RANCHING OF GENERALIZED TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS BASED ON RANK, MODE, DIVERGENCE AND SPREAD

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ABSTRACT : In the present paper we complete the method for ranking of generalized exponential trapezoidal intuitionistic fuzzy numbers based on rank, mode, divergence and spread. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

KEY WORDS : Intuitionistic fuzzy numbers, Ranking function, Mode, Divergence, Spread, Generalized exponential intuitionistic trapezoidal fuzzy numbers.

1. INTRODUCTION

A generalized fuzzy number $a = \langle a,b,c,d \;; w_a \rangle$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_a(x) \quad \begin{matrix} x-a \;\; w_a \,/\, \; b-d & a \le x < b \\ w_a & b \le x < c \\ d-x \;\; w_a \,/\, \; d-c & c < x \le d \end{matrix}$$

S. Rezvani, 2012 proposed a method for ranking of generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread

$$a \;=\; \frac{w_a \;\; a+b+d-c}{2},$$

$$\text{Mode } \; a \; = \frac{w_a \;\; c+b}{2}$$

Divergence $a \; = w_a \;\; d-a$

Left spread $a \; = w_a \;\; b-a$

Right spread $a \; = w_a \;\; d-c$

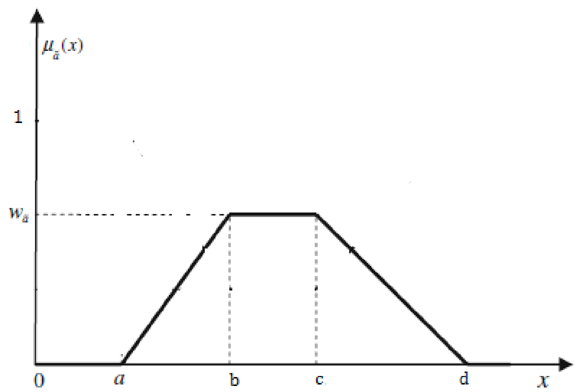

Figure 1. A trapezoidal intuitionistic fuzzy number $a = \langle a,b,c,d \;; w_a \rangle$

The intuitionistic fuzzy numbers (IFNs) on a universe X was introduced by K. Atanassov in 1983 as a generalization of FNs. Therefore, the concept of the triangular intuitionistic fuzzy number is a generalization of that of the triangular fuzzy number.

A trapezoidal intuitionistic fuzzy number is a special intuitionistic fuzzy set on the real number set R, whose membership and nonmembership functions are defined as follows:

$$\mu_a(x) \quad \begin{array}{ll} x-a \;\; w_a \,/\, b-d & a \le x < b \\ w_a & b \le x < c \\ d-x \;\; w_a \,/\, d-c & c < x \le d \\ & d \end{array}$$

$$\nu_a(x)$$

$$a$$

$$\begin{array}{ll} b-x+u_a \;\; x-a \;\; / \;\; b-a & a \le x < b \\ u_a & b \le x \le c \\ x-c+u_a \;\; d-x \;\; / \;\; d-c & c < x \le d \end{array}$$

where w_a and u_a represent the maximum membership degree and the minimum nonmembership degree of a such that they satisfy the conditions: $0 \le w_a \le 1, 0 \le u_a \le 1$, and $0 \le w_a + u_a \le 1$. [b; c] are called the mean interval and the lower and upper limits of the trapezoidal intuitionistic fuzzy number a , respectively.

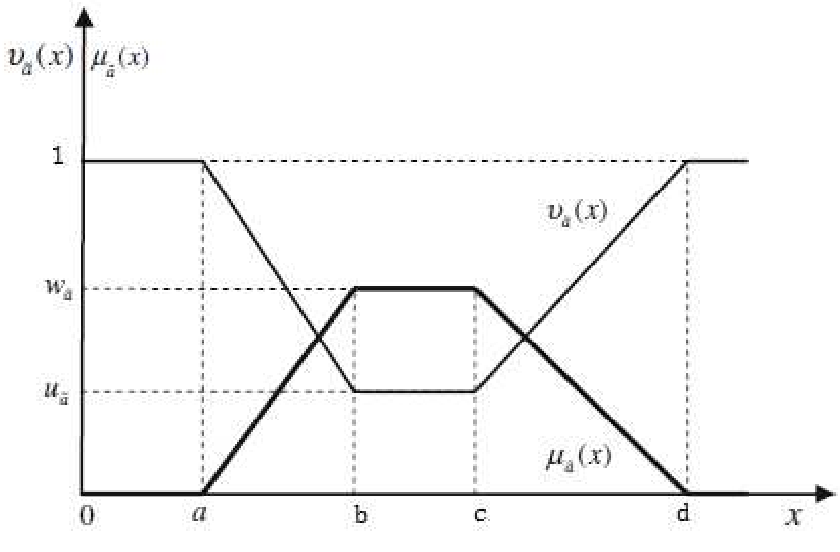

Figure 2. A trapezoidal intuitionistic fuzzy number $a = \langle \; a, b, c, d \;\; ; w_a, u_a \rangle$

2. FIND VALUE OF a , *Mode* a , *Divergence* a , LEFT AND RIGHT SPREAD.

Ranking fuzzy numbers were first proposed by (Jain 1976) for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set.

Starting from [13] will demonstrate some properties for generalized exponential intuitionistic trapezoidal fuzzy numbers. Let two monotonic functions be

$$L\;x = w_a u_a e^{-\frac{a-x}{c}},\; R\;x = w_a u_a e^{-\frac{x-b}{d}} \qquad (1)$$

The inverse functions of function L and R are respectively $L^{-1}\;x = a + c\ln\frac{w_a u_a}{x}$ and $R^{-1}\;x = b - d\ln\frac{w_a u_a}{x}$. The h-level graded mean value of generalized exponential fuzzy number $a = \langle a,b,c,d\;;w_a,u_a\rangle$ can be express as:

$$\frac{h[L^{-1}\;h + R^{-1}\;h\;]}{2}$$

Definition 1. [13] We call the graded mean integration representation of a the value:

$$P\;a = \frac{\int_0^{w_a} h\left(\frac{L^{-1}\;h + R^{-1}\;h}{2}\right)dh}{\int_0^{w_a} hdh} + \frac{\int_{u_a}^{1} h\left(\frac{L^{-1}\;h + R^{-1}\;h}{2}\right)dh}{\int_{u_a}^{1} hdh} \qquad (2)$$

In [13] have demonstrated that:

Theorem 1. Let $a = \langle a,b,c,d\;;w_a,u_a\rangle$, be generalized exponential number with $0 < w_a \le 1, 0 < u_a \le 1$ and c,d are positive real numbers, a,b are real numbers. Then the graded mean integration representation of a is:

$$P\;a = a - b - \frac{1}{2}\;d - c\;\ln u_a - \frac{1}{2}\;d - c$$
$$+\;d - c\;\;\ln w_a u_a - u_a{}^2 \ln w_a$$
$$= a - b - \frac{1}{2}\;d - c\;(\ln u_a + 1 + 2\;\ln w_a u_a - u_a{}^2 \ln w_a$$

Theorem 2. Let $a = \langle a,b,c,d\;;w_a,u_a\rangle$, be generalized exponential number with $0 < w_a \le 1, 0 < u_a \le 1$ and c,d are positive real numbers, a,b are real numbers. Then

1) a =

$$\frac{1}{2}\int_{u_a}^{w_a} L^{-1}\;h + R^{-1}\;h\;\;dh - \frac{c-d}{2}\ln\frac{w_a{}^{u_a}}{u_a{}^{w_a}} \qquad (3)$$

$$\text{2)Mode}\;a = \frac{w_a - u_a\;\;c + b}{2} \qquad (4)$$

$$\text{3)Divergence}\;a = w_a - u_a\;\;d - a \qquad (5)$$

$$\text{4)Left spread}\;a = w_a - u_a\;\;b - a \qquad (6)$$

5)Right spread a = 7)

Proof: 1) a =

$$\frac{1}{2}\int_{u_a}^{w_a}\left(a - c\ln\frac{w_a u_a}{h} + b + d\ln\frac{w_a u_a}{h}\right)dh \text{ -}$$
$$\frac{c-d}{2}\ln\frac{w_a{}^{u_a}}{u_a{}^{w_a}} =$$
$$\frac{w_a\;\;a+b+d-c}{2} - \frac{u_a\;\;a+b+d-c}{2} +$$
$$\frac{w_a d\ln u_a}{2} - \frac{w_a c\ln u_a}{2} + \frac{cu_a \ln w_a}{2} - \frac{du_a \ln w_a}{2}$$

$$\text{-}\frac{c-d}{2}\ln\frac{w_a{}^{u_a}}{u_a{}^{w_a}} =$$
$$\frac{w_a\;\;a+b+d-c}{2} - \frac{u_a\;\;a+b+d-c}{2}.$$

So

$$a = \frac{w_a \quad a+b+d-c}{2} - \frac{u_a \quad a+b+d-c}{2};$$

2) Mode $a = \frac{1}{2}\int_{u_a}^{w_a} b+c \ dh = \frac{1}{2} c\, w_a - \frac{1}{2} cu_a +$

$$+\frac{1}{2} bw_a - \frac{1}{2} bu_a = \frac{w_a \quad c+b}{2} - \frac{u_a \quad c+b}{2} =$$

$$\frac{w_a - u_a \quad c+b}{2};$$

3) Divergence $a = \int_{u_a}^{w_a} d-a \ dh =$

$d\, w_a - du_a + au_a - a\, w_a = \quad w_a - u_a \quad d-a \;;$

4) Left spread $a = \int_{u_a}^{w_a} b-a \ dh =$

$bw_a - bu_a + au_a - a\, w_a = \quad w_a - u_a \quad b-a \;;$

5) Right spread $a = \int_{u_a}^{w_a} d-a \ dh =$

$w_a - u_a \quad d-c \;;$

Proposition 1. Let
$a_1 = \langle a_1, b_1, c_1, d_1 \; ; w_{a_1}, u_{a_1} \rangle$,
$a_2 = \langle a_2, b_2, c_2, d_2 \; ; w_{a_2}, u_{a_2} \rangle$ be two generalized exponential number with
$0 < w_{a_1} \le 1, 0 < u_{a_1} \le 1$, $0 < w_{a_2} \le 1, 0 < u_{a_2} \le 1$
and c_1, d_1, c_2 , d_2 are positive real numbers, a_1, a_1, b_2 , b_2 are real numbers such that

i) $a_1 = a_2$

ii) Mode a_1 = Mode a_2 ;

iii) Divergence a_1 = Divergence a_2 ;

Then

1) Left spread a_1 >Left spread a_2 iff

$w_{a_1} - u_{a_1} \quad b_1 > \quad w_{a_2} - u_{a_2} \quad b_2$;

2) Left spread a < Left spread $\tilde{b}$ iff

$w_{a_1} - u_{a_1} \quad b_1 < \quad w_{a_2} - u_{a_2} \quad b_2$;

3) Left spread a = Left spread $\tilde{b}$ iff

$w_{a_1} - u_{a_1} \quad b_1 = \quad w_{a_2} - u_{a_2} \quad b_2$

Proof: i) We have $a_1 = a_2$ i.e.

$$\frac{w_{a_1} \quad a_1+b_1+d_1-c_1}{2} - \frac{u_{a_1} \quad a_1+b_1+d_1-c_1}{2} = \frac{w_{a_2} \quad a_2+b_2+d_2-c_2}{2} - \frac{u_{a_2} \quad a_2+b_2+d_2-c_2}{2}$$

(8)

ii) Mode a_1 = Mode a_2 i.e.

$$w_{a_1} - u_{a_1} \quad c_1 + b_1 \;=\; w_{a_2} - u_{a_2} \quad c_2 + b_2 \qquad (9)$$

iii) Divergence a_1 = Divergence a_2 i.e.

$$w_{a_1} - u_{a_1} \quad d_1 - a_1 \;=\; w_{a_2} - u_{a_2} \quad d_2 - a_2 \qquad (10)$$

Solving (8), (9) and (10) we get

$$w_{a_1} - u_{a_1} \quad a_1 = \quad w_{a_2} - u_{a_2} \quad a_2 \qquad (11)$$

$$w_{a_1} - u_{a_1} \quad d_1 = \quad w_{a_2} - u_{a_2} \quad d_2 \qquad (12)$$

$$w_{a_1} - u_{a_1} \quad b_1 + c_1 \;=\; w_{a_2} - u_{a_2} \quad b_2 + c_2 \qquad (13)$$

1) Left spread a_1 >Left spread a_2 iff

$w_{a_1} - u_{a_1} \quad b_1 - a_1 \;>\; w_{a_2} - u_{a_2} \quad b_2 - a_2$ iff $\Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 - \quad w_{a_1} - u_{a_1} \quad a_1$

$> \quad w_{a_2} - u_{a_2} \quad b_2 - \quad w_{a_2} - u_{a_2} \quad a_2 \Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 > \; w_{a_2} - u_{a_2} \quad b_2$.

(I used the relationship (11)).

2) Left spread a_1 < Left spread a_2 $\Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 - a_1 \; > \; w_{a_2} - u_{a_2} \quad b_2 - a_2 \quad \Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 < \; w_{a_2} - u_{a_2} \quad b_2$.

3) Left spread a = Left spread $\tilde{b}$ $\Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 - a_1 \; = \; w_{a_2} - u_{a_2} \quad b_2 - a_2 \; \Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 = \; w_{a_2} - u_{a_2} \quad b_2$ because we have (11).

Corollary 1. All the results of Proposition 1 is true and right spread.

Proposition 2. Let

$a_1 = \langle a_1, b_1, c_1, d_1 \;; w_{a_1}, u_{a_1} \rangle$,

$a_2 = \langle a_2, b_2, c_2, d_2 \;; w_{a_2}, u_{a_2} \rangle$ be two generalized exponential number with

$0 < w_{a_1} \leq 1, 0 < u_{a_1} \leq 1$, $0 < w_{a_2} \leq 1, 0 < u_{a_2} \leq 1$

and c_1, d_1, c_2 , d_2 are positive real numbers, a_1, a_1, b_2 , b_2 are real numbers such that

i) a_1 = a_2

ii) Mode a_1 = Mode a_2 ;

iii) Divergence a_1 = Divergence a_2 ;

Then

1) Left spread a_1 >Left spread a_2 iff Right spread a_1 > Right spread a_2 ;

2) Left spread a_1 < Left spread a_2 iff Right spread a_1 < Right spread a_2 ;

3) Left spread a_1 = Left spread a_2 iff Right spread a_1 = Right spread a_2 ;

Proof: From Proposition 1, we have

$w_{a_1} - u_{a_1} \quad a_1 = \; w_{a_2} - u_{a_2} \quad a_2$

$w_{a_1} - u_{a_1} \quad d_1 = \; w_{a_2} - u_{a_2} \quad d_2$

$w_{a_1} - u_{a_1} \quad b_1 + c_1 \; = \; w_{a_2} - u_{a_2} \quad b_2 + c_2$

1) Left spread a_1 >Left spread a_2 $\Leftrightarrow$

$w_{a_1} - u_{a_1} \quad b_1 > \; w_{a_2} - u_{a_2} \quad b_2 \; \Leftrightarrow$

$w_{a_1} - u_{a_1} \quad c_1 < \; w_{a_2} - u_{a_2} \quad c_2$ because

$w_{a_1} - u_{a_1} \quad b_1 + c_1 \; = \; w_{a_2} - u_{a_2} \quad b_2 + c_2$

$\Leftrightarrow - \; w_{a_1} - u_{a_1} \quad c_1 > - \; w_{a_2} - u_{a_2} \quad c_2 \; \Leftrightarrow$

$w_{a_1} - u_{a_1} \quad d_1 - c_1 \; > \; w_{a_2} - u_{a_2} \quad d_2 - c_2$

because $w_{a_1} - u_{a_1} \quad d_1 = \; w_{a_2} - u_{a_2} \quad d_2 \; \Leftrightarrow$

Right spread a_1 > Right spread a_2 ;

In the same way demonstrates the other items.

4. CONCLUSION

Starting from [13] We extended the method for ranking of generalized exponential trapezoidal intuitionistic fuzzy numbers based on rank, mode, divergence and spread..

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