

STUDY REGARDING THE DETERMINING OF THE NATURAL FREQUENCIES AND MODAL SHAPES OF THE COLUMN TYPE STRUCTURES WITH ADDITIONAL MASS

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ABSTRACT: In this paper there is presented a study regarding the analysis of the dynamic behavior of column type structures with additional mass. By this analysis, we have followed the determining of the natural frequencies and modal shapes via the analytical computing and the simulating with the finite element method (FEM) in the ANSYS program. The results obtained regarding the computed natural frequencies are highlighted in tables and in graphics, whereas concerning the modal shapes for the 9 modes that were analytically and through FEM determined, these are drafted with the help of the MathCad, respectively ANSYS programs.

KEY WORDS: Natural frequencies, modal shapes, column type structures, additional mass.

1. INTRODUCTION

In literature, elastic structures are defined as solids [1], with various studies on their analysis as well [2] - [9]: column or beam type structures, spring, pipes, plates etc.

For the calculation of natural frequencies on structures without additional mass added to the free end of a column there are various mathematical relations [10].

In the case of columns with additional or extra mass, appear additional charges both axial and transverse [11].

According to the citation [12] it's presents a discreet model with two degrees of freedom and additional mass at the free end for calculating the natural frequencies.

This discreet model with two degrees of freedom, involves a cranked beam (Figure 1), which is composed of a vertical element (called column) and a horizontal element (called beam), both of length L , having at the free a concentrated mass noted with m .

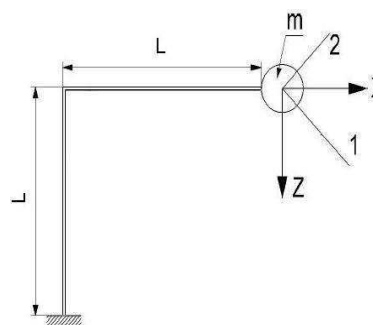


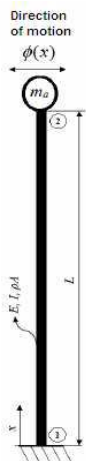
Figure 1. Cranked beam with two degrees of freedom and concentrated mass

In this paper, interested only the column type structures at the end of which it will be add an additional mass, where the important aspect in this study is the behavior of the column regarding the determination of natural frequencies and modal shapes.

2. DESCRIBING THE COLUMN TYPE STRUCTURES WITH ADDITIONAL MASS

The structure of the column type, which will be analyzed, is shown in Figure

2, with the following boundary conditions in the two bearings [10]:



Boundary conditions:

1. $\phi = 0; \phi' = 0$
(the arrow and the rotation are null)
2. $\phi'' = 0;$
 $\phi''' = \frac{m_a}{m} \alpha \cdot L$
(The bending moment and the cutting force are null)

Figure 2. Column with additional mass

This considered column has a constant section $b \cdot h$ (where: $b = 0,005$ m, and $h = 0,05$ m) with the length $L_s = 0,6$ m.

The column moment of inertia is $I = 5.208 \cdot 10^{-10} \text{ m}^4$, area is $A = 2.5 \cdot 10^{-4} \text{ m}^2$, and the mass of the column is $m_b = 2,1391$ kg, corresponding to a material density of $\rho = 7850 \text{ kg/m}^3$.

It's want to analyze this column with additional or extra mass as a segment mass with the length of 0,25 m, from several possible variants.

In the following table is randomly considered, several possible variants regarding length, mass and equivalent density.

Table 1. Some possible loading variants

Case	l^* [mm]	m [kg]	Equivalent density ρ^* [kg/m ³]
1	10	0.099	78500
2	150	0.297	235500
3	250	0.495	392500
4	300	0.594	471000
5	350	0.693	549500

The equivalent density has been computed with the relation [10]:

$$\rho^* = \frac{h \cdot \rho}{l^*} \quad (1)$$

where: ρ^* - is the equivalent density; h - height of the column; l^* - equivalent length; length: $l = (10, 150, 250, 300, 350 \text{ mm})$, and ρ is the chosen Steel density having the value of 7.850 kg/m^3 .

Further, the proper study will address only the case 3 of Table 1, where the mechanical properties for the column material are shown in Table 2.

Table 2. Mechanic properties for the column material

Yield Strength [MPa]	Tensile Strength [MPa]	Elastic modulus [MPa]	Poisson Ratio [-]
250	460	2×10^5	0.3

3. NATURAL FREQUENCIES AND MODAL SHAPES

Next, it will examine the case where the segment length is 250 mm, i.e. 50 times greater, by two methods: analytical method and the FEM method.

The simplified relation for the calculation of αL [10] that is the root of the equation used to represent the natural frequencies is as follows:

$$\frac{\cos(\alpha L) \cosh(\alpha L) + 1}{\sin(\alpha L) \cosh(\alpha L) - \cos(\alpha L) \sinh(\alpha L)} = \frac{m_b}{m_a} (\alpha L) \quad (2)$$

The modal shape is determined by replacing the value of $\alpha_n L$ [10] in the following relation:

$$\phi_n(x) = \left\{ \begin{array}{l} \frac{\cos(\alpha_n L) + \cosh(\alpha_n L)}{\sin(\alpha_n L) + \sinh(\alpha_n L)} \\ \left[\sin\left(\alpha_n \frac{x}{L}\right) - \sinh(\alpha_n \cdot x) \right] - \cos(\alpha_n \cdot x) + \cosh(\alpha_n \cdot x) \end{array} \right\} \quad (3)$$

In the simulation, the calculation of natural frequencies was determined using the static analysis module and the modal analysis of ANSYS program.

After opening the Workbench in ANSYS, it's setting the parameters (Fig. 3), then it will be a model created to analyze and it's run the Setup for the modal analysis [12].

The selected column has the base fixed, but the upper end is free. The

reference temperature at which the numerical simulations were carried out was 22 °C.

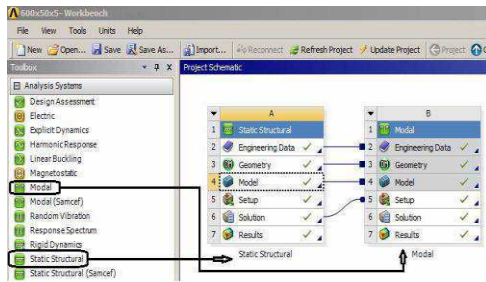


Figure 3. Choosing the modules in the ANSYS program

The mesh of the column was done with cubic elements, with finite elements dimensions average size equal to 2 mm, resulting 116364 nodes and 22500 elements, leading to a corresponding accuracy [9].

Subsequently, the selected additional mass was 50 times higher than the equivalent of a segment of 10 mm length located at the free end (Figure 4).

Therefore, in the simulation, the density has increased the amount of 7850 to 392500 kg/m³ for this segment.

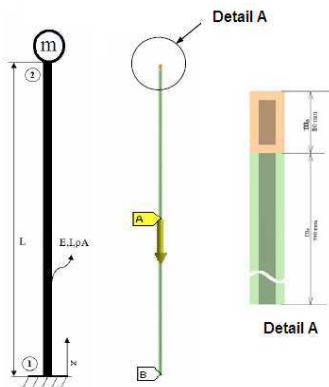


Figure 4. The column with the additional mass placed at the upper end

Finally, determining the natural frequencies was achieved for nine separate cases (9 modes).

The computed frequencies by the analytical method and by the finite element method (FEM) are given in Table 3, which is reflected in the relative deviation calculated by the formula [10]:

$$\text{Deviation}_n = \frac{f_{n \text{ ANALYTIC}} - f_{n \text{ AFEM}}}{f_{n \text{ ANALYTIC}}} \cdot 100 \text{ [\%]} \quad (4)$$

Table 3. Analytically computed frequencies and through FEM respectively the deviation

Mode	Analytic f_n [Hz]	FEM f_{FEMn} [Hz]	Relative deviation [%]
1	5.179	5.461	-5.435
2	52.534	53.859	-2.521
3	164.161	167.742	-2.181
4	339.075	345.774	-1.975
5	577.536	573.616	0.678
6	879.559	892.969	-1.524
7	1245.151	1259.921	-1.186
8	1674.316	1685.880	-0.69
9	2167.055	2150.915	0.744

The frequencies analytically obtained as well as through simulation are graphically presented in Figure 5.

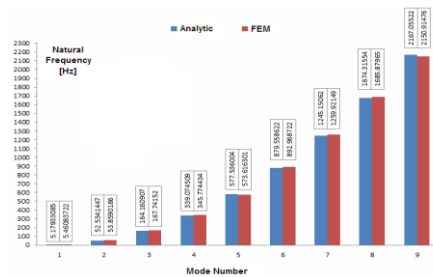
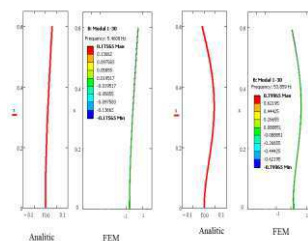
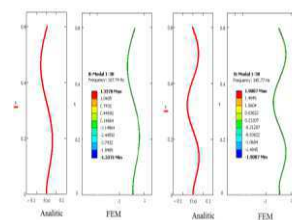


Figure 5. Comparison between natural frequencies analytically and through FEM

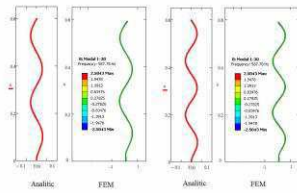
Further on, it's wish to present the modal shapes by the 2 methods in Figure 6 for the 9 chosen modes.



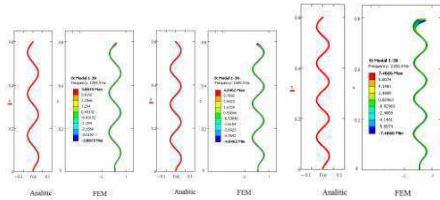
Modal shape for modes 1 and 2



Modal shape for modes 3 and 4



Modal shape for modes 5 and 6



Modal shape for modes 7, 8 and 9

Figure 6. Modal shape for modes 1÷9 analytically and through FEM determined

4. CONCLUSION

The modal shape for the 9 modes to form their natural frequencies described above, was determined by substituting the value of α_n in relation 3. It notes that the maximum and minimum values were generally smaller than the deviations of $\pm 3\%$, only in mode 1 are higher.

Because the Euler Bernoulli model disregards the shear force that can become important if at the free end appears a additional mass, the coefficients determined from the characteristic equation that allows calculation with sufficient accuracy to the natural frequencies and plotting mode shapes as represented in the figures.

In all cases above the 9 modal shapes are alike, and hence there has been found some analytical relations for the column with additional mass that can be used to detect columns defects with additional mass located at the free end.

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