BOOL ALGEBRA APPLIED IN PSEUDO BL-ALGEBRAS

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ABSTRACT: In this article we study Boolean algebra in terms of pseudo BL-algebras using our reflexomorfisme

KEY WORDS: Boole algebras

Pseudo-BL algebra

Definitia 1 A pseudo-BL algebra is an algebra $(A,\vee,\wedge.?,\rightarrow,?0,1)$ type $(2,2,2,2,2,0,0)$ that satisfies the following conditions:

$(psBL_1)$ $(A,\vee,\wedge,0,1)$ It is a lattice bordered;

$(psBL_2)$ $(A,?,1)$ is a monoid;

$(psBL_3)$ $a?\ \le b$ ▫

$a \le b \rightarrow c \Leftrightarrow b \le a?c$ (1)

for any $a,b,c \in A$;

$(psBL_4)$ $a \wedge b = (a \rightarrow b)?a = a?(a?b)$; (2)

$(psBL_5)$

$(a \rightarrow b)\vee(b \rightarrow a) = (a?b)\vee(b?a) = 1$ (3)

for any $a,b \in A$.

operations $\wedge,\vee,?$ have higher priority than the operations $\rightarrow,?$.

Either $(A,?,\oplus,^-,^\sim,0,1)$ a pseudo MV-algebra and be $\rightarrow,?$ two implications are defined by:

$x \rightarrow y = y \oplus x^-$ (4)

$x?y = x^\sim \oplus y$ (5)

then $(A,\vee,\wedge,?\rightarrow,?,0,1)$ It is a pseudo-BL algebra. For each pseudo-BL algebra A notam

$G(A) = \{x \in A : x?x = x\}$ (6)

$M(A) = \{x \in A : x = (x^\sim)^- = (x^-)^\sim\}$ (7)

Either $B(A)$ a Boolean algebra that contains all of the components of the distributive lattice of elementary $L(A) = (A,\vee,\wedge,0,1)$ the pseudo-BL algebra A .
So $B(A) = B(L(A))$

Teorema 2 *Be a pseudo-BL algebra* $PBL = (\mathbf{PBL},\vee,\wedge.?,\rightarrow,?0,1)$ *, type* $(2,2,2,2,2,0,0)$ *that define the following:*

$a \wedge b = a?(a?b) = (a \rightarrow b)?a$ (8)

$$a \vee b = (a^- \wedge b^-)^\sim = (a^- \odot (a^- \odot b^-))^\sim = ((a^\sim \rightarrow b^\sim)^- \odot a^\sim)^- \quad (9)$$

$0_{R(\mathbf{PBL})} = 0_{\mathbf{PBL}}$ (10)

$a \to b = (a^{-} \vee b)^{?} = (a \vee b^{\sim})^{\doteq}$
$= ((a^{\sim} \to b^{\sim}) \odot a^{\sim})^{-}$ (11)

R($\mathbf{PBL}$)={($a^{-})^{\sim} = (a^{\sim})^{-}$ | a $\in \mathbf{PBL}$}
(12)

D($\mathbf{PBL}$)={a $\in \mathbf{PBL}$ |a $? a = a$} (13)

Then pseudo-BL algebra satisfying the following conditions:

i) In relation to the order induced from $\mathbf{PBL}, R(\mathbf{PBL})$ Boolean algebra in which becomes for

$x, y \in R(\mathbf{PBL}), \quad x \vee y \quad (in \quad R(\mathbf{PBL}))$
$= ((x \vee y)^{-})^{\sim} = (x^{-} \wedge y^{-})^{\sim}$, (14)

$x \wedge y \quad (in \quad R(\mathbf{PBL})) = x \wedge y,$
$0_{R(A)} = 0_{A}(1)$ (15)
iar

$x \to y (in \quad R(\mathbf{PBL})) = ((x^{-} \vee y)^{-})^{\sim}$. (16)

ii) function $r_{\mathbf{PBL}} : \mathbf{PBL} \to R(\mathbf{PBL})$,
$r_{\mathbf{PBL}}(x) = x ? x$ for any $x \in A$ is the Group homomorphism surjective in $\mathbf{PBL}$ iar $\mathbf{PBL} / D(\mathbf{PBL}) \approx R(\mathbf{PBL})$ (in $\mathbf{PBL}$).

iii) Daca $D(\mathbf{PBL}) = \{1\}$, atunci $\mathbf{PBL} \in \mathbf{B}$.

Proof.

Let the following:

{ $R(\mathbf{PBL}), \wedge, \vee, ^{*}, 0\}$ is the Boolean algebra;

a $\to e = (a ? e^{\sim})^{-} = a^{-} \vee e$ (17)

a $? e = (e^{-} ? a)^{\sim} = e \vee a^{-}$ (18)

$^{*} = ^{\sim} = ^{-}$ (19)

$\phi_{R}(\mathbf{PBL})(x) = x ? x$ (20)

Define $\Phi_{R}(\mathbf{PBL}) : \mathbf{PBL} \to R(\mathbf{PBL})$ through $\Phi_{R}(\mathbf{PBL})(x) = x ? x$, for any $x \in \mathbf{PBL}$.

We infer that the $\Phi_{R}(A)$ It is a group homomorphism in PBL ,

If $A, A' \in Ob(PBL)$ si $f \in Hom_{BL}(A, A')$, then

$R(f)(x \vee y) = R(f)(((x^{\sim} \to y^{\sim})^{-} \odot x^{\sim})^{-})$
$= (f((x \to y) \odot x))^{-} = (f(x \wedge y))^{-}$
(21)

$= (f(x) \wedge f(y))^{-} = f(x)^{-} \vee f(y)^{-}$
$= (R(f)(x^{-})) \vee (R(f)(y^{-}))$ (22)

And

$(R(f)(x^{-})^{\sim}) = (f(x^{-}))^{\sim} = (f(x)^{-})^{\sim} = (R(f)(x))^{\sim}$
. (23)

Indeed, if $x, y \in A$, then

$R(f)(x \vee y) = R(f)(((x^{\sim} \to y^{\sim})^{-} \odot x^{\sim})^{-}) = R(f)(((x \wedge y))^{-})$
$= (f((x \to y) \odot x))^{-} = (f(x \wedge y))^{-} = (f(x) \wedge f(y))^{-}$
(24)

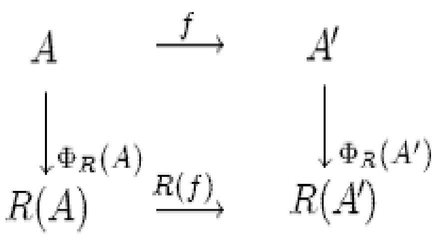
$= f(x)^{-} \vee f(y)^{-} = (R(f)(x^{-})) \vee (R(f)(y^{-}))$
(25)

And

$(R(f)(x^{-})^{\sim}) = (f(x^{-}))^{\sim} = (f(x)^{-})^{\sim} = (R(f)(x))^{\sim}$
. (26)

Thus, we obtain a functor $R : PBL \to \mathbf{B}$.

To prove that is a reflector, consider the diagram:



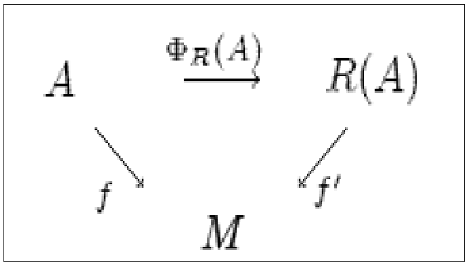
with $A, A' \in Ob(\mathbf{PBL})$.

Daca $x \in A$, then

$(\Phi_R(A') \circ f)(x) = \Phi_R(A')(f(x)) = ((f(x))^-)^\sim$
(27)

And
$(R(f) \circ \Phi_R(A))(x) = R(f)(\Phi_R(A)(x)) = R(f)((x^-)^\sim)$
$= (f(x^-))^\sim = (f(x)^-)^\sim = ((f(x))^-)^\sim$
(28)
where
$\Phi_R(A') \circ f = R(f) \circ \Phi_R(A)$ (29)

because the diagram is commutative.
Be now $A \in Ob(\mathbf{PBL})$, $M \in Ob(\mathbf{B})$ and $f : A \to M$ a group homomorphism in PBL .



For $x \in A$, define $f'(x^-) = f(x^-) = f(x)^-$ (where $f' = f_{|R(A)}$).
As stated above we have: $x, y \in A$,
$f'(x^- \vee y^-) = f'((x \wedge y)^-) = f'((x \wedge y))^-$,
$= (f(x) \wedge f(y))^- = f(x)^- \vee f(y)^-$
(30)
$f((x^-)^\sim) = (f(x)^-)^\sim = f(x) = (f(x^-))^\sim$ (31)
And
$f'(0) = f'(1^-) = f'(1)^- = 1^- = 0$ (32)

where f' It is a group homomorphism in $\mathbf{B}$. since

$(f' \circ \Phi_R(A))(x) = f'(\Phi_R(A)(x))$
$= f'((x^-)^\sim) = (f(x)^-)^\sim = f(x)$ (33)

, We infer that the

$f' \circ \Phi_R(A) = f$. (1)

If we again $f'' : R(A) \to M$ It is a group homomorphism in $\mathbf{B}$ so $f'' \circ \Phi_R(A) = f$, then for any $x \in A$, ($f'' \circ \Phi_R(A))(x^-) = f(x^-)$ (34)
, where
$f''(x^-) = f(x^-) = f'(x^-)$ (35)
so
$f'' = f'$. (36)
Be now $f : A \to A'$ a group homomorphism in $\mathbf{PBL}$ and $x, y \in A$ so $R(f)(x^-) = R(f)(y^-)$.
Then $f(x^-) = f(y^-)$, where $x^- = y^-$, ie $R(f)$ It is a group homomorphism in $\mathbf{B}$.

ii) The fact that the $r_{\mathbf{PBL}}$ It is a group homomorphism in $Ld(0,1)$ It is immediately. Either $x, y \in \mathbf{PBL}$. Then
$r_{\mathbf{PBL}}(x) \to^{R(\mathbf{PBL})}$
$r_{\mathbf{PBL}}(y) = x \odot x \to^{R(\mathbf{PBL})} y \odot y$
$= ((x \odot x)^- \vee y \odot y) \odot ((x \odot x)^- \vee y \odot y)$
(37)
$= (x^- \vee y \odot y) \odot (x \vee y \odot y) = (x \odot x \vee y^-)^-$
$= (x \to y) \odot (x \to y) = r_{\mathbf{PBL}}(x \to y)$
(38)
, I mean
$r_{\mathbf{PBL}} = PBL(\mathbf{PBL}, R(\mathbf{PBL}))$. (39)

We have isomorphism
$H \;/\; Ker(r_{\mathbf{PBL}}) \approx \mathrm{Im}(r_{\mathbf{PBL}}) = R(PBL)$.
(40)

We infer that the

$D(PBL) = Ker(r_{\mathbf{PBL}})$ (41)
such as
$PBL \;/\; D(PBL) \approx R(PBL)$. (42)
iii) If $D(PBL) = \{1\}$, then from ii) deduce that $PBL \approx R(PBL)$.

CONCLUSIONS

The main issue is topical because it approaches the fuzzy systems that satau based artificial intelligence that is implemented in the economic and industrial machines.

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