PLOTTING TRAJECTORIES ALONG A MOBILE PLAN, AT A MECHANISM WITH GEARS

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Abstract. Usually the coupler curves are plotted by mechanisms along fixed plans. The paper deals with the original idea of plotting coupler curves along mechanisms' mobile plans. A study relative to a particular case was performed. It is concerned with the situation when the tracer point belongs to an element involved in a rotation and the plane on which the curve is plotted is also rotating. An experimental stand was realized. It consists of a train of gears on which various curves were plotted. Afterward the curves were compared to the calculated ones. The method was also applied to the mechanisms with bars.

Kewords: coupler curves along fixed plans, trajectories of mechanisms

1. Introduction

At plan mechanisms, the coupler curves are described by points from certain elements found in plan movement. The trajectory of a point from a coupler is plotted in the mechanism's fix plan (cradle).

The specialty literature contains many papers presenting coupler curves plotted by various mechanisms. Some of them will be mentioned below. [4] presents variants of coupler curves for an articulated patrulater, whose basis is length is varied. The method of geometric distances applied to the plotting of coupler curves is applied in [5]. A chapter from [1] is dedicated to coupler curves. In this way one performs an analysis of these curves' particularities (in the case of an articulated patrulater). The equations are provided and various coupler curves are included as well. [10] presents studies related to the trajectories of mass centers for articulated patrulater mechanism and for the couple-crank mechanism, providing geometrical constructions and examples. For all cases the coupler curves are plotted along fixed planes.

The problems approached by us can be formulated as follows: which is the type of the plotted curve, if a mobile plane is used instead a fix plan? As far as we are concerned, no other authors were concerned before with this problem. We will present our studies related to this matter for a mechanism with gears. Our previous studies [6], [7], [8], [9] reveal how we applied this idea to mechanisms with bars.

2. The studied mechanism

Fig. 1 depicts the synthesis of the trains of gears. This time the tracer point is rotating . The same type of movement is decribed by the mobile plane hosting the

curve. The method can be extended to other mechanisms at which the element containing the tracer point and the element hosting the plot are describing a planar movement.

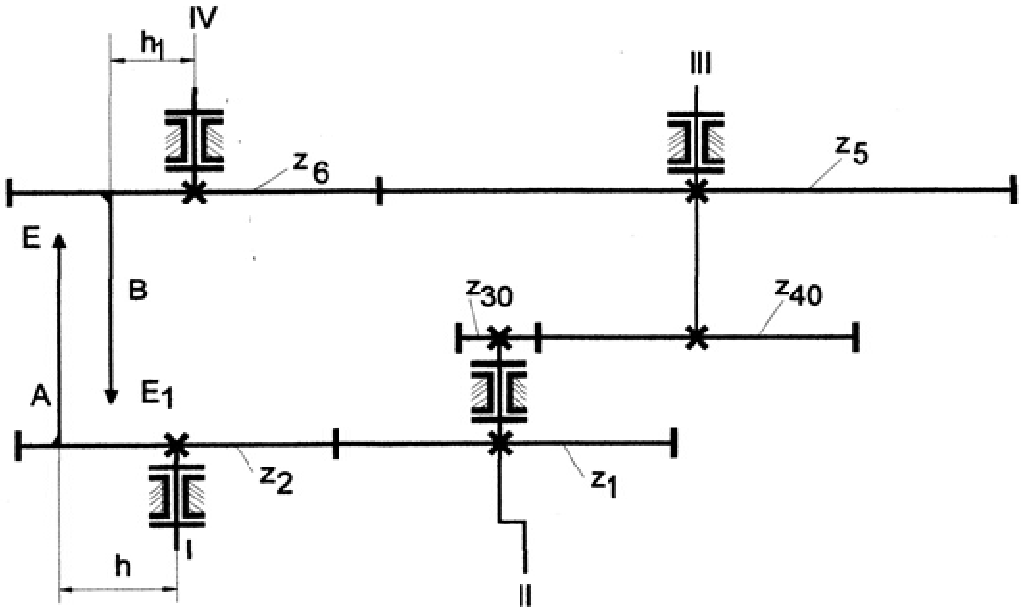


Fig. 1.

The crank II induces rotation to the wheel z_1, which at its turn is moving the wheel z_2, which is hosting the pencil A whose top is denoted by E. The distance h can be varied.

The same crank is transmitting the movement toward the wheels z_{30} si z_{40} and afterward to the wheels z_5 and z_6.

A wooden disk is placed on the wheel , in order to provide support for the paper used to plot the trajectories described by the pencil AE.

As a constructive detail, we should mention that for all toothed wheels, the geometric rotational axes are placed in the same horizontal plan, in order to facilitate the measuring of the distance between axes.

The model was designed such as to allow the realization of several approaches for the linking of a set of toothed wheels. In this sense, the positions of bearings can be varied (using the T channels from the device's table).

In order to obtain a more significant number of variants, we also conceived a distinct working frame, in which the pencil can be placed in the position BE ' whilst the wooden disk with the paper is to be placed on the wheel z_2.

Fig. 2 depicts different views of this device.

Although it is not impressive, the device allowed the obtaining of some satisfying results from the accuracy point of view.

The accuracy could have been improved providing that another system would have been conceived for the curve's plotting (for example a ray on a photographic paper, or the use of a metallic tracer on a deformable support etc.), but our goals were related to the validation of the algorithm and program. They were accomplished with the selected experimental solution.

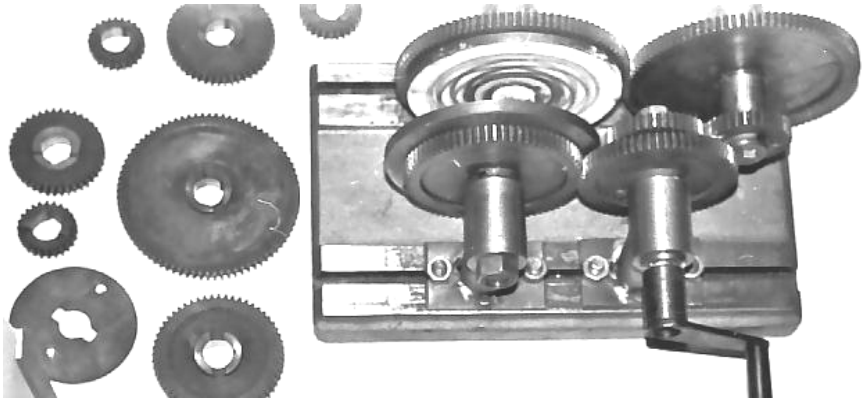


Fig. 2.

3. Mathematical support

The following equations can be written considering Fig. 1:

$$i_{II-I} = \frac{\omega_1}{\omega_2} = -\frac{z_2}{z_1} \tag{1}$$

$$i_{II-III} = \frac{\omega_{30}}{\omega_{40}} = -\frac{z_{40}}{z_{30}} \tag{2}$$

$$i_{III-IV} = \frac{\omega_5}{\omega_6} = -\frac{z_6}{z_5} \tag{3}$$

$$i_{26} = \frac{\omega_2}{\omega_1}.\frac{\omega_{30}}{\omega_{40}}.\frac{\omega_5}{\omega_6} = -\frac{z_1}{z_2}.\left(-\frac{z_{40}}{z_{30}}\right).\left(-\frac{z_6}{z_5}\right) \tag{4}$$

$$\omega_{30} = \omega_1 \tag{5}$$

$$\omega_5 = \omega_{40} \tag{6}$$

$$\frac{\omega_2}{\omega_6} = \frac{-z_1 z_{40} z_6}{z_2 z_{30} z_5} \tag{7}$$

In this way we could obtain the correlation of the movement of the wheels z_2 and z_6 from Fig. 1.
In order to plot the trajectory of the pencil belonging to the wheel no. 2 along the wheel no. 6 we adopted two systems of axis, as one can see in Fig. 3. One of them is fixed (it is passing through O) and the other is mobile, being solidar to the wheel z_6.

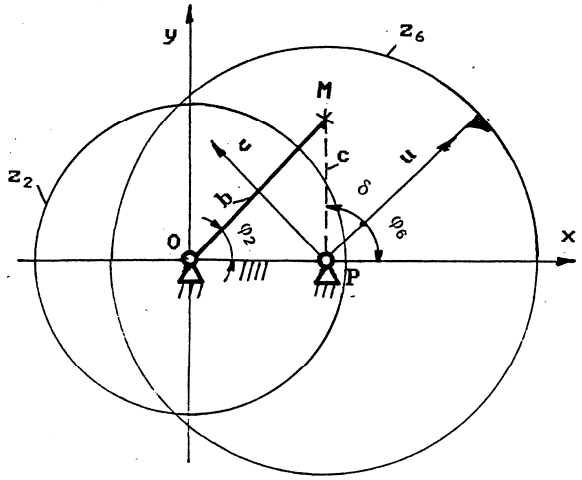


Fig. 3.

Based on the Figs. 3 and 4 one can write the equations:

$$\begin{cases} x_M = b.\cos\varphi_2 \\ y_M = b.\sin\varphi_2 \end{cases} \tag{8}$$

$$\begin{cases} x_M = x_P + c.\cos(\varphi_6 + \delta) \\ y_M = y_P + c.\sin(\varphi_6 + \delta) \end{cases} \tag{9}$$

$$\begin{cases} u_M = c.\cos\delta \\ v_M = c.\sin\delta \end{cases} \tag{10}$$

$$\begin{cases} x_M = x_P + c.(\cos\varphi_6.\cos\delta - \sin\varphi_6.\sin\delta) \\ y_M = y_P + c(\sin\varphi_6.\cos\delta + \cos\varphi_6.\sin\delta) \end{cases} \tag{11}$$

$$\begin{cases} x_M = x_P + u_M.\cos\varphi_6 - v_M.\sin\varphi_6 \\ y_M = y_P + u_M.\sin\varphi_6 + v_M.\cos\varphi_6 \end{cases} \tag{12}$$

$$tg\varphi_6 = \frac{y_M - y_P - v_M.\cos\varphi_6}{x_M - x_P + v_M.\sin\varphi_6} \tag{13}$$

$$\begin{cases} v_M = \dfrac{tg\varphi_6.(x_M - x_P) - y_M + y_P}{-\cos\varphi_6 - tg\varphi_6.\sin\varphi_6} \\ u_M = \dfrac{x_M - x_P + v_M.\sin\varphi_6}{\cos\varphi_6} \end{cases} \tag{14}$$

In order to calculate the distance OP one can use Fig. 4. It comes out that:

$$x_P = OP = r_1 + r_2 + r_{40} + r_{30} - r_5 - r_6 \tag{15}$$

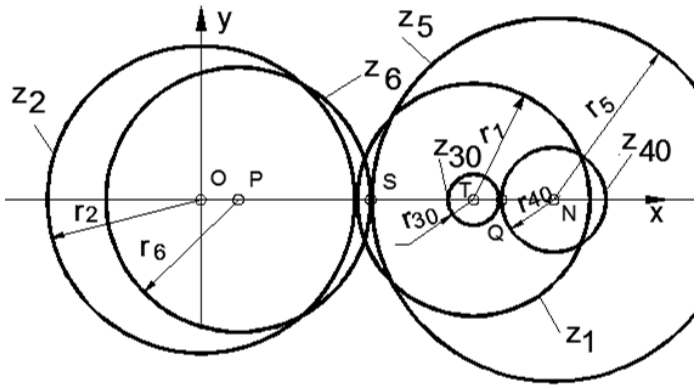


Fig. 4.

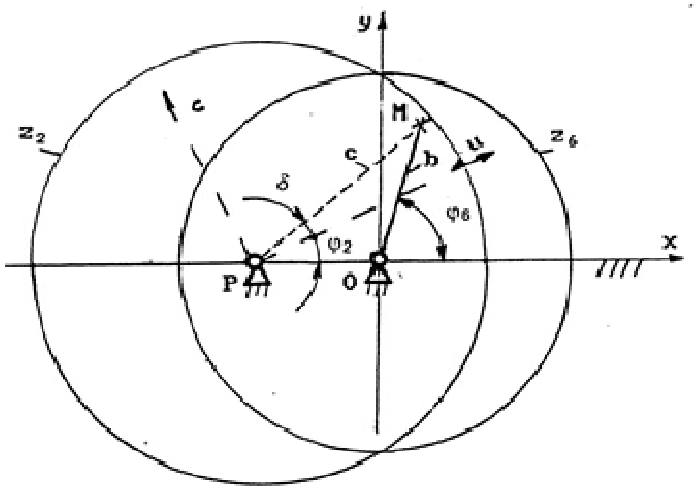


Fig. 5.

For the case when the tracer pencil is placed in the position BE_1 from Fig. 1, the positions of the centers of the wheels z_2 and z_6 will be changed, as depicted by Fig. 5. For this case the following equations can be written:

$$\begin{cases} x_M = b.\cos\varphi_6 = -x_P + c.\left(\cos\varphi_2.\cos\delta - \sin\varphi_2.\sin\delta\right) \\ y_M = b.\sin\varphi_6 = -y_P + c.\left(\sin\varphi_2.\cos\delta + \cos\varphi_2.\sin\delta\right) \end{cases} \tag{16}$$

$$\begin{cases} u_M = c.\cos\delta \\ v_M = c.\sin\delta \end{cases}$$

(17)

$$\begin{cases} x_M = -x_P + u_M.\cos\varphi_2 - v_M.\sin\varphi_2 \\ y_M = -y_P + u_M.\sin\varphi_2 + v_M.\cos\varphi_2 \end{cases} \tag{18}$$

$$tg\varphi_2 = \frac{y_M + y_P - v_M.\cos\varphi_2}{x_M + x_P + v_M.\sin\varphi_2} \tag{19}$$

$$\begin{cases} v_M = \dfrac{tg\varphi_2.(x_M + x_P) - y_M - y_P}{-\cos\varphi_2 - tg\varphi_2.\sin\varphi_2} \\ u_M = \dfrac{x_M + x_P + v_M.\sin\varphi_2}{\cos\varphi_2} \end{cases} \tag{20}$$

4. Conclusions

- A study was made with respect to the generation of couple curves along mobile plans.
- The analyzed cases refer to two elements: the first one is tracing the gear curve, whilst the second one is hosting the curve. Both of them are involved in rotation movements.
- For a specific case we determined the equations of trajectories for different alternatives.
- The method was also extended to elements with plan movements.

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