# APPLICATIONS OF THE THEORY OF PROBABILITIES INTO THE THEORY OF FIABILITY

**Miodrag IOVANOV, *„Constantin Brâncuşi" University of Târgu-Jiu,***

**ABSTRACT**.The theory of fiability (the theory of the safety during operation) aims at finding the admission laws of the equipments' or tools' failure. Thus, the equipment or the tool can be: turning machine, tractor, vehicle, industrial apparatus, factory, plant, computer, etc.By the quality of the equipment we understand the number of the features which define the degree of the utility in exploitation. The reliability of the equipment is the capacity of the equipment to preserve its quality under the conditions determined by exploitation.

**KEY WORDS:** class of function, analitic
.

The duration of operation until the first failure.

In the case of the complex systems both the reliability of the whole system and the reliability of some elements considered stand-alone entities are studied. An indivisible part of the system or studied as a whole which is independent on its elements, will be called element. In the case of some equipments or of some elements, the period of time from starting up to the occurance of the failure coincides with the length of life of the equipment or of the element .

Let us consider as initial moment the moment when an element is started up and let us mark with z the duration of operation until the occurance of the failure. By duration of operation we understand the period of actual functioning, eliminating the periods of deliberate interruption. Z is a stochastic variable whose function of distribution will be marked with Q:

$$Q(t) = P( z < t ) , ( t > 0).$$

We'll assume that the că function Q(t) is derivable in any point t > 0 and we mark q(t) = Q'(t).

The probability that the element is operating at the point t (or it operates without failure for a longer time than t) is the following:

$$\Phi(t) = P ( z < t )=1-P(t) , ( t > 0 ).$$

The fun ction P(t) is called *function of safety.*

From the general features of the functions of distribution and from the conditions imposed to Q we can imediately deduce the features of the function of safety Φ: it is continual and derivable in any t > 0, $\Phi(0) = 1$ ; $\lim_{t\to\infty}\phi(t) = 0$.

The average value of the duration of operation without failure is

$$M(z) = \int_0^\infty tq(t)\,dt = \int_0^\infty \phi(t)dt - m^2$$

where m = M(z).

In practice there can be found numerous examples in which it is important that failures are prevented. In this case a computing and experience based limit of functioning $t_0$ is established.

This means that regardless of the condition in which the element or the equipment are found at a certain time of $t_0$, it is taken out. If z were the life duration of such equipment without imposing a maximum duration of operation, then the real value of this duration would be $z^* = \min(z, t_0)$.

If $Q^*$ is the distribution function of $z^*$ there can be noticed that for any t $\geq 0$:

$$Q^*(t) = P(z^* < t) = \begin{cases} Q(t), & t \leq t_0 \\ 1, & t > t_0, \end{cases}$$

and concordantly

$$\Phi^*(t) = 1 - Q^*(t) = \begin{cases} \Phi(t), t \leq t_0, \\ 0, t > t_0. \end{cases}$$

The average value of a variable $z^*$ is

$$m^* = \int_0^\infty \Phi^*(t)dt = \int_0^{t_0} \Phi(t)dt$$

And the dispersion of this variable:

$$D^2(z^*) = 2\int_0^{t_0} t\Phi(t)dt - m^{*2}.$$

*The function of failure risk.* Let us consider the following:

A: the element operates without failure before the moment t;

B: the element does not fail between the moments t and t + h. It is noticed that A$\cap$B is the event " the element operates without failure until the moment t + h". We obtain:

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(z > t + h)}{P(z > t)} = \frac{\Phi(t + h)}{\Phi(t)}.$$

In other words, if the element does not fail before the moment t, the probability that it won't fail before the moment t + h is $\frac{\Phi(t+h)}{\Phi(t)}$.

This means that in the same assumption the probability that it won't fail before the moment t + h is:

$$1 - \frac{\Phi(t + h)}{\Phi(t)} = \frac{\Phi(t) - \Phi(t + h)}{\Phi(t)}.$$

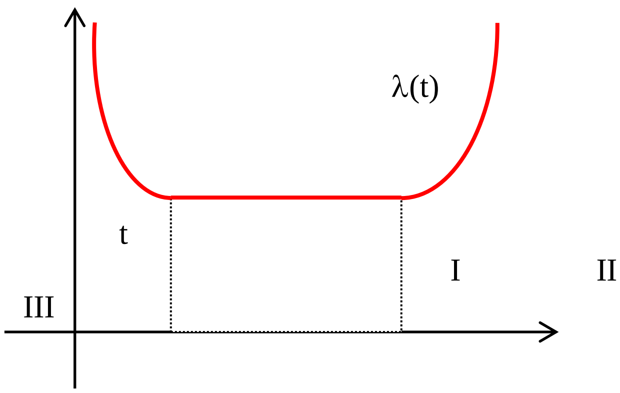
If h is small then

$$\Phi(t) - \phi(t + h) \cong h\phi'(t)$$

so for such a h

$$P(B/A) \cong -\frac{\Phi'(t)}{\Phi(t)} \cdot h = \lambda(t) \cdot h.$$

The function $\lambda(t)$ is called *failure risk*. The diagram of the empirical failure risk function obtained by processing the statisticall data has the following shape:




This shape of the diagram suggests the existence of three distinct periods during exploatation. In the first period (I in the diagram) the failure risk decreases in time. When the equipment is put into service, the hidden manufacturing defects begin to manifest themselves. Those who work with certain tools know that the failure risk is lower after some time from putting into service. This is the running in period. The second period (II in the diagram) is *the period of normal operation*. After the running in period there follows a period when the failure risk stabilises and practically it doesn't depend on time.

The third (III in the diagram) is *the aging period* of the equipment. Under the influence of some physical and chemical factors the elements degrade irreversibly and the failure risk increases with in time.

If we consider as initial moment the moment when the running in period finishes and the period of normal operation begins, for a long period of time the failure risk will practically be constant. For many times it cannot be reached too far into the third period, the equipment being replaced in order to prevent damage or moral wear before it becomes incapable of operating. If $\lambda(t) = \lambda$ , $\lambda > 0$ this means that

$$\frac{\Phi'(t)}{\Phi(t)} = -\lambda$$

from where it results $\Phi(t) = e^{-\lambda t}$. The distribution function of the duration of operation without failure is

$$Q(t) = 1- e^{-\lambda t}, \; t > 0,$$

That is to say it has exponential distribution with the parameter $\lambda$.

This law of reliability is not universal. In practice there can frequently be found situations in which the experimental data don't coincide with the model above. A law of probability which appears increasingly often in the theory of reliability is *Weibull* distribution. If z has Weibull distribution with parameters $\lambda$ and $\alpha$, that is to say its function of distribution is

$$Q(t) = 1- e^{-\lambda t^{\alpha}}, \; t > 0.$$

Then the corresponding safety function is

$$\phi(t) = e^{-\lambda t^{\alpha}}$$

and the function of failure risk $\lambda(t) = \lambda \alpha t^{\alpha-1}$ will correspond to it.

The Weibull law is more general than the exponential law. Depending on two parameters, it can comprise a much bigger number of concrete cases than the exponential law.

If the failure risk is proportional to the time:

$$\lambda(t) = 2\lambda t, \quad \lambda > 0 \text{ constant},$$

then from the following relations

$$\frac{\phi'(t)}{\phi(t)} = -2\lambda t; \phi(0) = 1$$

it results:

$$\Phi(t) = e^{-\lambda t^{2}}$$

and we have the case of a Weibull law.

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