GĂSIŢI LIMITA

FIND THE LIMIT

Miodrag Iovanov, Universitatea "Constantin Brâncuşi" din Tg-Jiu

Miodrag Iovanov, University Constantin Brancusi of Tg-Jiu

REZUMAT: *In aceasta lucrare vom determina cu ajutorul calculului integral o limita propusa la OIM 2011*

ABSTRACT: *In this paper we determine the full calculation using the ILO 2011 proposed limit*

Cuvinte cheie: limita, integrale Riemann

KEY WORDS: limit, Riemann integrals

1. INTRODUCERE.

În această lucrare se determină limita unui şir folosind inegrala Riemann.

1. INTRODUCTION.

In this paper we determine the limit of a string using Riemann blackened.

2. MODUL DE LUCRU

Găsiţi limita

2. METHOD OF WORK

Find the limit

$$\lim_{n\to\infty}\left(\prod_{k=1}^{n}\frac{k}{\sqrt{k^2+n^2}}\right)^{(e^{\frac{a}{n}}-1)}$$

Caz special un 2009 =

Soluţia este

Particular case a=2009

Solution Let

$$y_n=\left(\prod_{k=1}^{\infty}\frac{k}{\sqrt{k^2+n^2}}\right)^{x_n},\ x_n=e^{\frac{a}{n}}-1$$

Logaritmând ecuaţia obţinem:

Trough logarithming the equation we will obtain:

$$\ln y_n=x_n\sum_{k=1}^{n}\ln\frac{k}{\sqrt{k^2+n^2}} \quad \text{or} \quad \ln y_n=nx_n\sum_{k=1}^{n}\frac{1}{n}\ln\sqrt{\frac{\left(\frac{k}{n}\right)^2}{1+\left(\frac{k}{n}\right)^2}}\,.$$

Formula devine:

Last formula yelds :

$$\ln\left(\lim_{n\to\infty}y_n\right)=\lim_{n\to\infty}nx_n\cdot\frac{1}{2}\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\ln\sqrt{\frac{\left(\frac{k}{n}\right)^2}{1+\left(\frac{k}{n}\right)^2}} \tag{1}$$

Pentru relaţia (1) obţinem:

From relation (1) we obtain:

$$\lim_{n\to\infty}nx_n=\lim_{x\to\infty}x(e^{\frac{a}{x}}-1)\underset{e^{\frac{a}{x}}-1=t}{\overset{\uparrow}{=}}\lim_{t\to 0}\frac{at}{\ln(1+t)}=a$$

şi and

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\ln\sqrt{\frac{\left(\frac{k}{n}\right)^2}{1+\left(\frac{k}{n}\right)^2}}=\int_0^1\ln\frac{x^2}{1+x^2}dx$$

Avem We have

$$\int_0^1\ln\frac{x^2}{1+x^2}dx=x\ln\frac{x^2}{1+x^2}\Big|_0^1-2\int_0^1\frac{x}{1+x^2}dx=\ln\frac{1}{2}-2arctg(x)\Big|_0^1=-\left(\ln 2+\frac{\pi}{2}\right)$$

$$(\lim_{x\to 0}(x\ln x)=0)\ .$$

Obţinem We obtain

$$\ln\left(\lim_{n\to\infty}y_n\right)=-\frac{a}{2}\left(\ln 2+\frac{\pi}{2}\right)$$

de unde from where

$$\lim_{n\to\infty}y_n=e^{-\frac{a}{2}\left(\ln 2+\frac{\pi}{2}\right)} \qquad (2)$$

3. CONCLUZII

Pentru cazul particular a= 2009 obţinem

3. CONCLUSIONS

The particular case a=2009 we obtain

$$e^{-\left(\frac{2009}{2}\right)\left(\ln 2+\frac{\pi}{2}\right)}$$

Pentru a=1999 şi

For a=1999 and

$$"\prod_{k=1}^{n}\frac{k}{k+n}"$$

vom obţine prima problema de la editia 24 a AVJMC, Ostrava, 2009.

we obtain the 1st problem from the 24 rd edition of the A.V.J.M.C.,Ostrava, 2009 .

BIBLIOGRAFIE:

1. I. Bucur – *Curs de analiză matematică*, Editura Matrix Rom, Bucureşti, 2000

2. C. Drăguşanu ş.a. – *Analiză matematică*, Editura Teora, Bucureşti, 1991

3. Fihtenholţ G. M. – *Curs de calcul diferenţial şi întegral*, (vol. I, II), Editura Tehnică, Bucureşti, 1964-1965

REFERENCES:

1. I. Bucur - Course in mathematical analysis, Matrix Rom Publishing House, Bucharest, 2000

2. C. Dragusanu etc. - Mathematical analysis, Teora Publishing House, Bucharest, 1991

3. Fihtenholţ GM - Course of differential and integral calculus, (Vol. I, II), Technical Publishing House, Bucharest, 1964-1965