

## DEFUZZIFICATION OF TIFNs

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**ABSTRACT :** In the present paper we complete the above fuzzy assessment approach by presenting a defuzzification method of TIFNs based on the Center of Gravity (COG) technique, which enables the required comparison.

**KEY WORDS :** Intuitionistic Fuzzy Numbers (IFNs), Center of Gravity (COG) defuzzification technique, Triangular (TIFNs) IFNs.

### 1. INTRODUCTION

The intuitionistic fuzzy number is a very important concept of the intuitionistic fuzzy modeling and optimization, which may be used to quantify an ill-known quantity in some real management situations [5, 6]. In this section, we will firstly introduce the concept of an intuitionistic fuzzy number. We will discuss special forms of intuitionistic fuzzy numbers such as triangular intuitionistic fuzzy numbers [4,7,11]. The intuitionistic fuzzy numbers (IFNs) on a universe  $X$  was introduced by K. Atanassov in 1983 as a generalization of FNs. Therefore, the concept of the triangular intuitionistic fuzzy number is a generalization of that of the triangular fuzzy number.

The additional parameters  $p_a \in [0,1]$  and  $q_a \in [0,1]$  are introduced to reflect the confidence level and nonconfidence level of the triangular intuitionistic fuzzy number  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$ , respectively. Thus, the triangular intuitionistic fuzzy number may express more uncertainty information than the triangular fuzzy number.

### 2. DEFINITIONS

The definitions are consistent with [4]. Let  $p_a \in [0,1]$  and  $q_a \in [0,1]$  be any real numbers, which satisfy the condition:  $0 \leq p_a + q_a \leq 1$ . An intuitionistic fuzzy number  $a$  is a special intuitionistic fuzzy set on the set of real numbers  $R$ , whose membership function  $\mu_a : R \rightarrow [0, p_a]$  and nonmembership function  $\nu_a : R \rightarrow [q_a, 1]$  satisfy the four conditions (1)–(4) as follows:

- (1) There exist at least two real numbers  $x$  and  $y$  such that  $\mu_a(x) = p_a$ ,  $\nu_a(x) = q_a$
- (2)  $\mu_a$  Is quasi concave and upper semicontinuous on  $R$ ,
- (3)  $\nu_a$  Is quasi convex and lower semicontinuous on  $R$  and
- (4) The support of  $a$  (i.e.,  $a_{(0,1)} = \{x | \mu_a(x) \geq 0, \nu_a(x) \leq 1, x \in R\}$ ) is compact.

From the above definition of the intuitionistic fuzzy number, we can easily construct a general intuitionistic fuzzy number

$a = \langle (\underline{a}_1, a_{1l}, a_{1r}, \bar{a}_1), p_a; (\underline{a}_2, a_{2l}, a_{2r}, \bar{a}_2), q_a \rangle$ , whose membership and nonmembership functions are given as follows:

$$\mu_a(x) = \begin{cases} 0, & x < \underline{a}_1 \\ f_{\mu l}(x), & \underline{a}_1 \leq x < a_{1l} \\ p_a, & a_{1l} \leq x \leq a_{1r} \\ f_{\mu r}(x), & a_{1r} < x \leq \bar{a}_1 \\ 0, & x > \bar{a}_1 \end{cases}$$

$$\nu_a(x) = \begin{cases} 1, & x < \underline{a}_2 \\ f_{\nu l}(x), & \underline{a}_2 \leq x < a_{2l} \\ q_a, & a_{2l} \leq x \leq a_{2r} \\ f_{\nu r}(x), & a_{2r} < x \leq \bar{a}_2 \\ 1, & x > \bar{a}_2 \end{cases}$$

respectively, where the functions

$$f_{\mu l} : [\underline{a}_1, a_{1l}] \rightarrow [0, p_a]$$

$$f_{\mu r} : [a_{1r}, \bar{a}_1] \rightarrow [0, p_a]$$

$$f_{\nu r} : [a_{2r}, \bar{a}_2] \rightarrow [q_a, 1]$$

$$f_{\nu l} : [\underline{a}_2, a_{2l}] \rightarrow [q_a, 1]$$

$f_{\mu l}, f_{\nu r}$  are continuous and nondecreasing,

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and satisfy the conditions:  $f_{\mu l}(\underline{a}_1) = 0$ ,

$$f_{\mu l}(a_{1l}) = p_a, \quad f_{\nu r}(a_{2r}) = q_a, \quad f_{\nu r}(\bar{a}_2) = 1,$$

$$f_{\mu r}(a_{1r}) = p_a, \quad f_{\mu r}(\bar{a}_1) = 0, \quad f_{\nu l}(\underline{a}_2) = 1,$$

$$f_{\nu l}(a_{2l}) = q_a$$

$[a_{1l}, a_{1r}]$ ,  $\underline{a}_1$ ,  $\bar{a}_1$  are called the mean interval and the lower and upper limits of the general intuitionistic fuzzy number  $a$  for the membership function, respectively.

$[a_{2l}, a_{2r}]$ ,  $\underline{a}_2$ ,  $\bar{a}_2$  are called the mean interval and the lower and upper limits of the general intuitionistic fuzzy number  $a$  for the nonmembership function, respectively.  $p_a$  and  $q_a$  are called the maximum membership degree and minimum nonmembership degree, respectively.

For some specific values of the parameters  $\underline{a}_1, a_{1l}, a_{1r}, \bar{a}_1, \underline{a}_2, a_{2l}, a_{2r}, \bar{a}_2$  we can further construct some particular forms of intuitionistic fuzzy numbers such as triangular intuitionistic fuzzy numbers.

A triangular intuitionistic fuzzy number  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$  is a special intuitionistic fuzzy set on the real number set  $R$ , whose membership and nonmembership functions are defined as follows:

$$\mu_a(x) = \begin{cases} 0, & x < \underline{a} \\ (x - \underline{a}) p_a / (a - \underline{a}), & \underline{a} \leq x < a \\ p_a, & x = a \\ (\bar{a} - x) p_a / (\bar{a} - a), & a < x \leq \bar{a} \\ 0, & x > \bar{a} \end{cases}$$

$$\nu_a(x) = \begin{cases} 1, & x < \underline{a}_2 \\ (a - x + q_a(x - a)) / (a - \underline{a}), & \underline{a} \leq x < a \\ q_a, & x = a \\ (x - a + q_a(\bar{a} - x)) / (\bar{a} - a), & a < x \leq \bar{a} \\ 1, & x > \bar{a} \end{cases}$$

We call degrees of indeterminacy of an element  $x$  in the triangular intuitionistic fuzzy number  $a$ . It is the degree of indeterminacy membership of the element  $x$  to  $a$ .

$$\pi_a(x) = 1 - \mu_a(x) - \nu_a(x)$$

It is a hesitancy degree of  $x$  to  $a$  [1-4]. It is obvious that

$$0 \leq \pi_a(x) \leq 1, \forall x \in X$$

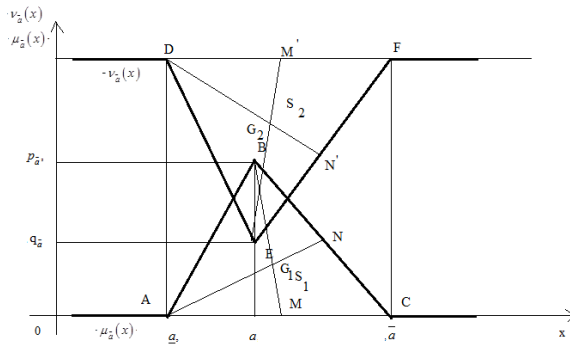


Figure 1[4] A triangular intuitionistic fuzzy number  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$

Let  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$  and  $\tilde{b} = \langle (\underline{b}, b, \bar{b}); p_b, q_b \rangle$  be any triangular intuitionistic fuzzy numbers and  $k \neq 0$  be any real number. The algebraic operations are stipulated as follows:

- The sum  
 $a + \tilde{b} = \langle (\underline{a} + \underline{b}, a + b, \bar{a} + \bar{b}); p_a \wedge p_b, q_a \vee q_b \rangle$
- The difference  
 $a - \tilde{b} = \langle (\underline{a} - \underline{b}, a - b, \bar{a} - \bar{b}); p_a \wedge p_b, q_a \vee q_b \rangle$
- $k + a = \langle (k + \underline{a}, k + a, k + \bar{a}); p_a, q_a \rangle, k > 0$   
 $k + a = \langle (k + \bar{a}, k + a, k + \underline{a}); p_a, q_a \rangle, k < 0$
- $k \cdot a = \langle (k\underline{a}, ka, k\bar{a}); p_a, q_a \rangle, k > 0$   
 $k \cdot a = \langle (k\bar{a}, ka, k\underline{a}); p_a, q_a \rangle, k < 0$

Let  $a_i = \langle (\underline{a}_i, a_i, \bar{a}_i); p_{a_i}, q_{a_i} \rangle, i = 1, \dots, n$

be TIFNs, where  $n$  is a nonnegative integer,  $n \geq 2$ . Then we define the mean value of the

$a_i$ 's to be the:  $a = \frac{1}{n}(a_1 + a_2 + \dots + a_n)$ .

### 3. DEFUZZIFICATION OF TIFNs

In this section we shall use the COG technique for defuzzifying a given TIFN.

Starting from [8], We extended the proposition 1 for ITNF.

**Proposition** The coordinates  $(X, Y)$  of the COG of the graph of the TIFN  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$  are calculated by the

formulas 
$$X = \frac{\underline{a} + a + \bar{a}}{3},$$

$$Y = \frac{p_a^2 - q_a^2 - q_a + 2}{3(p_a + 1 - q_a)}$$

**Proof:** The graph of the TIFN  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$  is the triangle ABC and the triangle DEF of Figure 1, where A ( $\underline{a}, 0$ ), B ( $a, p_a$ ) and C ( $\bar{a}, 0$ ) respectively D( $\underline{a}, 1$ ), E ( $a, q_a$ ) and F ( $\bar{a}, 1$ ). Then, the COG, say G, of ABC is the intersection point of its

medians AN and BM, where N  $\left( \frac{a + \bar{a}}{2}, \frac{p_a}{2} \right)$

and M  $\left( \frac{\bar{a} - \underline{a}}{2}, 0 \right)$ . Therefore the equation

of the straight line on which AN lies is

$$\frac{x - \underline{a}}{a + \bar{a} - \underline{a}} = \frac{y}{\frac{p_a}{2}}, \text{ or } \frac{a + \bar{a}}{2} - \underline{a} = \frac{p_a}{2}$$

$$x p_a - (a + \bar{a} - 2\underline{a})y = p_a a \quad (1).$$

In the same way one finds that the equation of the straight line on which BM lies is  $2x p_a + (\bar{a} - \underline{a} - 2a)y = p_a (\bar{a} - \underline{a} - 2a) \quad (2).$

The linear system of (1) and (2) has a unique solution with the respect to the variables  $x$  and  $y$  determining the coordinates of the

triangle's COG,  $G_1 \left( \frac{\underline{a} + a + \bar{a}}{3}, \frac{p_a}{3} \right)$

Similar to determine the coordinates of the triangle's COG,  $G_2 \left( \frac{\underline{a} + a + \bar{a}}{3}, \frac{2 + q_a}{3} \right).$

Further, the areas of the two triangles

are equal to  $S_1 = \frac{p_a(\bar{a} - \underline{a})}{2}$  and  $S_2 =$

$\frac{(1 - q_a)(\bar{a} - \underline{a})}{2}$  respectively. Then the

coordinates of the COG of the TIFN, being

the resultant of the COGs  $G_i (x_i, y_i)$ , for  $i=1, 2$  are calculated by the formulas

$$X = \frac{1}{S} \sum_{i=1}^2 S_i x_i, \quad Y = \frac{1}{S} \sum_{i=1}^2 S_i y_i, \quad S = S_1 + S_2 = \frac{(\bar{a} - \underline{a})(p_a + 1 - q_a)}{2}$$

is the area of the TIFN  $a = \langle (\underline{a}, a, \bar{a}); p_a, q_a \rangle$

$$X = \frac{1}{S} \sum_{i=1}^2 S_i x_i = \frac{2}{(\bar{a} - \underline{a})(p_a + 1 - q_a)} \left[ \frac{p_a(\bar{a} - \underline{a})}{2} \cdot \frac{\underline{a} + a + \bar{a}}{3} - \frac{(1 - q_a)(\bar{a} - \underline{a})}{2} \cdot \frac{\underline{a} + a + \bar{a}}{3} \right] = \frac{\underline{a} + a + \bar{a}}{3}$$

$$Y = \frac{1}{S} \sum_{i=1}^2 S_i y_i = \frac{2}{(\bar{a} - \underline{a})(p_a + 1 - q_a)} \left[ \frac{p_a(\bar{a} - \underline{a})}{2} \cdot \frac{p_a}{3} - \frac{(1 - q_a)(\bar{a} - \underline{a})}{2} \cdot \frac{2 + q_a}{3} \right] = \frac{p_a^2 - q_a^2 - q_a + 2}{3(p_a + 1 - q_a)}$$

#### 4. CONCLUSION

Decision and games with intuitionistic fuzzy sets are remarkably different from fuzzy

decision and games. Starting from [8] We extended the proposition 1 for ITNF.

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