A FUZZY MULTI-ATTRIBUTE DECISION MAKING ALGORITM BASED ON INTUITIONISTIC FUZZY SETS

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Abstract: *The proposed method enables decision makers to choose the most important of the criteria in making a decision using degrees of membership and non-membership of the criteria to the fuzzy concept "importance.". The proposed method uses degrees of satisfiability and non-satisfiability of each alternative the truth-membership function and non-truth membership function. The problem ultimately lies in solving a linear programming model. The proposed method differs from previous approaches for multicriteria fuzzy decision-making not only due to the fact that the proposed method use intuitionistic fuzzy set theory rather than fuzzy set theory, but also due to the degree of importance of the criteria are not constant and the calculation is smaller.*

Key words: *Intuitionistic Fuzzy Sets, Decision making, Membership function, Degree of membership, Degree of non-membership, Degrees of indeterminacy.*

1. INTRODUCTION

Considering the unpredictable factors in decision-making, Zadeh [9] introduced the idea of fuzzy set which has a membership function that assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. Atanassov [1] subsequently proposed the concept of intuitionistic fuzzy set (IFS) by bringing a non-membership function together with the membership function of the fuzzy set introduced earlier by Zadeh [9]. Among the various notions of higher-order fuzzy sets, IFS proposed by Atanassov[2] provides a flexible framework to elaborate uncertainty and vagueness. This idea of IFS seems to be resourceful in modeling many real life situations like negotiation processes, psychological investigations, reasoning, medical diagnosis among others .This paper is organized as follows among others This paper is organized as follows. I will first present the definition and properties of intuitionistic fuzzy sets. I will continue to multicriteria decision-making method based on intuitionistic fuzzy sets and the corresponding linear programming model. Finally I will present a numerical example and a short conclusion.

2. INTUITIONISTIC FUZZY SETS

The concept of intuitionistic fuzzy sets was introduced by K.T. Atanassov [1] as a generalization of the notion of a fuzzy set

$$A=\{(x,\mu_A(x))|x\in X\} \qquad (1)$$

An intuitionistic fuzzy set (IFS) A in X is given by

$$A=\{(x,\mu_A(x),\nu_A(x))|x\in X\} \qquad (2)$$

where $\mu_A : X \to [0,1]$ is called degree of membership and $\nu_A : X \to [0,1]$ is called degree of non-membership, with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X \qquad (3)$$

We call degrees of indeterminacy of *x* to *A*, for each *A* in *X* the numbers [2]:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \forall x \in X \qquad (4)$$

Given two IFSs *A* and *B* over an universe of discourse *X*, one can define the following relations:

A ⊂ B iff ∀ x ∈ X, $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$

A = B iff A ⊂ B and B ⊂ A

as well as the following operations [1]:

$$\bar{A} = \{(x, \nu_A(x), \mu_A(x)) | x \in X\}$$

$$A \cap B = \{(x, \mu_{A\cap B}(x), \nu_{A\cap B}(x)) | x \in X\} \qquad (5)$$

where

$$\mu_{A\cap B} = \min\{\mu_A(x), \mu_B(x)\} \qquad (6)$$

and

$$\nu_{A\cap B} = \max\{\nu_A(x), \nu_B(x)\} \qquad (7)$$

$$A \cup B = \{(x, \mu_{A\cup B}(x), \nu_{A\cup B}(x)) | x \in X\} \qquad (8)$$

where

$$\mu_{A\cup B} = \max\{\mu_A(x), \mu_B(x)\} \qquad (9)$$

and $\nu_{A\cup B} = \min\{\nu_A(x), \nu_B(x)\}$ (10)

The degrees of indeterminacy are the numbers $\pi_{ij}(x) = 1 - \mu_{ij}(x) - \nu_{ij}(x)$, $\forall x \in X$. Values $\pi_{ij}(x)$ is the higher a hesitation margin of the decision maker of the

2. MULTICRITERIA FUZZY DECISION MAKING BASED ON INTUITIONISTIC FUZZY SETS

2.1. Presentation of the Problem

In [3] we applies the fuzzy multi-attribute decision making approach to the process of products selection based on quality of product, wich can select the most appropriate one with the highest degree of membership belonging to the positive ideal soltion. I thought four important quality criteria: production cost (shortly cost), time, form, popular. Extending this hypothesis, suppose that for analyzed we have a set of *m* alternative $A = \{A_1, A_2, ..., A_m\}$ from which you must select one, which to expand the number of criteria to *n*: $C = \{C_1, C_2, ..., C_n\}$ .We will use to evaluate each alternative $A_i \in A$ satisfies the criteria $C_j \in C$.

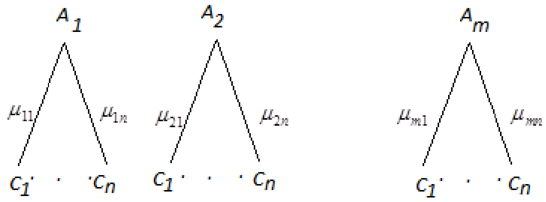

Figure 1.

Assume that μ_{ij} and υ_{ij} are the degrees of membership respectively non-membership of the alternative *Ai* ∈ *A* (Figure 1) which satisfy the criterion *Cj* ∈ *C*, respectively, where $0 \le \mu_{ij}(x) + \nu_{ij}(x) \le 1$, $0 \le \mu_{ij}(x) \le 1$ and $0 \le \upsilon_{ij}(x) \le 1$. In other words, the evaluation of the alternative A_i with respect to the criterion C_j is an intuitionistic fuzzy set:

$$A_i = \{(C_j, \mu_{ij}(x), \nu_{ij}(x)) | x \in X\} \qquad (11)$$

$1 \le i \le m$, $1 \le j \le n$

alternative A_i with respect to the criterion C_j whose intensity is given by μ_{ij} .
With this index aims to increase the quality of the assessment process. In [5], is proposed to assess location in the closed interval $[\mu_{ij}^l, \mu_{ij}^u] = [\mu_{ij}, 1-\upsilon_{ij}]$.
Obviously, $1 \le \mu_{ij}{}^l + \mu_{ij}{}^u \le 2$ for all $A_i \in A$ and $C_j \in C$.
In [5] the authors presented (11) another form as follows for the sake of performing the decision-maker's evaluation more directly
$A_i = \left\{ \left(C_j, [\mu_{ij}^l, \mu_{ij}^u] \right) \middle| x \in X \right\}$.
The authors start from the assumption that there is a decision-maker who wants to choose an alternative which satisfies the criteria C_j, C_k, . . . , C_p or which satisfies the criteria C_s .
But in reality must choose between two sets of criteria. So in this paper we assume that who wants to choose an alternative which satisfies the criteria $C_j, C_k, ..., C_p$ or which satisfies the criteria $C_r, C_t, ..., C_s$.
This decision-maker's requirement is represented by the following expression:

(C_j AND C_k AND . . . AND C_p)OR (C_r AND C_t AND . . . ANDC_s). (12)

Therefore, the degree of evaluation the fact that, which the alternative A_i satisfies and does not satisfy the decision-maker's requirement can be measured by the evaluation function F:

$$
\begin{aligned}
F(A_i) &= \left(C_j \cap C_k \cap ... \cap C_p\right) \cup \left(C_r \cap C_t \cap ... \cap C_s\right) \\
&= [\mu^l_{i,C_j \cap C_k \cap ... \cap C_p}, \mu^u_{i,C_r \cap C_t \cap ... \cap C_s}] \\
&= \left([\mu_{ij}^l, \mu_{ij}^u] \cap [\mu_{ik}^l, \mu_{ik}^u] \cap ... \cap [\mu_{ip}^l, \mu_{ip}^u]\right) \\
&\cup \left([\mu_{ir}^l, \mu_{ir}^u] \cap [\mu_{it}^l, \mu_{it}^u] \cap ... \cap [\mu_{is}^l, \mu_{is}^u]\right) \\
&= \left[\min\left\{\mu_{ij}^l, \mu_{ik}^l, ..., \mu_{ip}^l\right\}, \min\left\{\mu_{ij}^u, \mu_{ik}^u, ..., \mu_{ip}^u\right\}\right] \\
&\cup \left[\min\left\{\mu_{ir}^l, \mu_{it}^l, ..., \mu_{is}^l\right\}, \min\left\{\mu_{ir}^u, \mu_{it}^u, ..., \mu_{is}^u\right\}\right] \\
&= [\max\left\{\min\left\{\mu_{ij}^l, \mu_{ik}^l, ..., \mu_{ip}^l, \mu_{ir}^l, \mu_{it}^l, ..., \mu_{is}^l\right\}\right\}, \\
&\max\left\{\min\left\{\mu_{ij}^u, \mu_{ik}^u, ..., \mu_{ip}^u, \mu_{ir}^u, \mu_{it}^u, ..., \mu_{is}^u\right\}\right\}] \\
&= [\mu_{A_i}^l, \mu_{A_i}^u]
\end{aligned}
$$

$1 \le i \le m$ (see relationships (5), (6), (7), (8), (9), (10)).
Let A=$[\mu_A(x), 1-\nu_A(x)]$, where $\mu_A(x) \in [0,1]$, $\nu_A(x) \in [0,1]$, $0 \le \mu_A(x) + \nu_A(x) \le 1$. The score of A can be evaluated by the score function S shown as $S(A) = \mu_A(x) - \nu_A(x)$, where $S(A) \in [-1, 1]$. Next, [6] define an accuracy function H to evaluate the degree of accuracy of *IFS* as follows: $H(A) = \mu_A(x) + \nu_A(x)$ where $H(A) \in [0, 1]$. $H(A) = 1 - \pi_A(x)$. This results in the fact that as H is higher, the more the degree of accuracy of the *IFS A*. Using the two functions, [4] and then [5] consider a function W, which can measure the degree of alternatives satisfy the decision-maker's requirement.

$$
\begin{aligned}
W\left(E(A_i)\right) &= S\left(E(A_i)\right) + \frac{1 - H\left(E(A_i)\right)}{2} \\
&= \mu_A - \nu_A + \frac{1 - \mu_A - \nu_A}{2} = \frac{\mu_A - 3\nu_A + 1}{2}
\end{aligned}
$$

$W(E(A_i)) \in [-1, 1]$. The larger the value of $W(E(Ai))$, the more the suitability to which the alternative A_i satisfies the decision-maker's requirement.

In [4] the authors presented a weighted technique for handling multicriteria fuzzy decision-making problems, but they assumed that the degree of importance of the criteria entered by the decision-maker are constant, it is hard to do in reality. In [5] the authors extend this hypothesis and assume that each criterion to have a different degree of importance, actually more common in reality. So, in [5] the authors assume that ρ_j and τ_j are the degrees of membership and non-membership of the criteria *Cj* ∈ *C* to the fuzzy concept "importance," respectively, where $0 \le \rho_j \le 1$, $0 \le \tau_j \le 1$ and $0 \le \rho_j + \tau_j \le 1$.

The authors consider intuitionistic indices $\xi_j = 1 - \rho_j - \tau_j$ are such that the larger ξ_j the higher a hesitation margin of decision-maker as to the "importance" of the criteria *Cj* whose intensity is given by ρ_j. These two new quantities are used to calculate the biggest weight (and the smallest one) we can expect in a process leading to a final decision.

During the process the decision-maker can change his evaluating weights in the following way.

It is considered that these values are included in the closed interval $[\omega_j^l, \omega_j^u] = [\rho_j, 1-\tau_j]$. Obviously, $0 \le \omega_j^l \le \omega_j^u \le 1$ for each criterion *Cj* ∈ *C*. In addition, it is assumed that $\sum_{j=1}^{n} \omega_j^l \le 1$ and $\sum_{j=1}^{n} \omega_j^u \ge 1$ in order to find optimal weights satisfying $\omega_j^l \le \omega_j \le \omega_j^u$ and $\sum_{j=1}^{n} \omega_j = 1$.

As mentioned before, I will add in addition to the [4] and [5], a decision-maker who wants to choose an alternative which satisfies the criteria $C_j, C_k, ..., C_p$ or which satisfies the criteria $C_r, C_t, ..., C_s$. This decision-maker's requirement can be represented by (12).

The degree of importance of the criteria $C_j, C_k, ..., C_p$ entered by the decision-maker are $\omega_j, \omega_k, ..., \omega_p$, the degree of importance of the criteria $C_r, C_t, ..., C_s$ entered by the decision-maker are $\omega_r, \omega_t, ..., \omega_s$ where $\omega_j^l \le \omega_j \le \omega_j^u$, $\omega_k^l \le \omega_k \le \omega_k^u, \dots,$ $\omega_p^l \le \omega_p \le \omega_p^u$, $\omega_r^l \le \omega_r \le \omega_r^u$, $\omega_t^l \le \omega_t \le \omega_t^u$, …, $\omega_s^l \le \omega_s \le \omega_s^u$ $\omega_j + \omega_k + ... + \omega_p = 1$, and, $\omega_r + \omega_t + ... + \omega_s = 1$.

The weighting functions *H* and *S* with degrees of importance $\omega_j, \omega_k, ..., \omega_p$. Let

$$T(A_i) = H([\mu_{ij}^l, \mu_{ij}^u]) * \omega_j + H([\mu_{ik}^l, \mu_{ik}^u]) * \omega_k + ... + H([\mu_{ip}^l, \mu_{ip}^u]) * \omega_p \quad (13)$$

$$W(A_i) = S([\mu_{ij}^l, \mu_{ij}^u]) * \omega_j + S([\mu_{ik}^l, \mu_{ik}^u]) * \omega_k + ... + S([\mu_{ip}^l, \mu_{ip}^u]) * \omega_p \quad (14)$$

$$U(A_i) = H([\mu_{ir}^l, \mu_{ir}^u]) * \omega_r + H([\mu_{it}^l, \mu_{it}^u]) * \omega_t + ... + H([\mu_{is}^l, \mu_{is}^u]) * \omega_s \quad (15)$$

$$V(A_i) = S([\mu_{ir}^l, \mu_{ir}^u]) * \omega_r + S([\mu_{it}^l, \mu_{it}^u]) * \omega_t + ... + S([\mu_{is}^l, \mu_{is}^u]) * \omega_s \quad (16)$$

where $T(A_i) \in [0,1], U(A_i) \in [0,1]$, $W(A_i) \in [-1,1]$, $V(A_i) \in [-1,1]$, $1 \le i \le m$.

Then the degree of suitability that the alternative *Ai* satisfies the decision-maker's requirement can be measured by the following function:

$$F(A_i)=\max\left\{W(A_i)+\frac{1-T(A_i)}{2},\mathrm{U}(A_i)+\frac{1-V(A_i)}{2}\right\} \quad (17)$$

$F(A_i)\in[-1,1]$, $1\le i\le m$. The larger the value of *R(Ai)*, the more the suitability to which the alternative *Ai* satisfies the decision-maker's requirement. In Eq. (17), we know that the value of *F(Ai)* bases on the value of $T(A_i), W(A_i), \mathrm{U}(A_i), V(A_i)$.

To obtain weights $\omega_j,\omega_k,...,\omega_p$ corresponding criteria $C_j, \mathrm{C}_k,...,\mathrm{C}_p$ and the weights $\omega_r,\omega_t,...,\omega_s$ corresponding criteria $C_r,\mathrm{C}_t,...,\mathrm{C}_s$ [5] proposes to determine maximum function *F* using linear programming. If it fails to transpose that requirement a linear programming problem then the solution will be obtained using Simplex method. The optimal weights value can be computed via linear programming problem is:

The objective function:

$$\max F(A_i)=\frac{\mu_{ij}^l-3\mu_{ij}^u+1}{2}*\omega_j+\frac{\mu_{ij}^l-3\mu_{ij}^u+1}{2}*\omega_k+ \\ ...+\frac{\mu_{ij}^l-3\mu_{ij}^u+1}{2}*\omega_p+\frac{\mu_{it}^l-3\mu_{it}^u+1}{2}*\omega_t+ \\ +\frac{\mu_{ir}^l-3\mu_{ir}^u+1}{2}*\omega_r+...+\frac{\mu_{is}^l-3\mu_{is}^u+1}{2}*\omega_s \quad (18)$$

restrictions problem:

$$\omega_j+\omega_k+...+\omega_p=1 \quad (19)$$

$$\omega_r+\omega_t+...+\omega_s=1 \quad (20)$$

non-negativity conditions:

$$\omega_j^l\le\omega_j\le\omega_j^u,\ \omega_k^l\le\omega_k\le\omega_k^u,\ldots,\ \omega_p^l\le\omega_p\le\omega_p^u, \quad (21)$$

$$\omega_r^l\le\omega_r\le\omega_r^u,\ \omega_t^l\le\omega_t\le\omega_t^u,\ \ldots,\ \omega_s^l\le\omega_s\le\omega_s^u \quad (22)$$

As I said, this linear programming problem is solved using Simplex method.

3. A NUMERICAL EXAMPLE

Next we will present a decision-making problem, where the characteristics of the alternatives are represented by intuitionistic fuzzy sets. Consider the problem of selection of cars *A= {a_1, a_2, a_3, a_4, a_5}*. The criteria : *C={c_1 (price), c_2 (consumption), c_3(utility) }* are taken into consideration in the selection problem. Using statistical methods, the degrees μ_{ij} of membership and the degrees υ_{ij} of non-membership for the alternative $a_i \in A$ satisfies the criterion $c_j \in C$ can be obtained, respectively. Namely,

$$\left(\mu_{ij}(x),\nu_{ij}(x)\right)_{3\times5}=$$

$$=\begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix}\begin{pmatrix} (0.7,0.2) & (0.8,0.1) & (0.5,0.3) & (0.4,0.45) & (0.6,0.3) \\ (0.5,0.3) & (0.9,0.1) & (0.7,0.1) & (0.5,0.4) & (0.4,0.5) \\ (0.8,0.2) & (0.6,0.3) & (0.7,0.2) & (0.5,0.3) & (0.8,0.1) \end{pmatrix}$$

$$\quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$$

$$\left([\mu_{ij}^l,\mu_{ij}^u]\right)_{3\times5}=$$

$$=\begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix}\begin{pmatrix} (0.7,0.8) & (0.8,0.9) & (0.5,0.7) & (0.4,0.55) & (0.6,0. \\ (0.5,0.7) & (0.9,0.9) & (0.7,0.9) & (0.5,0.6) & (0.4,0. \\ (0.8,0.8) & (0.6,0.7) & (0.7,0.8) & (0.5,0.7) & (0.8,0. \\ a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix}$$

In a similar way, the degrees ρ_j of membership and the degrees τ_j of non-membership for the three criteria $c_j \in C$ to the fuzzy concept "importance" can be obtained, respectively, where j = 1, 2, 3. Namely,

$$\left((\rho_j,\tau_j)\right)_{1\times3}=$$

$$=\left((0.25,0.3)\quad(0.3,0.35)\quad(0.35,0.55)\right)$$

Therefore, criteria weights lie in the closed interval as follows,

$$\left([\omega_{ij}^l,\omega_{ij}^u]\right)_{1\times3}=$$

$$=\left((0.25,0.7)\quad(0.3,0.65)\quad(0.35,0.45)\right)$$

According to Eq. (18)-(22), the linear programming can be obtained:

$$\max F\left(A_1\right)=1.975*\omega_1+1.9*\omega_2+2.55*\omega_3$$

$$\omega_1+\omega_2+\omega_3=1$$
$$0.25\le\omega_1\le 0.7$$
$$0.3\le\omega_1\le 0.65$$
$$0.35\le\omega_1\le 0.45$$

Using Simplex method to solve the above linear programming, its optimal solution can be obtained as ω_1 = 0.25, ω_2 = 0.3, ω_3 = 0.45. Then by applying Eq. (13), we can get

$$F\left(a_1\right)=\left(\frac{1}{2}0.7+\frac{3}{2}*0.8-1\right)0.25$$
$$+\left(\frac{1}{2}*0.5+\frac{3}{2}*0.7\text{-}1\right)0.3+\left(\frac{1}{2}0.8+\frac{3}{2}*0.8-1\right)0.4$$
$$=0.4975$$

$$F\left(a_2\right)=\left(\frac{1}{2}0.8+\frac{3}{2}*0.9-1\right)0.25$$
$$+\left(\frac{1}{2}*0.9+\frac{3}{2}*0.9\text{-}1\right)0.3+\left(\frac{1}{2}0.6+\frac{3}{2}*0.7-1\right)0.4$$
$$=0.585$$

$$F\left(a_3\right)=\left(\frac{1}{2}0.5+\frac{3}{2}*0.7-1\right)0.25$$
$$+\left(\frac{1}{2}*0.7+\frac{3}{2}*0.9\text{-}1\right)0.3+\left(\frac{1}{2}0.7+\frac{3}{2}*0.8-1\right)0.4$$
$$=0.5325$$

$$F\left(a_4\right)=\left(\frac{1}{2}0.4+\frac{3}{2}*0.55-1\right)0.25$$
$$+\left(\frac{1}{2}*0.5+\frac{3}{2}*0.6\text{-}1\right)0.3+\left(\frac{1}{2}0.5+\frac{3}{2}*0.7-1\right)0.4$$
$$=0.1862$$

$$F\left(a_5\right)=\left(\frac{1}{2}0.6+\frac{3}{2}*0.7-1\right)0.25$$
$$+\left(\frac{1}{2}*0.4+\frac{3}{2}*0.5\text{-}1\right)0.3+\left(\frac{1}{2}0.8+\frac{3}{2}*0.9-1\right)($$
$$=0.41$$

Therefore, we can see that the alternative a_2 is the best choice. And the optimal ranking order of the alternatives is given by $a_2\succ a_3\succ a_1\succ a_5\succ a_4$. From the process of calculation, we can see that the method present in this paper is easier than that in [4].

4. CONCLUSION

In this paper it has been analyzed that many decision making approach where the characteristics of the alternatives are represented by intuitionistic fuzzy sets. The difference from other authors remains that set of criteria that must be fulfilled each alternative is of the form (12).

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