deducem ca

$$(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(1)=\min\{\tilde{\alpha}_A(1),\tilde{\beta}_B(1)\}$$
$$\geq\min\{\tilde{\alpha}_A(x),\tilde{\beta}_B(x)\}=(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(x),$$
$$(\tilde{\lambda}_A\vee\tilde{\upsilon}_B)(1)=\max\{\tilde{\lambda}_A(1),\tilde{\upsilon}_B(1)\}$$
$$\leq\max\{\tilde{\lambda}_A(x),\tilde{\upsilon}_B(x)\}=(\tilde{\lambda}_A\vee\tilde{\upsilon}_B)(x).$$

And for any $x,y\in X$, we deduce that

$$\begin{aligned}(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(x)&=\min\{\tilde{\alpha}_A(x),\tilde{\beta}_B(x)\}\\&\geq\min\{\min\{\tilde{\alpha}_A(N(Nx*Ny)),\tilde{\alpha}_A(y)\},\min\{\tilde{\beta}_B(N(Nx*Ny)),\tilde{\beta}_B(y)\}\}\\&=\min\{\min\{\tilde{\alpha}_A(N(Nx*Ny)),\tilde{\beta}_B(N(Nx*Ny)\},\min\{\tilde{\alpha}_A(y),\tilde{\beta}_B(y)\}\}\\&=\min\{(\tilde{\alpha}_A\wedge\tilde{\beta}_B)(N(Nx*Ny)),(\tilde{\alpha}_A\wedge\tilde{\beta}_B(y)\}\end{aligned}$$

$$\begin{aligned}(\tilde{\lambda}_A\vee\tilde{\upsilon}_B)(x)&=\max\{\tilde{\lambda}_A(x),\tilde{\upsilon}_B(x)\}\\&\geq\max\{\max\{\tilde{\lambda}_A(N(Nx*Ny)),\tilde{\lambda}_A(y)\},\max\{\tilde{\upsilon}_B(N(Nx*Ny)),\tilde{\upsilon}_B(y)\}\}\\&=\max\{\max\{\tilde{\lambda}_A(N(Nx*Ny)),\tilde{\upsilon}_B(N(Nx*Ny)\},\max\{\tilde{\lambda}_A(y),\tilde{\upsilon}_B(y)\}\}\\&=\max\{(\tilde{\lambda}_A\wedge\tilde{\upsilon}_B)(N(Nx*Ny)),(\tilde{\lambda}_A\wedge\tilde{\upsilon}_B(y)\}.\end{aligned}$$

Acest lucru inseamna ca $A\times B=(\tilde{\alpha}_A\wedge\tilde{\beta}_B,\tilde{\lambda}_A\vee\tilde{\upsilon}_B)$ este un ideal dual i-v IF al lui X.

This shows that $A\times B=(\tilde{\alpha}_A\wedge\tilde{\beta}_B,\tilde{\lambda}_A\vee\tilde{\upsilon}_B)$ is an i-v IF dual ideal of X.

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