METHODS OF THE THEORY OF MECHANISMS USEFUL IN GEOMETRICAL LOCI PROBLEMS SOLVING

Liliana LUCA[1], Iulian POPESCU[2]
[3)] University Constantin Brancusi of Targu-Jiu, Faculty of Engineering and Sustainable Development, Romania
[4)] University of Craiova, Faculty of Mechanical, Craiova, Romania

Abstract. It was started from a parallelogram completed with a straight line whose ends move in a circle and a line, being requested the intersection locus of two straight lines. The problem was solved by using the methods of the Theory of mechanisms. The searched geometrical locus is a curve looking like a "banana" that changes its position and dimensions by changing the lengths of some straight lines.

Keywords: geometric locus; kinematics of plane mechanisms; articulated parallelogram mechanism

1. INTRODUCTION

In geometry the geometric locus is defined as the set of points which satisfy a certain condition [1]. By geometric and algebraic methods it is determined this geometric locus, in a general case this one being a curve. Usually it is searched the equation of the geometric locus, following that by graphical methods to be drawn this curve. Often the geometric locus is defined as the intersection of the two families of curves, as a problem solved in Cartesian or polar coordinates. There are not only spatial geometric loci, but also plane ones. In [2] there are given many examples of mathematical curves generated by the mechanisms using the geometric properties of these curves. In [3, 4] there are studied the singularities of the 3-RPR mechanisms, optimizing the minimum lengths for imposed workspaces. In [5] it is achieved the optimizing of the parallel mechanisms of 3-RPR type, studying the singularities in the workspace, establishing the maximum permissible length of the elements.

In [6] it is presented the synthesis of numerous mechanisms generating complex geometric curves. It was started from the analytical relations of these curves and some geometrical considerations, building generating mechanisms and tracing numerous curves for exemplifying them.

2. THE PROPOSED PROBLEM OF GEOMETRIC LOCUS

It is considered the parallelogram ABCD and a point E on CD. From E it is traced the straight line EF, F slipping on the straight line AF which is perpendicular to AD. It is required the geometric locus of the intersection point of the lines BC and EF (Fig. 1)

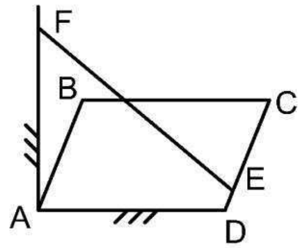

Fig. 1

3. EQUIVALENT MECHANISM SYNTHESIS

It is originally built the articulated parallelogram mechanism ABCD from fig. 2, where: AB = CD; BC = AD.

It is taken a point E on CD and it is marked the straight line EF, with F on the straight line AF. The abscissa of the axis system is DC, and the ordinate line AF is perpendicular to AD. The lengths of the mechanism elements are constant. For moving F point on the straight line AF, F is positioned on a slideway in F and a rotating hitch. Further the point of intersection of the two rods has to be defined precisely so that into G two slideways are connected by a rotating hitch which materializes the tracing point. In this way the issue of geometric locus became an issue of mechanisms.

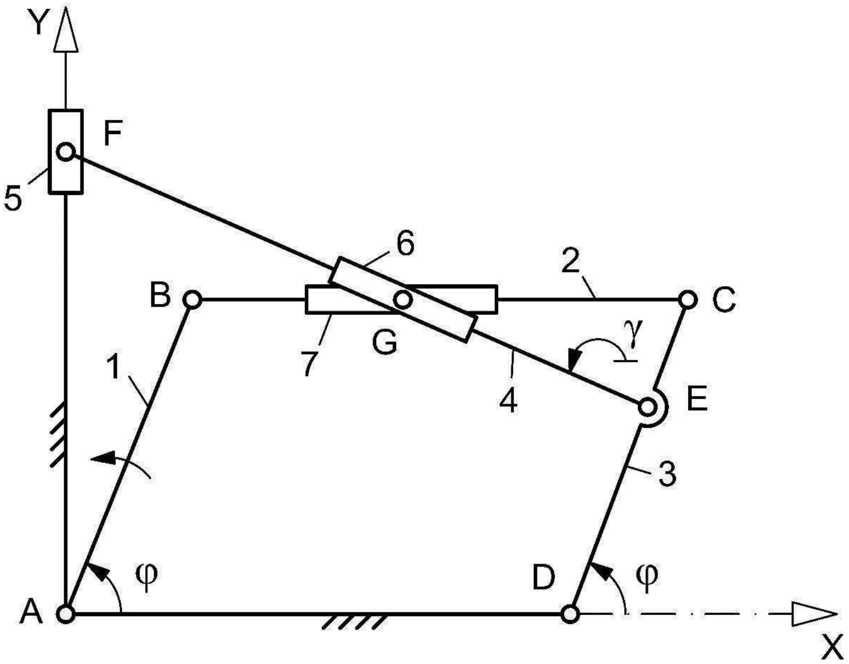

Fig. 2

4. STRUCTURAL AND KINEMATIC ANALYSIS OF THE MECHANISM

The structural diagram of the mechanism of fig. 2 is given in fig. 3 where it is also shown the decomposition in kinematic groups. It results that the mechanism has a rotation driving element with three dyads, being of type: RRP-R-PRP-RRR.

It uses the contours method applied on the two drivelines. For the chain ABCD it is known that AB and CD elements remain parallel, as BC and AD. so the reciprocating rod BC performs circular translations, her points describing circles. Based on fig. 2 there are written the following relationships to outline EF and also the specific relations of the dyad PRP:

$X_F = X_E + EF\cos\gamma = 0$

$Y_F = Y_E + EF\sin\gamma$

$X_G = AB\cos\varphi + BG\cos\alpha = X_E + EG\cos\gamma$

$Y_G = AB\sin\varphi + BG\sin\alpha = Y_E + EG\sin\gamma$

With relations (1) and (2) it is solved the contour kinematics of EF.There are known the lengths of the elements and it is calculated the angle γ. From (3) and (4) result the tracer point coordinates, G, calculating initial the variable lengths BG and EG.

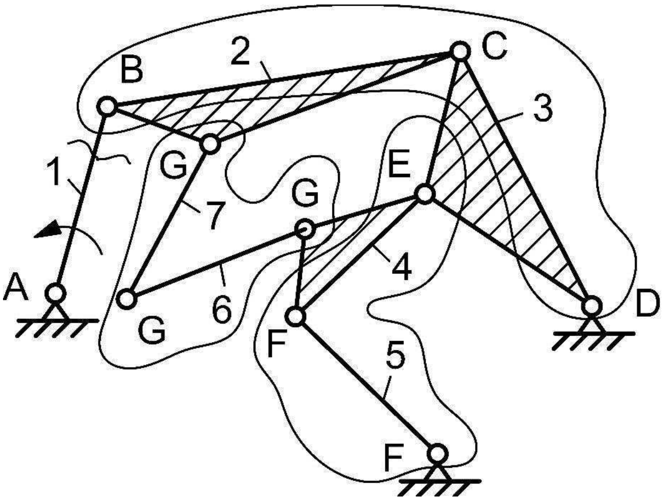

Fig. 3

5. OBTAINED RESULTS

There were taken as initial data the following values of constant length:
AB = 22, BC = 80: DC = AB: XD = BC: DC = AB: AD = BC: DE = DC / 2: EF = 95.

In fig. 4 it is shown the kinematic chain ABCD in a position, and also the trajectory of the center of the reciprocating rod BC, which is a circle.

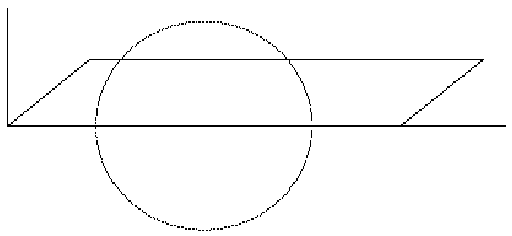

Fig. 4

The successive positions of this chain are given in FIG. 5.

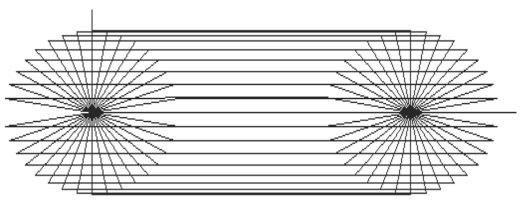

Fig. 5

The successive positions of the entire mechanism can be seen in Fig. 6.

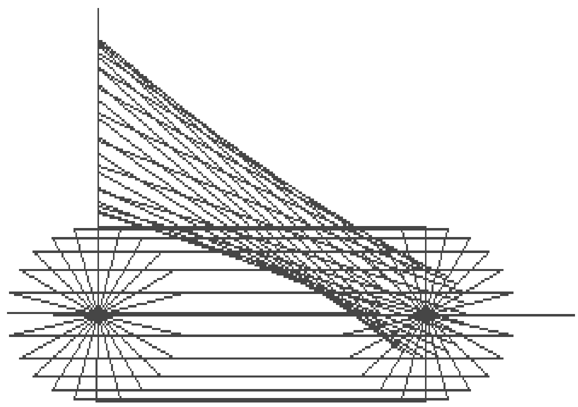

Fig. 6

In Fig. 7 it is shown the mechanism for a position and the trajectory of the centre of the reciprocating rod BC (a circle), the trajectory of the centerof the reciprocating rod EF (a kind of ellipse curve), the geometric locus of the intersection point of the straight lines BC and EF (a curve like a "banana).

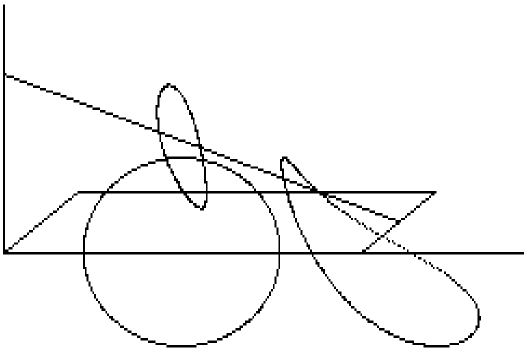

Fig. 7

The variations of the coordinates of the intersection point G (the searched geometric locus) are given in fig. 8.

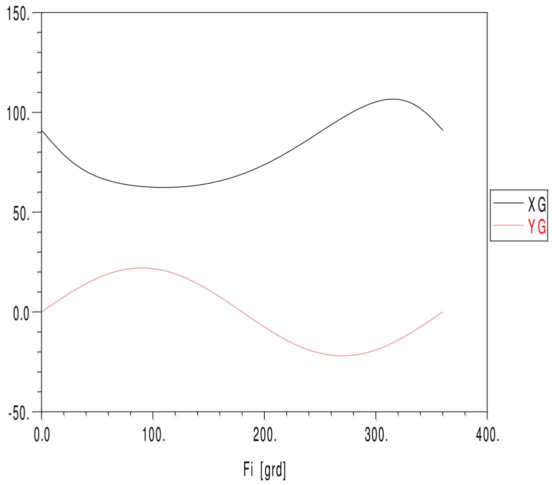

Fig. 8

YG's curve is symmetrical. Both curves have fine fluctuating lines without jumps.

6. GEOMETRIC LOCI FOR OTHER DIMENSIONS OF THE MECHANISM

There were determined also the geometric loci for other cases, in which some dimensions of the mechanism are changed, resulting in the curves in Fig. 9 (EF = 105), fig. 10 (EF = 110) fig. 11 (EF = 115), fig. 12 (EF = 120) fig. 13 (EF = 130).

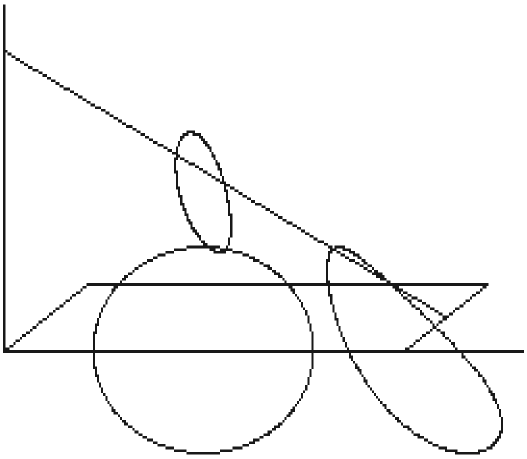

Fig. 9

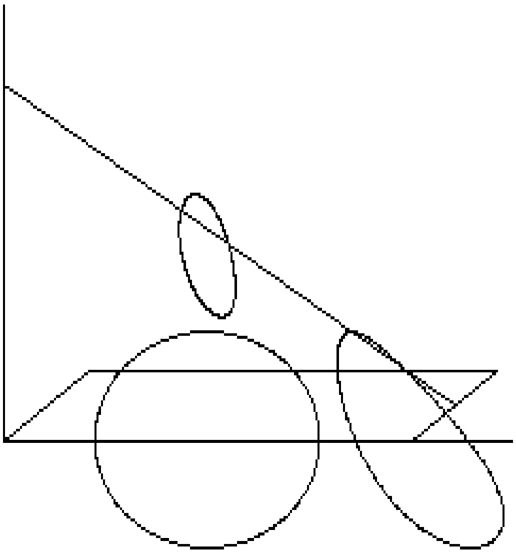

Fig. 10

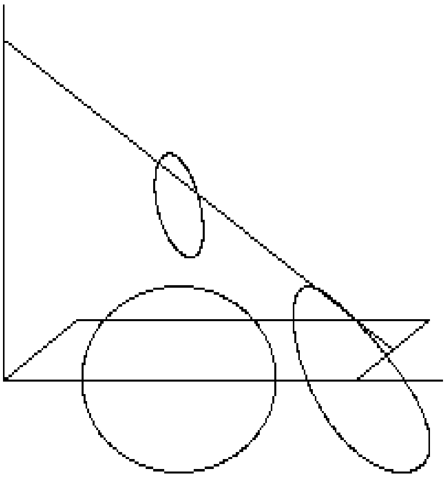

Fig. 11

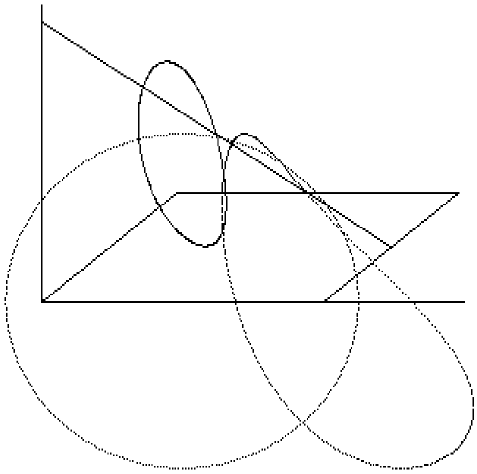

Fig. 12

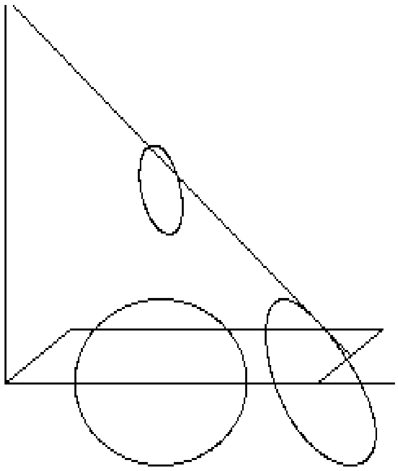

Fig. 13

It is found that the same types of curves result, but with other dimensions and otherwise positioned.

The resulting algebraic system using the contours method has two solutions, thereby resulting two positions of the mechanism for a given position of the driving element. These are the curves resulted for the second position: fig. 14 (EF = 100), fig. 15 (EF = 115).

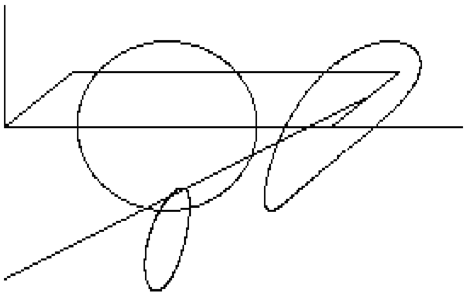

Fig. 14

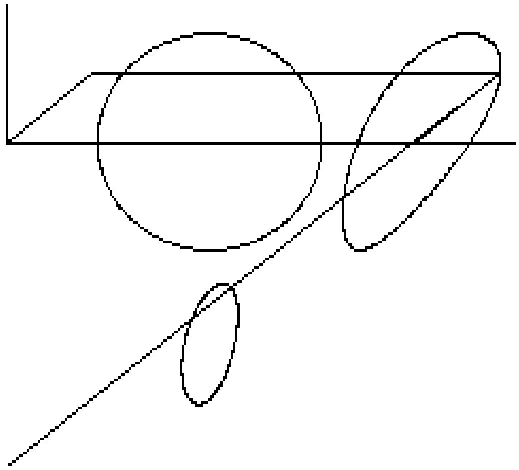

Fig. 15

The curves are similar to those from the first position but positioned below the abscissa.

There have been tried other values, too; Thus, for EF = 80, there were obtained the curves of fig. 16 which are incomplete and crossed.

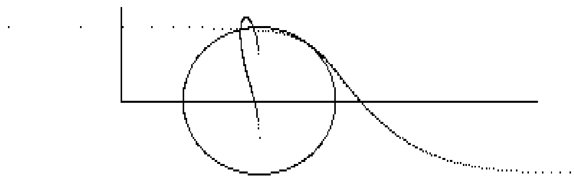

Fig. 16

When the AB=80 and EF = 100, there are obtained the curves of fig. 17, and incomplete intersection.

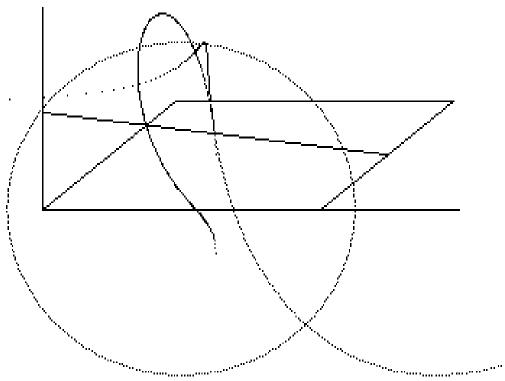

Fig. 17

For AB = 80 and EF = 75 resulted the curves in fig. 18 and the diagram of fig. 19, ascertaining that the movement is locked in a cycle subinterval.

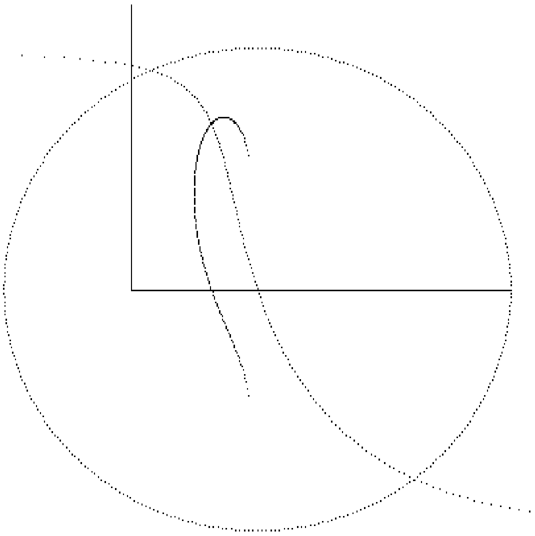

Fig. 18

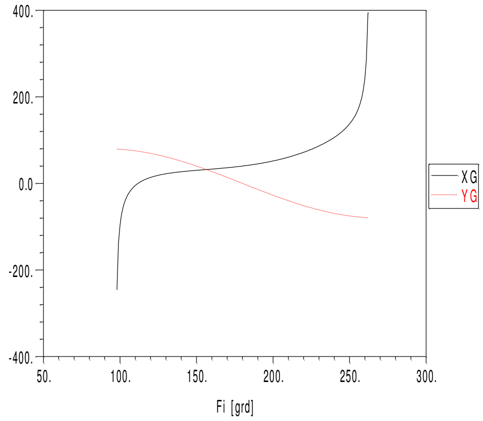

Fig. 19

Referring to fig. 19 it is revealed that there xG tends to infinity xG at ends**.**

7. CONCLUSIONS

It was started from a question of geometric locus that was resolved by the methods of the Theory mechanisms. It was found the equivalent mechanism and studied this structurally and cinematically. There were repesented the trajectories of the centers of the connecting rods and their point of intersection, that is, the searched geometric locus. It was found that this geometric locus is a curve like a "banana". For other sizes there are obtained the same types of curves, but with other dimensions and other positions.

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