EXTENSIONS OF BOOLEAN ALGEBRA IN PSEUDO MV-ALGEBRAS

Bogdan P. Constantin, Universitatea din Craiova
Pecingina Olimpia, Universitatea "Constantin Brancusi"Tg-jiu

ABSTRACT In this article we study Boolean algebra in terms of pseudo MV-algebras using our refelexomorfisme.

Key words Boole algebras

Pseudo MV algebras

We consider an algebra $\Re = (A, \oplus, ^-, ^\sim, 0, 1)$ of type $(2,1,1,0,0)$.

We can define: $y ? x = (x^- \oplus y^-)^\sim$ operation (1)

We believe that the operation ? takes precedence before the surgery $\oplus$.

Definitia 1 *A pseudo MV-algebra is an algebra* $\Re = (A, \oplus, ^-, ^\sim, 0, 1)$ *of type* $(2,1,1,0,0)$ *satisfying the following axioms:*

$(psMV_1)$ $x \oplus (y \oplus)z = (x \oplus)$ (2)

$(psMV_2)$ $x \oplus 0 = 0 \oplus x = x;$ (3)

$(psMV_3)$ $x \oplus 1 = 1 \oplus x = 1;$ (4)

$(psMV_4)$ $1^\sim = 0; 1^- = 0;$ (5)

$(psMV_5)$ $(x^- \oplus y^-)^\sim = (x^\sim \oplus y^\sim)^-;$ (6)

$(psMV_6)$

$x \oplus x^\sim ? y = y \oplus y^\sim ? x = x ? y^- \oplus y = y ? x^- \oplus x;$ (7)

$(psMV_7)$ $x ?(x^- \oplus y) = (x \oplus y^\sim) ? y;$ (8)

$(psMV_8)$ $(x^-)^\sim = x$, for any $x, y, z \in A$. (9)

Note a pseudo MV-algebra $\Re = (A, \oplus, ^-, ^\sim, 0, 1)$ through the universe, or A .

We can define two corresponding implications two negations:

$x \to y := x^- \oplus y$ (10)

and

$x ? y := y \oplus x^\sim$ (11)

for any $x, y \in A$.

If $A^\wedge \subseteq A$ We write $A^\wedge \leq A$ to indicate that the $A^\wedge$ It is a pseudo MV-subalgebra of A .

A lot with a single element $\{0\}$ is a trivial example of a pseudo MV-algebra. A pseudo MV-algebra is non-trivial, provided that the universe or to have more than one element. To consider an l-group arbitrary $(G, +, -, 0, \leq)$.

For each $u \in G$, $u > 0$, fie $[0, u] = \{x \in G \ : \ 0 \leq x \leq u\}$

and for each $x, y \in [0, u]$, either

$x \oplus y = u \wedge (x + y)$ (12)

$(x ? y) = (x - u + y) \vee 0$ (13)

$x^{-} = u - x$ si $x^{\sim} = -x + u$. Then $([0,u], \oplus, ?, ^{-}, ^{\sim}, 0, u)$ It is a pseudo MV-algebra. We notice that the order is the order of restriction of relations G. more $(x^{-})^{-} = u + x - u$ si $(x^{\sim})^{\sim} = -u + x + u$, for any $x, y \in [0,u]$.

Dvurečenskij has shown that each pseudo MV-algebra is izomorfa with an interval in $l - grup$.

Teorema 2

Be a pseudo MV-MV algebra $= (\mathbf{PMV}, \oplus, ?, ^{-}, ^{\sim}, 0, 1)$ *, type* $(2,2,1,1,0,0)$ *that define the following:*

$$x \vee y = x \oplus x^{\sim} \odot y$$
$$= y \oplus y^{\sim} \odot x = x \odot y^{-} \oplus y \quad (14)$$
$$= y \odot x^{-} \oplus x,$$

$$x \wedge y = x \odot (x^{-} \oplus y) = y \odot (y^{-} \oplus x)$$
$$= (x \oplus y^{\sim}) \odot y = (y \odot x^{\sim}) \odot x$$
(15)

$0_{R(\mathbf{PMV})} = 0_{\mathbf{PMV}}$ (16)

R(**PMV**)={ $a^{-} = a^{\sim}$ | a $\in \mathbf{PMV}$} (17)

D(**PMV**)={a $\in \mathbf{PMV}$ |a $\oplus a = a$} (18)

Then pseudo MV-algebra satisfying the following conditions: (i)) In relation to the order induced from $\mathbf{PMV}, R(\mathbf{PMV})$ Boolean algebra in which becomes for

$x, y \in R(\mathbf{PMV}),\quad x \vee y \quad (in \quad R(\mathbf{PMV})) = ((x \vee y)^{-})^{\sim} = (x^{-} \wedge y^{-})^{\sim},$ (19)

$x \wedge y \quad (in \quad R(\mathbf{PMV})) = x \wedge y,$ (20)

$0_{R(A)} = 0_A$ (21)

iar

$x \rightarrow y (in \quad R(\mathbf{PMV})) \ni \quad \bar{x} \vee (\quad \bar{y}\ ^{\sim}$.
(22)

ii) Function $r_{\mathbf{PMV}}$: $\mathbf{PMV} \rightarrow R(\mathbf{PMV})$,

$r_{\mathbf{PMV}}(x) = x ? x$ (23)

for any $x \in A$ is the Group homomorphism surjectiv in **PMV** and $\mathbf{PMV} / D(\mathbf{PMV}) \approx R(\mathbf{PMV})$ (in **PMV**).

iii) If $D(\mathbf{PMV}) = \{1\}$, then $\mathbf{PMV} \in \mathbf{B}$.

Consider the following customisations: proof.

{ $R(\mathbf{PMV}), \wedge, \vee, ^{*}, 0\}$ is the Boolean algebra;

a $\rightarrow e = (a ? e^{\sim})^{-} = a^{-} \oplus e$ (24)

a $? e = (e^{-} ? a)^{\sim} = e \oplus a^{\sim}$ (25)

$^{*} = ^{\sim} = ^{-}$ (26)

$\phi_R(\mathbf{PMV})(x) = x \oplus x$ (27)

$(R(f)(x^{-})^{\sim}) = (R(f)(x))^{\sim}$ (28)

Define $\Phi_R(\mathbf{PMV})$: $\mathbf{PMV} \rightarrow R(\mathbf{PMV})$ through $\Phi_R(\mathbf{PMV})(x) = x \oplus x$, for any $x \in \mathbf{PMV}$.

We infer that the $\Phi_R(A)$ *It is a group homomorphism in PMV,*

If $A, A' \in Ob($ PMV $)$ si $f \in Hom_{BL}(A, A')$, then

$$R(f)(x \vee y) = R(f)(x \oplus x^{-} \odot y) = (f(x \odot (x^{-} \oplus y))^{-} = (f(x \wedge y))^{-} =$$

(29)

$$(f(x) \wedge f(y))^{-} = f(x)^{-} \vee f(y)^{-} = (R(f)(x^{-})) \vee (R(f)(y^{-}))$$

(30)

and

$(R(f)(x^{-})^{\sim}) = (f(x^{-}))^{\sim} = (f(x)^{-})^{\sim} = (R(f)(x))^{\sim}$
. (31)

Indeed, if $x, y \in A$, then

$R(f)(x \oplus x^- \odot y) = R(f)(((x \wedge y))^-) = (f(x \odot (x^- \oplus y))^- =$

(32)

$(f(x \wedge y))^- = (f(x) \wedge f(y))^- = f(x)^- \vee f(y)^-$

(33)

$R(f)(x \oplus x^- \odot y) = R(f)(((x \wedge y))^-) = (f(x \odot (x^- \oplus y))^- =$

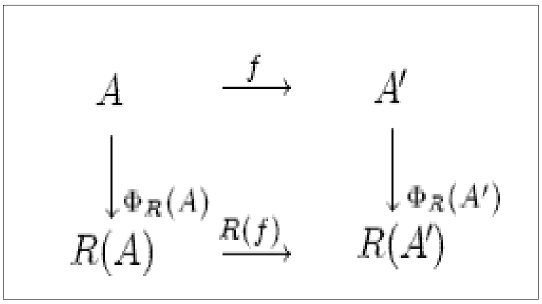
(34)

$(f(x \wedge y))^- = (f(x) \wedge f(y))^- = f(x)^- \vee f(y)^-$

(35)

si
$(R(f)(x^-)^{\sim}) = (f(x^-))^{\sim} = (f(x)^-)^{\sim} = (R(f)(x))^{\sim}$
. (36)

Thus, we obtain a functor R : PMV $\rightarrow \mathbf{B}$
.
To demonstrate that R is a reflector, consider the diagram:



with $A, A' \in Ob(\mathbf{PMV})$.
If $x \in A$, then

$(\Phi_R(A') \circ f)(x) = \Phi_R(A')(f(x)) = ((f(x))^-)^{\sim}$
(37)

And

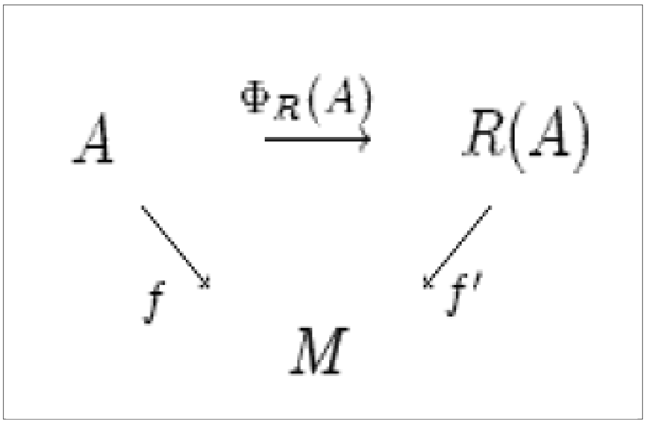
$(R(f) \circ \Phi_R(A))(x) = R(f)(\Phi_R(A)(x))$
$= R(f)((x^-)^{\sim}) = (f(x^-))^{\sim} = (f(x)^-)^{\sim} = ((f(x))^-)^{\sim}$
(38)

Where

$\Phi_R(A') \circ f = R(f) \circ \Phi_R(A)$ (39)

because the diagram is commutative.

Be now $A \in Ob(\mathbf{PMV})$, $M \in Ob(\mathbf{B})$ and $f : A \rightarrow M$ a group homomorphism in PMV.



For $x \in A$, define

$f'(x^-) = f(x^-) = f(x)^-$ (40)
(where $f' = f_{|R(A)}$).
As stated above we have: $x, y \in A$,

$f'(x^- \vee y^-) = f'((x \wedge y)^-) = f'((x \wedge y))^-$,
$= (f(x) \wedge f(y))^- = f(x)^- \vee f(y)^-$
(41)
$f((x^-)^{\sim}) = (f(x)^-)^{\sim} = f(x) = (f(x^-))^{\sim}$ si
$f'(0) = f'(1^-) = f'(1)^- = 1^- = 0$ (42)

, where f' It is a group homomorphism in $\mathbf{B}$. Because

$$(f' \circ \Phi_R(A))(x) = f'(\Phi_R(A)(x)) = f'((x^-)^-) = (f(x)^-)^- = f(x) \quad (43)$$

, We infer that the

$f' \circ \Phi_R(A) = f$. (44)

If we again $f'' : R(A) \to M$ It is a group homomorphism in **B** so

$f'' \circ \Phi_R(A) = f$, (45)

then for any

$x \in A$, ($f'' \circ \Phi_R(A))(x^-) = f(x^-)$ (46)

, where

$f''(x^-) = f(x^-) = f'(x^-)$ (47)

, Thus $f'' = f'$.
Be now $f : A \to A'$ a group homomorphism in **PMV** and $x, y \in A$ so $R(f)(x^-) = R(f)(y^-)$.
Then $f(x^-) = f(y^-)$, where $x^- = y^-$, ie $R(f)$ It is a group homomorphism in **B** .
ii) The fact that the $r_{\mathbf{PMV}}$ It is a group homomorphism in $Ld(0,1)$ It is immediately. Either $x, y \in \mathbf{PMV}$. Then

$r_{\mathbf{PMV}}(x) \to^{R(PMV)}$ (48)

$r_{\mathbf{PMV}}(y) = x \oplus x \to^{R(\mathbf{PMV})} y \oplus y$

$= ((x \oplus x)^- \vee y \oplus y) \oplus ((x \oplus x)^- \vee y \oplus y)$ (6)

$= (x^- \vee y \oplus y) \odot (x \vee y \oplus y)$

$$r_{\mathbf{PMV}}(y) = x \oplus x \to^{R(\mathbf{PMV})} y \oplus y = ((x \oplus x)^- \vee y \oplus y) \oplus ((x \oplus x)^- \vee y \oplus y) = (x^- \vee y \oplus y) \odot (x \vee y \oplus y)$$

$= (x \oplus x \vee y^-)^- = (x \to y) \oplus (x \to y) = r_{\mathbf{PMV}}(x \to y)$
(49)

, I mean
$r_{\mathbf{PMV}} = PMV(\mathbf{PMV}, R(\mathbf{PMV}))$. (50)

We have isomorphism
$PMV / Ker(r_{\mathbf{PMV}}) \approx \mathrm{Im}(r_{\mathbf{PMV}}) = R(PM$
(51)

We infer that the $D(PMV) = Ker(r_{\mathbf{PMV}})$ such as $PMV \,/\, D(PMV) \approx R(PMV)$.
iii) If $D(PMV) = \{1\}$, then from ii) deduce that $PMV \approx R(PMV)$.

CONCLUSIONS

The main issue is topical because it approaches the fuzzy systems underlying the artificial intelligence that is implemented in the economic and industrial machines.

BIBLIOGRAPHI

[1] Balbes si Dwinger,The curatours of the University of Missouri,1974
[2] Dumitru Buşneag ,Categories of Algebric Logic, Editura Academiei Române, Bucureşti 2006
[3] George Gratzer, Lattice teory,1971
[4] Piciu Dana,Categories of Fuzzy Logic, Editura Academiei Române, Bucureşti 2007