HOMOGENIZATION-FUZZY METHOD OF THE DURALUMIN COMPOSITE

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Abstract: From theoretical point of view this work contains a double optimization on "material constants" of the dielectric composite. The first concerns on the homogenization method based on asymptotic development theory and the calculus of variations, and the second on the application of the FAHP (Fuzzy HIERCHY ANALYTIC PROCESS) algorithm. These two optimization methods are not mutually exclusive, rather they complement. If at least one component of the composite is non-homogeneous, the mixing method is applied first and then the FAHP algorithm. If all components of composite are homogeneous then applies directly the FAHP method.

Keywords: composite, asymptotic expansion, variational equations, boundary problems, fuzzy numbers, optimization, material constant, convex combination.

1. INTRODUCTION

Processing through *EDM* (Electrical Discharge Machining) the metals and dielectric alloys is a non-conventional technology process. This aspect sends us in the nanomaterials' area. For this reason, any optimization of the finished quality of the process is important. From a theoretical perspective, the paper contains a double optimization, the first is optimization through modern mathematical method of composite components, here we refer to those composites that have at least one non-homogeneous component. In this way we obtain for each component of the composite material the same constant in each of its points.

The second part of the optimization concerns the determination of a single constant of material for entire composite in each of its points which is optimal from a certain point of view.

This optimality is obtained using fuzzy logic numbers algorithm and analytical hierarchy of the process. Applying this theory, denoted short FAHP, enables the researcher to introduce linguistic information about the properties of various types of composite materials and components influence each other.

Assign fuzzy numbers to each feature is a subjective process and it depends on the designer's expertise. In this paper we proposed a defuzzification method by embedding fuzzy numbers in intervals on the real axis using fuzzy numbers cut which provide us a parameter $\alpha \in (0,1)$ that can be used to obtain material constants leading to results as close as possible to reality. The intervals embedding themselves in real numbers *"crisp"*, through convex combination of the interval ends. In this process is obtained a parameter, $\beta \in (0, 1,)$ which can be used to improving results. Validation of the results can be achieved using these two parameters *α, β*.

FAHP algorithm described in the paper on a case study on the determination of the constant of material for duralumin composite, which has six components made of metals: aluminium, copper, magnesium, manganese, iron and silicium.

It is clear that the six components participating in different made in the global behavior of the composite. Therefore the proposed study finds full application.

The proposed mathematical theory is not specific for dielectric composites only, it can be applied to any type of composite material.

2. THE DURALUMIN COMPOSITE

The attempts to obtain some new performance materials led to the development of a class of products known as composite materials.

Composite materials we can find in nature in various forms and examples of their use are found both birds, animals, insects and humans: the nests of birds are constructed from fibres (straw, branches and others) and clay, the nests of ants or termites are made of fibres (straw, branches, leaves) and clay, the peasant homes were made of adobe (clay and straw) and others. The clay acts as a binder, comprising materials of reinforcement, giving rigidity to the realized body.

Currently, because of their properties, there is not any area where composite materials have no application: electrical, electronics, civil, transport (road, rail, marine, cable, air and space) and others.

Composite materials belong to compound material, which are made of two phases: a continuous one, named matrix, having a low resistance, and a dispersed phase which constitutes the reinforcing material with outstanding resistance. In the category of new materials, who replace metals, given the characteristics and their future prospects, attention ought to be paid to composites, known until recently consolidated plastics. Composite materials are the first materials whose internal structural layout conceived by man, not only in their molecular linkage, but giving them preferential resistance in facilities directions. As a general definition, composite materials are mixtures of two or more different components whose properties complement each other, results a material with superior properties to those specific to each component alone. Thus, these components will work together, the deficiencies of some components being filled by the qualities of others, give the whole property that no component can't be taken separately.

From the technical point of view, the term refers to composite materials have the following properties:

- artificially created by combining different components;
- represent a combination of at least two different materials from chemical point of view, which have a distinct separation surface;
- present properties that any component can't be taken separately.

Major advantage of composites is the possibility of modifying properties and obtain in this way a very large range of materials whose use can be extended to almost all technical fields. In most cases, the composite material comprises a basic material, named matrix, in which is dispersed a supplementary material in the form of particles or fiber, and the main properties that are intended to be obtained in an enhanced form are as follows: breaking strength, strength the wear density, high temperature resistance, superficial hardness, dimensional stability, vibration damping ability.

In accordance with the definitions of composite, materials characterization can take a number of criteria for the classification of these materials as follows:

- By way of dispersion of the phases:

a) Composites with fine scattering:

- Natural composite materials;
- Micro-composite materials;

According to the definition of composites defined as "a material composed of several materials of different natures and compositions that has properties and characteristics" on this topic can define a series of natural composites such as wood, bone, muscle and others.

In micro-composites assemblage, from the class of composites with fine dispersion are materials with structure at the microscopic level are emerging such as metal alloys.

b) Composites with coarse scattering;

- According to the form, size and distribution of the two or more phases in the composite material:

- Continuous fibres in the matrix;
- Short fibres in the matrix;
- Particles dispersed in the matrix;
- Lamellar structure;
- Interpenetrated networks;
- A multi-fibre particles.

Continuous fibres arranged in a composite matrix give rise to what today are called *"high performance composites"*. In case of the short fibres-reinforced composites, results in terms of resistance are not comparable to those of *HPC*, but has the advantage of a less developed manufacturing process.

After structures form components:

- Fibre composite made of fibres included in a matrix;
- Composite lamination - layers of material arranged one above another;
- Composites in the form of particles

3. HOMOGENIZATION BASED ON ASYMPTOTIC DEVELOPMENTS

We assume that the composite material has non-homogeneous components. Let *D*, $D \subset \Re^3$, the space occupied by the composite and $D_0 = D \times \Re^3$. We define an asymptotic development by:

$$u_0(x) + \varepsilon \cdot u_1(x,y) + \varepsilon^2 \cdot u_2(x,y) + ... \qquad (1)$$

where: $u_0: D \to \Re$,

$u_k: D_0 \to \Re$, $u_0 = u_0(x)$, $x \in D$

$u_k = u_k(x, y)$, $x \in D$, $y = \dfrac{1}{\varepsilon} x$.

If, $x = (x_1, x_2, x_3)$ then $y = (\dfrac{x_1}{\varepsilon}, \dfrac{x_2}{\varepsilon}, \dfrac{x_3}{\varepsilon})$ şi $y_k = \dfrac{x_k}{x}$, $k=1\div3$.

Therefore, $u_k(x, y) = u_k^*(x)$.

$$\frac{\partial u_k^*}{\partial x_i} = \frac{\partial u_k}{\partial x_i} + \frac{1}{\varepsilon} \cdot \frac{\partial u_k}{\partial y_i}, \; i=1\div3, \; k=1\div\infty \qquad (2)$$

We suppose further that $\varepsilon > 0$ is a small enough number.

In the event of sufficient smoothness of the coefficients of the (1) series and that it is uniformly convergent to the function *u(x, y)*, that is:

$$u(x,y) = u_0(x) + \sum_{n \geq 1} \varepsilon^n u_n(x,y) \qquad (3)$$

In the equality (3) will construct the homogenization method.

"Material's constants" of the composite are connected to Dirichlet' problems or Dirichlet – Neumann's problem occurring for example translates or thermal field problem. We illustrate this method on homogeneous Dirichlet's problem.

Let $f : D \to \Re$, continuous, and $\left(a_{ij}(x)\right)_{m\times n}$ is the matrix of material constants. If we denote by $u : D \to \Re$, the movement in $x \in D$, then the homogeneous Dirichlet problem is:

$$\begin{cases} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right) = f(x), \quad x \in D \\ x\,|_0 = 0 \\ a_{ij}\cdot\xi_i\cdot\xi_j \geq \gamma\cdot\|\xi\|_{\Re^3}, \quad \xi = (\xi_1,\xi_2,\xi_3) \end{cases} \quad (4)$$

In (4) we used Einstein's mute index notation.
If noted in (4):

$$p_i(x) = \sum_{j=1}^{3} a_{ij}\frac{\partial u}{\partial x_j} \quad (5)$$

and develop in asymptotic series:

$$p_i(x,y) = p_{i,0}(x) + \sum_{n\geq 1}\varepsilon^n\cdot p_{i,n}(x,y) \quad (6)$$

At this point we are making a first approximation identifying in (4) to (6), which leads to:

$$\sum_{i=1}^{3}\frac{\partial p_i(x,y)}{\partial y_i} = 0 \quad (7)$$

$$-\sum_{i=1}^{3}\left(\frac{\partial p_{i,0}}{\partial x_i} + \frac{\partial p_{i,1}}{\partial y_i}\right) = f \quad (8)$$

Equality (7) is called the microscopic equation, and (8) macroscopic equation. The approximation we referred tonappears using integral averaging:

$$\langle g\rangle = \frac{1}{V}\iiint_D g(x,y,z)dxdydz \quad (9)$$

where $V = vol(D)$.
Applying successively (9) in (7) and (8) we are led to homogenized coefficients, that is to the constant coefficients:

$$a_{ij}^* = \langle a_{ij}(x)\rangle + \sum_{i=1}^{3}\sum_{j=1}^{3}\left\langle a_{ij}\frac{\partial w_i}{\partial y_j}\right\rangle \quad (10)$$

The functions w_k, $k=1\div 3$, are solutions of the variational equations:

$$\sum_{i,j=1}^{3}\iiint_D a_{ij}(y)\frac{\partial w_k}{\partial y_j}\cdot\frac{\partial\vartheta}{\partial y_i}dy_1dy_2dy_3 = \sum_{i=1}^{3}\iiint_D a_{ik}(y)\cdot\vartheta(y)dy_1dy_2dy_3 \quad k = 1\div 3 \quad (11)$$

To approximate variational solutions $w_k(y)$ we propose the method of Galerkin considering that each component has been embedded in the cube having the vertex (0,0,0), denoted $\hat{D}$. For $\hat{D}$ we use the six vertexis as nodes and we associate the orthonormatized functions α_α, $k = 1\div 8$, by:

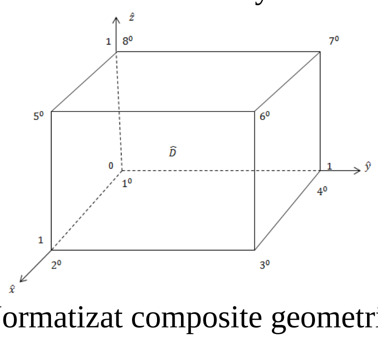


Fig. 1 Normatizat composite geometric image

The orthonormalized functions are given by:

$$\begin{cases} \alpha_1(\hat{x},\hat{y},\hat{z}) = -(\hat{x}-1)(\hat{y}-1)(\hat{z}-1) \\ \alpha_2(\hat{x},\hat{y},\hat{z}) = \hat{x}\cdot(\hat{y}-1)(\hat{z}-1) \\ \alpha_3(\hat{x},\hat{y},\hat{z}) = -\hat{x}\cdot\hat{y}\cdot(\hat{z}-1) \\ \alpha_4(\hat{x},\hat{y},\hat{z}) = (\hat{x}-1)\cdot\hat{y}\cdot(\hat{z}-1) \\ \alpha_5(\hat{x},\hat{y},\hat{z}) = -\hat{x}\cdot(\hat{y}-1)\cdot\hat{z} \\ \alpha_6(\hat{x},\hat{y},\hat{z}) = \hat{x}\cdot\hat{y}\cdot\hat{z} \\ \alpha_7(\hat{x},\hat{y},\hat{z}) = -(\hat{x}-1)\cdot\hat{y}\cdot\hat{z} \\ \alpha_8(\hat{x},\hat{y},\hat{z}) = (\hat{x}-1)(\hat{y}-1)\cdot\hat{z} \end{cases} \quad (12)$$

With this informations, using Galerkin's method we obtain the constants of the material which is its a homogeneous material and which in turn is an approximation of the used material. Let be these constants a_{ij}^*, $i, j = 1 \div m$. Applying the FAHP algorithm we obtain the weights of the best approximation denoted by $w_{i,j}$, $i, j = 1 \div m$. Therefore, the constant for the entire composite material is now:

$$a = \sum_{i=1}^{m}\sum_{j=1}^{m} w_{ij}a_{ij} \quad (13)$$

4. APPLICATION OF FAHP ALGORITHM TO DURALUMIN COMPOSITE

We describe the optimization algorithm steps on this case study. The first step is to define the characteristics of the composite and the fuzzy numbers assignment for each characteristic. In the present case this step is described in table 1.

Table 1. The duralumin characteristics'definition and loading with fuzzy numbers

CR_1	CR_2	CR_3	CR_4	CR_5	CR_6
Al	*Cu*	*Mg*	*Mn*	*Fe*	*Si*
94%	4%	0,5%	0,5%	0,5%	0,5%
$1\tilde{3}$	$\tilde{7}$	$\tilde{3}$	$\tilde{1}$	$\tilde{3}$	$\tilde{1}$

The second phase includes flowcharts hierarchy of characteristics that lead to the definition of *MOF* matrix (matrix of fuzzy ordering). This flowchart is shown in figure 2.

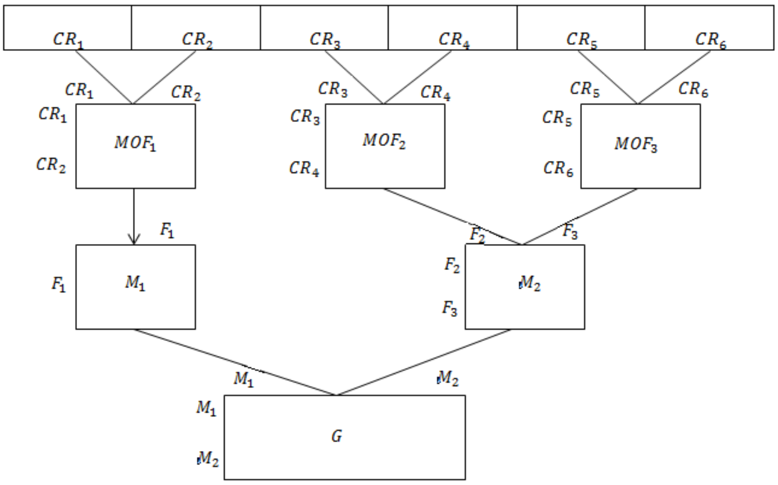


Fig. 2 Flowchart of the characteristics

The third stage involves the loading of *MOF* matrice, *M* and *G* matrixes, with fuzzy numbers ordered by rule:

$$\tilde{a}_{ij} = \begin{cases} \tilde{a}_i, & \tilde{a}_i > \tilde{a}_j \\ \tilde{a}_i^{-1}, & \tilde{a}_i < \tilde{a}_j \\ 1, & \tilde{a}_i = \tilde{a}_j \quad or \quad i = j \end{cases} \qquad (14)$$

Ordering fuzzy numbers can be made by many criteria. In this case we used the arithmetic mean criterion.

After loading *MOF* matrices, *M* and *G*, with fuzzy numbers, these are replaced with *"crisp"* real numbers were embedded. The embedding is made by the rule given in figure 3.

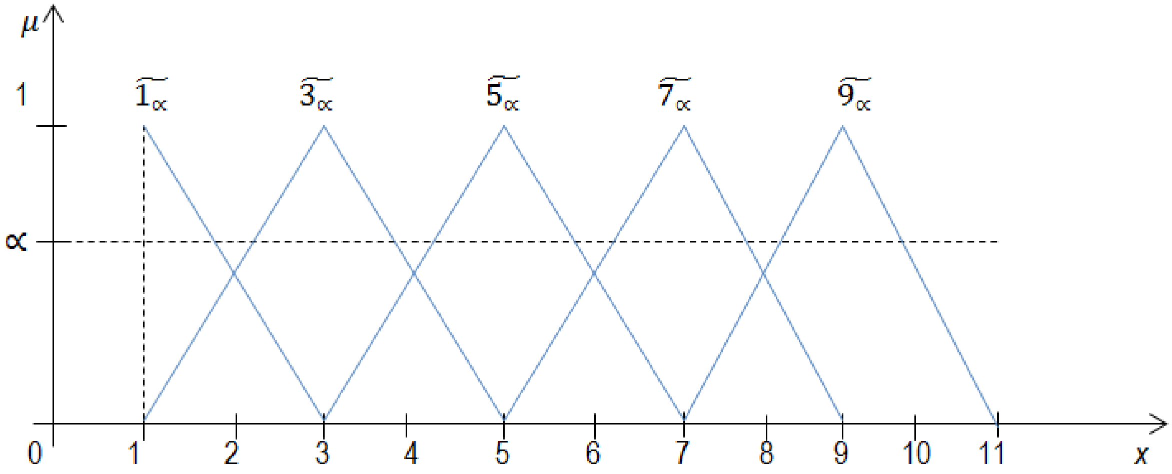


Fig. 3 Fuzzy numbers cut embedded in intervals

The embedded intervals in *"crisp"* real numbers obtained by:

$$[a,b], \quad \mu = \beta(1-\beta)\cdot a + \beta \cdot b, \qquad (15)$$

where, $\beta \in (0,1)$.

The fourth phase includes the determination of eigenvalues and eigenvectors of all matrices embedded in *"crisp"* real numbers and determine local and global weights by the rules:

If A is one of the matrices generated by the flowchart shown in Figure 2, and *"a"* is the largest positive eigenvalue, respectively $(x_1, x_2, ..., x_p)$ is the corresponding eigenvector, then the partial global weights are given by the formula:

$$w_k = \frac{x_k}{x_1 + x_2 + ... + x_p}, \quad k = 1 \div p \qquad (16)$$

Finally the global weights are obtained from the partial weights and for $\alpha = \frac{1}{2}$, $\beta = \frac{3}{4}$. The calculus of the global weights is realized in MATLAB. In this way we have the weights used in (13).

Table 2. Eigenvalues of the matrixes from the flowchart, fig. 2

C_1	C_2	MOF_1	MOF_2	G
2,5742	2,5742	2,4559	2,4998	2,4625

Table 3. Eigenvalues of the matrixes from the flowchart, fig. 2, variant A

C_1	C_2	MOF_1	MOF_2	G
0,4102	0,912	0,9942	0,9648	0,992

Table 4. Eigenvalues of the matrixes from the flowchart, fig. 2, variant B

C_1	C_2	MOF_1	MOF_2	G
0,912	0,4102	0,1072	0,2631	0,1262

Table 5. Partial eigenvalues, variant A

C_1	C_2	MOF_1	MOF_2	G
0,3102405	0,6897594	0,9026693	0,7857317	0,8871400

Table 6. Partial eigenvalues, variant B

C_1	C_2	MOF_1	MOF_2	G
0,6897594	0,3102405	0,0973306	0,2142682	0,1128599

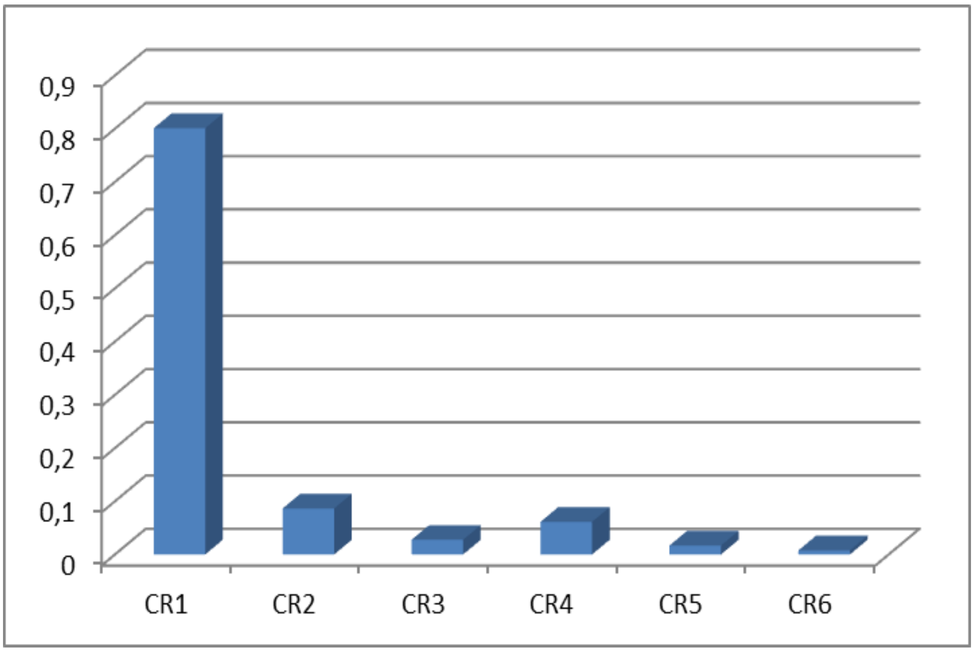
Table 7. Global weights

CR_1	CR_2	CR_3	CR_4	CR_5	CR_6
0,800794111	0,086345935	0,027511398	0,061166249	0,016679975	0,007502331

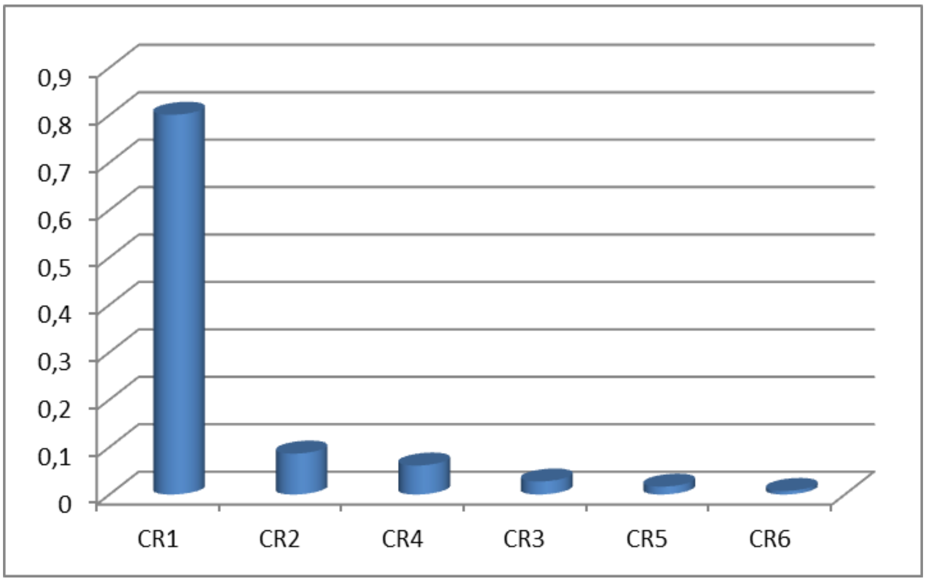
Table 8. Ordered global weights

CR_1	CR_2	CR_4	CR_3	CR_5	CR_6
0,8007	0,0863	0,0611	0,0275	0,0166	0,0075

Histogram 1. Histogram of the global weights



Histogram 2. Histogram of the ordered global weights



5. CONCLUSIONS

The approximation of the composites' constants is a problem in evolution. This statement is asserted by the mathematics of the asymptotic expansions.
If the components of the composite are non-homogeneous, then the first approximation means, in this paper too, the solution of the proposed variational problem.
In this case we have a simple function, which provides us discret constant values. Having these values we can apply *FAHP*, proposed in this paper.

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