BOOLEAN ALGEBRAS EXTENSIONS TO PSEUDO MV-ALGEBRAS AND PSEUDO BL-ALGEBRAS

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ABSTRACT: In this article we study Boolean algebra in terms of pseudo MV-algebrasand pseudo BL-algebras using our reflexomorfisme.

KEY WORDS: Boole algebras

Pseudo MV-algebras

Teorema 1
Category of pseudo PMV MV-algebras is reflexive subcategory of the category of pseudo PBL BL-algebras and reflector $R : PBL \to PMV$ keep monomorfismul.

Proof.
We can define:

$y ? x = (x^{-} \oplus y^{-})^{\sim}$ (1)

Either $(A, \wedge, \vee, ?, \to, ?, 0, 1) \in Ob(PBL)$ and we define:

R(**PBL**)={($a^{-})^{\sim} = (a^{\sim})^{-}$ | a $\in$ **PBL**} (2)

D(**PBL**)={a $\in$ **PBL** |a $? a = a\}$ (3)

We like $(R(A), \wedge, \vee, \oplus, *, 0)$ It is the largest subalgebra of MV- A through operations:

x $\to y = y \oplus x^{-}$ (4)

x $? y = x^{\sim} \oplus y$ (5)

0 $_{R(PBL)} = 0_{PBL}$ (6)

$x^{-} \oplus y^{-} = (x^{-})^{\sim} \to y^{-} = (x ? y)^{-} = ((x^{-})^{\sim} ?(y^{-})^{\sim})^{-}$; (7)

I Mean

$x \oplus y = x^{\sim} \to y$ (8)

$x^{-} \vee y^{-} = (x^{-} \to y^{-}) \to y^{-} = (y^{-} \to x^{-}) \to x^{-}$ (9)

$x \vee y = (x \to y) \to y = (y \to x) \to x$ (10)

And

$x \wedge y = (x^{\sim} \vee y^{\sim})$ (11)

$^{*} = ^{\sim} = ^{-}$ (12)

I also know that the operations are:

x $\wedge y = x ?(x ? y) = (x \to y) ? x$ (13)

x $\vee y = (x^{-} \to y^{-})^{\sim} = (x^{\sim} \to y^{\sim})^{-}$ (14)

Define $\Phi_R(A): A \to R(A)$ through $\Phi_R(A)(x) = x ? x$, for any $x \in A$.
We infer that the $\Phi_R(A)$ It is a group

homomorphism in PBL ,
If $A, A' \in Ob(PBL)$ si
$f \in Hom_{PBL}(A, A')$, then

$R(f)((x?y)^{-}) = (f(x?y))^{-} = (f(x)?f(y))^{-} = (f(y)^{-} \oplus f(x)^{-})^{\sim} = (R(f)(y^{-})) \oplus (R(f)(x^{-}))$
(15)

and

$(R(f)(x^{-})^{\sim}) = (f(x^{-}))^{\sim} = (f(x)^{-})^{\sim} = (R(f)(x))^{\sim}$
. (16)
I know that

$x^{\sim} \oplus y^{\sim} = (y^{\sim} ? x^{\sim})^{\sim} = (y^{\sim} ? x^{\sim})^{\sim}$ (17)

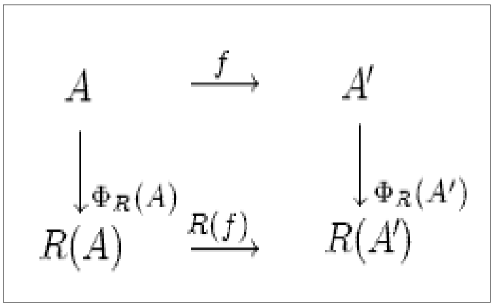
Indeed, if x, y ? A , then

$R(f)(x^{\sim} \oplus y^{\sim}) = R(f)((y^{\sim} \odot x^{\sim})^{\sim}) = (f(x \odot y))^{\sim}$
(18)

$= (f(x) \odot f(y))^{\sim} = f(y)^{\sim} \oplus f(x)^{\sim} = (R(f)(y^{\sim})) \oplus (R(f)(x^{\sim}))$
(19)

And

$R(f)((x^{-})^{\sim}) = (f(x^{-}))^{\sim} = (f(x)^{-})^{\sim} = (R(f)(x))^{\sim}$
. (20)
Thus, we obtain a functor
$R : PBL \to PMV$.
To demonstrate that R is a reflector, consider the diagram:



with $A, A' \in Ob(BL)$.
If $x \in A$, then

$(\Phi_R(A') \circ f)(x) = \Phi_R(A')(f(x))$
$= ((f(x))^{-})^{\sim} = f(x)$ (21)

and

$(R(f) \circ \Phi_R(A))(x) = R(f)(\Phi_R(A)(x))$
$= R(f)((x^{-})^{\sim}) = (f(x^{-}))^{\sim}$ (22)
$= (f(x)^{-})^{\sim} = ((f(x))^{-})^{\sim} = f(x)$

Where

$\Phi_R(A') \circ f = R(f) \circ \Phi_R(A)$ (23)

because the diagram is commutative.
For $x \in A$, define $f'(x^{-}) = f(x^{-}) = f(x)^{-}$
(where $f' = f_{|R(A)}$).
For $x, y \in A$, We have

$f'(x^{-} \oplus y^{-}) = f'((x \oplus y)^{-}) = f'((x \oplus y))^{-} = (f(x)?f(y))^{-} = f(x)^{-} \oplus f(y)^{-}$
(24)

$f((x^{-})^{\sim}) = (f(x)^{-})^{\sim} = f(x) = (f(x^{-}))^{\sim}$
(25)

And

$f'(0) = f'(1^{-}) = f'(1)^{-} = 1^{-} = 0$ (26)

, where f' is a morphism in PMV .
since

$(f' \circ \Phi_R(A))(x) = f'(\Phi_R(A)(x)) = f'((x^{-})^{\sim}) = f(x^{-})^{\sim} = f(x)$
(27)

We infer that the

$f' \circ \Phi_R(A) = f$. (28)

If we again $f'' : R(A) \to M$ It is a group homomorphism in PMV so

$f'' \circ \Phi_R(A) = f$ (29)

then for any

$x \in A$, ($f'' \circ \Phi_R(A))(x^-) = f(x^-)$ (30)

Where

$f''(x^-) = f(x^-) = f'(x^-)$ (31)

So

$f'' = f'$. (32)
Be now $f : A \to A'$ a group homomorphism in P BL and $x, y \in A$ so $R(f)(x^-) = R(f)(y^-)$.
Then $f(x^-) = f(y^-)$, where $x^- = y^-$, ie $R(f)$ It is a group homomorphism in PMV .

CONCLUSIONS

The problem is tackled by addressing the current satau fuzzy systems based on artificial intelligence that is implemented in the economic and industrial machines.

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