THE CATEGORY OF SUBGROUPOIDS OF TRIVIAL GROUPOIDS WITH GROUP Z AND SIMPLIFIED MODELS FOR DISCRETE DYNAMICAL SYSTEMS

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ABSTRACT: The purpose of this paper is to study the isomorphisms of the category of all subgroupoids of trivial groupoids on various sets X with group $\mathbb{Z}$. We also provide Maple procedures for obtain simplified models of a discrete dynamical system seen as a subgroupoid a trivial groupoid with group $\mathbb{Z}$.

KEY WORDS: trivial groupoid; groupois isomorphism; orbit; dynamical system; complexity reduction.

1. MOTIVATION

The mathematical setting for a discrete-time dynamical system is a space X (the space of all possible states of the system) and a map ψ:X→X that defines how the system changes in time: the state u∈X at time t = 0 progresses into $\psi(u)$ at t = 1, $\psi(\psi(u))$ at t = 2, and so on (see [5]). There is a groupoid (a small category in which every morphism is invertible) G(X, ψ) associated to such a system (see [4]). This groupoid is a subgroupoid of the trivial groupoid on X with group $\mathbb{Z}$, i.e. of the groupoid $X\times\mathbb{Z}\times X$ whose partial product and inverse map are given by

$$(u, n, v)(v, m, w) = (u, n+m, w)$$
$$(u, n, v)^{-1} = (v, -n, u)$$

Simulation the properties of the groupoid G(X, ψ) associated to a dynamic system (X, ψ) usually involves a discretization of the space X: $\{u_1,u_2,\ldots,u_n\}$. But $D=\{u_1,u_2,\ldots,u_n\}$ is not necessarily invariant with respect to the dynamics (the system (X, ψ) can not be reduced to (D, $\psi|_D$)). However always the reduction G(X, ψ)|D (contraction) of the groupoid G(X, ψ) to D has sense. G(X, ψ)|D is a subgroupoid of G(X, ψ) and consequently, a subgroupoid of $X\times\mathbb{Z}\times X$. We propose the following method for complexity reduction of G(X, ψ)|D (and consequently, of the dynamical system (X, ψ)): consider ≈ an equivalence relation on X (and hence on D) and σ:D/≈ → D be a section of the quotient map D→D/≈. Let S=σ(D/≈) and consider the groupoid G(X, ψ)|D|S= G(X, ψ)|S as a simplified model for (X, ψ). A question arises: if (X', ψ') is a dynamical system equivalent in certain sense to (X, ψ), D' is a discretization of X', σ':D'/≈' → D' a section of a quotient map D'→D'/≈' and S'=σ'(D'/≈'), are G(X, ψ)|S and G(X', ψ')|S' isomorphic groupoids? More generally, what is the form of homomorphisms between two subgroupoids G and G' of $X\times\mathbb{Z}\times X$, respectively $X'\times\mathbb{Z}\times X'$?

2. ISOMORPHISMS IN THE CATEGORY OF SUBGROUPOIDS OF TRIVIAL GROUPOIDS WITH GROUP $\mathbb{Z}$

Using the results in [1] (Section 2) and those in [2] (Section 2) we may conclude that any subgroupoid $G\subset X\times\mathbb{Z}\times X$ can be represented as $G = \bigcup_{f(u)=f(v)} G_v^u$ with

$G_v^u=\{(u, nk(f(u))+k(u)-k(v),v): n\in \mathbb{Z} \}$,

where f: X→X and k:X→$\mathbb{Z}$ are two functions with following properties:

1. f(f(u))=f for every u∈X.
2. k(f(u)) ≥0 for every u∈X.
3. If k(f(u)) ≠ 0 and u≠f(u), then k(u)∈{0,1,…, k(f(u))-1}.

The connection with the family $\{k_{u,v}\}_{(u,v)\in R}$ of integer numbers used in [1] is the following: f(u) is a representative in the orbit [u], $k(f(u))=k_{u,u}$ and $k(u) = k_{u,f(u)}$ for every u∈[f(u)] such that u≠f(u). Let us recall that for a groupoid G, $G^{(0)}$ denotes the unit space of G (set of objects in the category language) and the orbit of a unit $u\in G^{(0)}$ is

$[u]=\{v\in G^{(0)}: \exists\ \gamma\in G \text{ s.t. } r(\gamma)=u \text{ and } d(\gamma)=v\}$,

where r(γ) is the range (target) of γ and d(γ) is the domain (source) of γ. For a subgroupoid of the trivial groupoid X×$\mathbb{Z}$×X the unit space can be identified with a subset of X. Thus in this case the orbits can be viewed as subsets of X.

Notation 1. Let us denote by G(X,f,k) the subgroupoid of X×$\mathbb{Z}$×X characterized by the functions f: X→X and k:X→$\mathbb{Z}$ as above. Let us observe that if [u] is an orbit of G(X,f,k), then v∈[u] if and only if there is n∈ $\mathbb{Z}$ such that (u,n,v)∈G(X,f,k). Let us write

$$\overline{X} =\{[u],u\in G^{(0)}\}$$

Proposition 2. Let X×$\mathbb{Z}$×X and Y×$\mathbb{Z}$×Y be two trivial groupoids on X, respectively Y with group $\mathbb{Z}$ and G= G(X,f,k) be a subgroupoid of X×$\mathbb{Z}$×X. If

$$H: G(X,f,k) \to Y\times\mathbb{Z}\times Y$$

is a groupoid homomorphism (functor), then there are three functions h:X→Y, θ:X→$\mathbb{Z}$ and $\overline{\eta}:\overline{X}\to\mathbb{Z}$ such that

$$H(u, nk(f(u))+k(u)-k(v), v) = =(h(u),\theta(u)+n\overline{\eta}([u])-\theta(v),h(v))$$

for all (u, nk(f(u))+k(u)-k(v), v)∈ G(X,f,k).

Conversely, if h:X→Y, θ:X→$\mathbb{Z}$ and $\overline{\eta}$: $\overline{X}\to\mathbb{Z}$ are there arbitrary function, then H:G(X,f,k) → Y×$\mathbb{Z}$×Y defined by

$$H(u, nk(f(u))+k(u)-k(v), v) = =(h(u),\theta(u)+n\overline{\eta}([u])-\theta(v),h(v))$$

is a groupoid homomorphism and

$$h(u)=r(H(u,0,u))$$
$$\overline{\eta}([u])= pr_2(H(f(u)), k(f(u)), f(u)))$$
$$\theta(u)-\theta(f(u)) = pr_2(H(u,k(u),f(u)))-pr_2(H(f(u)), k(f(u)), f(u))).$$

Proof. The proof of this proposition is an adaptation to the current context of the proof of the general structure of homomorphisms of trivial groupoids.

Let us assume that

$$H: G(X,f,k) \to Y\times\mathbb{Z}\times Y$$

is a groupoid homomorphism. Let h:X→Y be defined by

$$h(u)=r(H(u,0,u)) \text{ for all } u\in X,$$

θ:X→$\mathbb{Z}$ be defined by

$$\theta(u)=pr_2(H(u,k(u),f(u))) \text{ for all } u\in X,$$

and ξ:$\mathbb{Z}\times\overline{X}\to\mathbb{Z}$ be defined by

$$\xi(n,[u])=pr_2(H(f(u),k(f(u))n,f(u)))$$

for all [u]∈ $\overline{X}$. Then we have

$$\begin{aligned}
&(h(f(u)), \xi(n+m,[u]), h(f(u))) =\\
&= H(f(u), (n+m)\, k(f(u)),f(u))\\
&= H(f(u), k(f(u))n, f(u)(f(u), k(f(u))m, f(u))\\
&= H(f(u), k(f(u))n, f(u))H((f(u),k(f(u))m, f(u))\\
&=(h(f(u)), \xi(n,[u]), h(f(u)))\ (h(f(u)), \xi(n,[u]), h(f(u)))\\
&=(h(f(u)), \xi(n,[u])+ \xi(m,[u]), h(f(u)))
\end{aligned}$$

for all for all [u]∈ $\overline{X}$ and m, n∈ $\mathbb{Z}$. Thus

$$\xi(n+m,[u])= \xi(n,[u])+ \xi(m,[u])$$

for all for all [u]∈ $\overline{X}$ and m,n∈ $\mathbb{Z}$. If we denote

$$\overline{\eta}([u])=\xi(1,[u])=pr_2(H(f(u)), k(f(u)), f(u))),$$

then ξ(n,[u]) = ξ(1+1+..+1,[u]) = ξ(1,[u])+ ξ(1,[u]) + … + ξ(1,[u])=nξ(1,[u])=n$\overline{\eta}$([u]) for all n∈$\mathbb{Z}$ and all [u]∈ $\overline{X}$ and consequently,

$$H(f(u),k(f(u))n,f(u)) = = (h(f(u)),n\overline{\eta}([u]), h(f(u))).$$

For all n∈$\mathbb{Z}$ and all u,v∈X we have

$$\begin{aligned}
&H(u, nk(f(u))+k(u)-k(v), v) =\\
&= H((u, k(u), f(u)(f(u), n\, k(f(u)), f(u))\ (f(u), -k(v),v))\\
&= H(u, k(u), f(u))H(f(u), nk(f(u)), f(u))\ H(f(u),-k(v),v)\\
&= H(u, k(u), f(u))H(f(u), nk(f(u)), f(u))\ H(v,k(v),f(v))^{-1}\\
&= (h(u),\theta(u),h(f(u)))(h(f(u)), n\overline{\eta}([u]), h(f(u)))\ (h(v), \theta(v), h(f(v)))^{-1}\\
&= (h(u),\theta(u),h(f(u)))(h(f(u)), n\overline{\eta}([u]), h(f(u)))\ (h(f(v)),-\theta(v),h(v))\\
&= (h(u),\theta(u)+n\overline{\eta}([u])-\theta(v),h(v)).
\end{aligned}$$

Let h:X→X, θ:X→$\mathbb{Z}$ and $\overline{\eta}:\overline{X}\to\mathbb{Z}$ be there arbitrary function, and

$$H:G(X,f,k) \to Y\times\mathbb{Z}\times Y$$

be defined by

H(u, nk(f(u))+k(u)-k(v), v) =

$=(h(u),\theta(u)+n\bar{\eta}([u])-\theta(v),h(v))$

for all $(u, nk(f(u))+k(u)-k(v), v)\in G(X,f,k)$. Then obviously, H is a groupoid homomorphism. For every $u\in X$ we have

$r(H(u,0,u))=r(h(u),0 ,h(0))$

$=h(u)$.

Also for every $u\in X$ we have

$pr_2(H(f(u)), k(f(u)), f(u)))=$

$=pr_2((h(f(u), \bar{\eta}([u]),h(f(u)))= \bar{\eta}([u])$,

and

$pr_2(H(u,k(u),f(u)))-pr_2(H(f(u)),k(f(u)),f(u)))$ =

$=pr_2((h(u), \theta(u)+ \bar{\eta}([u])-\theta(h(f(u))), h(f(u))) -pr_2((h(f(u), \bar{\eta}([u]),h(f(u)))$

$= \theta(u)+ \bar{\eta}([u])-\theta(h(f(u))) - \bar{\eta}([u])$

$= \theta(u)-\theta(h(f(u)))$.

Remark 3. The functions h, $\bar{\eta}$ in Proposition 2 are uniquely determined by $H:G(X,f,k)\to Y\times\mathbb{Z}\times Y$ and θ is determined modulo a function $\theta_1: \bar{X}\to\mathbb{Z}$

$\theta(u)=\theta_1([u])+ pr_2(H(u,k(u),f(u)))$ -

$- pr_2(H(f(u)), k(f(u)), f(u)))$, for all $u\in X$

($\theta_1([u]) = \theta(f(u))$, for all $u\in X$).

Corollary 4. Let $X\times\mathbb{Z}\times X$ and $Y\times\mathbb{Z}\times Y$ be two trivial groupoids on X, respectively Y with group $\mathbb{Z}$. Let $G_1=G(X,f_X,k_X)$ be a subgroupoid of the $X\times\mathbb{Z}\times X$ and $G_2=G(Y,f_Y,k_Y)$ be a subgroupoid of $Y\times\mathbb{Z}\times Y$. If

$H: G(X,f_X,k_X) \to G(Y,f_Y,k_Y)$

is a groupoid homomorphism, then there is a function $h:X\to Y$ with the property that

$h([u])\subset[h(u)]$

for all $u\in X$ and another two functions $\mu:X\to\mathbb{Z}$ and $\eta:\bar{X}\to\mathbb{Z}$ such that

$H(u,nk_X(f_X(u))+k_X(u)-k_X(v),v) =$

$=(h(u), (n\eta([u])+\mu(u)-\mu(v))k_Y(f_Y(h(u))) + k_Y(h(u))- k_Y(h(v)),h(v))$

for all $(u,nk_X(f_X(u))+k_X(u)-k_X(v),v)\in G_1$. Conversely, if $h:X\to Y$ is a function with the property that

$h([u])\subset[h(u)]$

for all $u\in X$ and $\mu:X\to\mathbb{Z}$ and $\eta:\bar{X}\to\mathbb{Z}$ are arbitrary functions, then

$H: G(X,f_X,k_X) \to G(Y,f_Y,k_Y)$

defined by

$H(u,nk_X(f_X(u))+k_X(u)-k_X(v),v) =$

$=(h(u), (n\eta([u])+\mu(u)-\mu(v))k_Y(f_Y(h(u))) + k_Y(h(u))- k_Y(h(v)),h(v))$

is a groupoid homomorphism and

$h(u)=r(H(u,0,u))$

$k_Y(f_Y(h(u)))\eta([u])=$

$=pr_2(H(f_X(u)), k_X(f(u)), f_X(u)))$

$\mu(u)-\mu(f_X(u)) = pr_2(H(u,k_X(u),f_X(u)))$ - $pr_2(H(f_X(u)), k_X(f_X(u)), f_X(u)))$.

Proof. Let us assume that

$H: G(X,f_X,k_X) \to G(Y,f_Y,k_Y)$

is a groupoid homomorphism. Since $G_2=G(Y,f_Y,k_Y)$ is a subgroupoid of $Y\times\mathbb{Z}\times Y$, by Proposition 2, there are three functions $h:X\to Y$, $\theta:X\to\mathbb{Z}$ and $\bar{\eta}:\bar{X}\to\mathbb{Z}$ such that

$H(u, nk_X(f_X(u))+k(u)-k(v), v) =$

$=(h(u),\theta(u)+n\bar{\eta}([u])-\theta(v),h(v))$

for all $(u, nk(f(u))+k(u)-k(v), v)\in G(X,f_X,k_X)$. Taking into account Remark 3 and choosing θ_1: $\bar{X}\to\mathbb{Z}$ defined by $\theta_1([u])= pr_2(H(f(u)), k(f(u)), f(u)))$ for all $[u]\in\bar{X}$, it follows that we can choose θ to be defined by

$\theta(u)=pr_2(H(u,k_X(u),f_X(u)))$ for all $u\in X$.

Since

$H(u,k_X(u),f_X(u))=(h(u), \theta(u),h(f_X(u))) \in G_2$,

it follows that there is $\mu(u)\in\mathbb{Z}$ such that

$\theta(u)=\mu(u)k_Y(f_Y(h(u)))+k_Y(h(u))-k_Y(h(f_X(u)))$

for all $u\in X$. Since

$(h(f(u)), \bar{\eta}([u]), h(f(u)))=$

$=H(f(u)), k_X(f(u)), f(u))\in G_2$,

it follows that there is $\eta([u])\in Z$ such that $\bar{\eta}([u])=\eta([u])k_Y(f_Y(h(u))$ for all $u\in X$. Thus

$H(u,nk_X(f_X(u))+k_X(u)-k_X(v),v) =$

$= (h(u),n\bar{\eta}([u])+(\mu(u)-\mu(v))k_Y(f_Y(h(u))) +k_Y(h(u))- k_Y(h(v)), h(v))$

$=(h(u),(n\eta([u])+\mu(u)-\mu(v)) k_Y(f_Y(h(u))) +k_Y(h(u))- k_Y(h(v)), h(v))$

Let $h:X\to Y$ be a function such that

$h([u])\subset[h(u)]$

for all $u\in X$ and $\mu:X\to\mathbb{Z}$ and $\eta:\bar{X}\to\mathbb{Z}$ be arbitrary functions. Let

$H: G(X,f_X,k_X) \to G(Y,f_Y,k_Y)$

be defined by

$H(u,nk_X(f_X(u))+k_X(u)-k_X(v),v) =$

$=(h(u), (n\eta([u])+\mu(u)-\mu(v))k_Y(f_Y(h(u))) + k_Y(h(u))- k_Y(h(v)),h(v))$

for all $(u,nk_X(f_X(u))+k_X(u)-k_X(v),v)\in G_1$. Then H is a correctly defined groupoid homomorphism. As in the proof of Proposition 2 for every $u\in X$ we have

$r(H(u,0,u))=h(u)$.

$pr_2(H(f_X(u)), k_X(f_X(u)), f_X(u)))=$

$= \eta([u])([u])k_Y(f_Y(h(u)))$,

and

$pr_2(H(u,k_X(u),f_X(u)))$-

$pr_2(H(f_X(u)),k_X(f_X(u)),f_X(u)))$
$= \mu(u)-\mu(f_X(u))$.

Proposition 5. Let $X\times\mathbb{Z}\times X$ and $Y\times\mathbb{Z}\times Y$ be two trivial groupoids on X, respectively Y with group $\mathbb{Z}$. Let $G_1=G(X,f_X,k_X)$ be a subgroupoid of the $X\times\mathbb{Z}\times X$ and $G_2=G(Y,f_Y,k_Y)$ be a subgroupoid of $Y\times\mathbb{Z}\times Y$. Then

$$H: G(X,f_X,k_X) \to G(Y,f_Y,k_Y)$$

is a groupoid isomorphism if and only if there are three functions $h:X\to Y$, $\mu:X\to\mathbb{Z}$ and $\eta: \bar{X}\to\{-1,1\}$ such that

h bijective, $h([u])=[h(u)]$ for all $u\in X$

and

$H(u,nk_X(f_X(u))+k_X(u)-k_X(v),v) =$
$=(h(u), (n\eta([u])+\mu(u)-\mu(v))k_Y(f_Y(h(u))) + k_Y(h(u))- k_Y(h(v)),h(v))$

for all $(u,nk_X(f_X(u))+k_X(u)-k_X(v),v)\in G_1$. The functions h and η are uniquely determined by H and μ is determine modulo a function $\bar{\mu}: \bar{X}\to\mathbb{Z}$.

Proof. Let us assume that H is a groupoid isomorphism. By Corollary 4 there are three functions $h:X\to Y$, $\mu:X\to\mathbb{Z}$ and $\eta: \bar{X}\to\mathbb{Z}$ such that

$h([u])\subset[h(u)]$ for all $u\in X$

and

$H(u,nk_X(f_X(u))+k_X(u)-k_X(v),v) =$
$=(h(u), (n\eta([u])+\mu(u)-\mu(v))k_Y(f_Y(h(u))) + k_Y(h(u))- k_Y(h(v)),h(v))$

for all $(u,nk_X(f_X(u))+k_X(u)-k_X(v),v)\in G_1$.
Since H is bijective it follows that h is bijective and $h([u])=[h(u)]$ for all $u\in X$. It remains to prove that $\eta(\bar{X})\subset\{-1,1\}$. If $k_Y(f_Y(h(u)))=0$, then the value of $\eta(u)$ does not matter. Hence we may assume $k_Y(f_Y(h(u)))\neq 0$. Since H is surjective and

$(h(u), k_Y(f_Y(h(u))), h(u))\in G_2$,

it follows that there is $n\in\mathbb{Z}$ such that $H(u,nk_X(f_X(u)),u)= (h(u), k_Y(f_Y(h(u))), h(u))$. Thus $(h(u), n\eta([u])k_Y(f_Y(h(u))),h(u)) = (h(u), k_Y(f_Y(h(u))), h(u))$. Consequently, $n\eta([u])=1$ and therefore $\eta([u])\in\{-1,1\}$.

Conversely, any there functions as in Proposition 5 define a groupoid isomorphism H. Let us remark that

$H^{-1}(s,nk_Y(f_Y(s))+k_Y(s)-k_Y(t),t) =$
$= (h^{-1}(s), (n-\mu(h^{-1}(s)) + \mu(h^{-1}(t)))\eta([h^{-1}(s)])k_X(f_X(h^{-1}(s))) + k_X(h^{-1}(s)) - k_X(h^{-1}(t)),h^{-1}(t))$.

3. MAPLE PROCEDURES FOR REDUCING COMPLEXITY

We propose the following method for complexity reduction of subgroupoid $G(X,f_X,k_X)$ of the trivial groupoid $X\times\mathbb{Z}\times X$: consider $\approx$ an equivalence relation on X and $\sigma:X/\!\approx\,\to X$ be a section of the quotient map $X\to X/\!\approx$. We assume that the relation $\approx$ is chosen such that $G(X,f_X,k_X)|\sigma(X/\!\approx)$ can be seen as simplified model for $(G(X,f_X,k_X)$. Of course it is desirable for a different section $\sigma':X/\!\approx\,\to X$ the groupoids $G(X,f_X,k_X)|\sigma(X/\!\approx)$ and $G(X,f_X,k_X)|\sigma'(X/\!\approx)$ to be isomorphic. According Proposition 5 this happens if $\{v:v\approx u\}\subset[u]$. That is why replace the relation $\approx$ with $\approx'$ where $u\approx' v$ iff $u\approx v$ and $v\in[u]$.

We use the same implementation of subgroupoid $G(X,f_X,k_X)$ of the trivial groupoid $X\times\mathbb{Z}\times X$ as in [2] (X is a finite set). The input parameters of the procedure reduce are gd (the data associated to the groupoid as in [2]) and relmat the matrix of the relation $\approx$. It returns $\sigma(X/\!\approx)$ for a certain section σ of the quotient map $X\to X/\!\approx$

```
reduce := proc (gd, relmat)
local n, i, j, k, Ar, mark;
n := op(2, op(2, gd[1]));
mark := array(1 .. n+1, [seq(0, i
= 1 .. n+1)]);
Ar := array(1 .. n);
k := 1; j := 0;
while k <= n do
 mark[k] := 1; j := j+1;
 Ar[j] := k;
 for i to n do
  if relmat[i, k] = 1 and
        gd[2][i] = gd[2][k] then
     mark[i] := 1
  end if
 end do;
 while mark[k] = 1 do
  k := k+1
 end do
end do;
RETURN(array(1 .. j, [seq(Ar[i],
i = 1 .. j)]))
end proc
```

We give below a variant of this procedure that have as input parameters the groupoid data gd and a function rel of two variable. We use the Roy-Warshall [6] algorithm for computing the transitive closure of the relation:

u ρ v iff. u=v or rel(u,v)=1 or rel(v,u)

```
reduce2 := proc (gd, rel)
```

```
local n, i, j, k, Ar, mat, mark;
n := op(2, op(2, gd[1]));
mat := array(1 .. n, 1 .. n);
for i to n do
 for j to n do mat[i, j] := 0 end
do end do;
for i to n do mat[i, i] := 1;
 for j to n do
  if     apply(rel,     gd[1][i],
gd[1][j])  =  1  and  gd[2][i]  =
gd[2][j]
  then
   mat[i, j] := 1;
   mat[j, i] := 1
  end if
 end do
end do;
for k to n do
 for i to n do
  for j to n do
   if mat[i, j] = 0 and k <> j and
i <> k
   then
    mat[i, j] := mat[i, k]*mat[k,
j]
   end if
  end do
 end do
end do;
mark := array(1 .. n+1, [seq(0, i
= 1 .. n+1)]);
Ar := array(1 .. n);
k := 1; j := 0; while k <= n do
mark[k] := 1; j := j+1;
Ar[j] := k;
for i to n do
  if mat[i, k] = 1 then
    mark[i] := 1
  end if
end do;
while mark[k] = 1 do
 k := k+1
end do end do;
RETURN(array(1  ..  j,  [seq(Ar[i],
i = 1 .. j)]))
end proc
```

We consider the discrete dynamical system associate to the function $\psi:\mathbb{R}\to\mathbb{R}$,

$\psi(u)=6.55u^2(1-u)$ for all $u\in\mathbb{R}$

reduction this groupoid to X={0, 0.01, 0.02, …, 1}. Using the procedure groupoid_data [] we construct the groupoid data associated to G($\mathbb{R}$,ψ)|X

```
>gd := groupoid_data(
x->6.55*x*(1-x)*x, ar-
ray([seq(i/(100.), i = 0 ..
100)]), 100):
```

We consider the following function

```
> relconnect := (x, y) ->piece-
wise(abs(evalf(y-x)) <= 1./100,
1, 0):
```

and apply the procedure reduce2

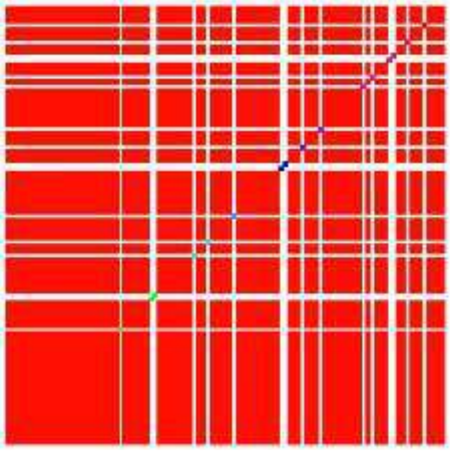
```
> S:=reduce2(gd,relconnect):
```

We construct the groupoid contraction G($\mathbb{R}$,ψ)|S

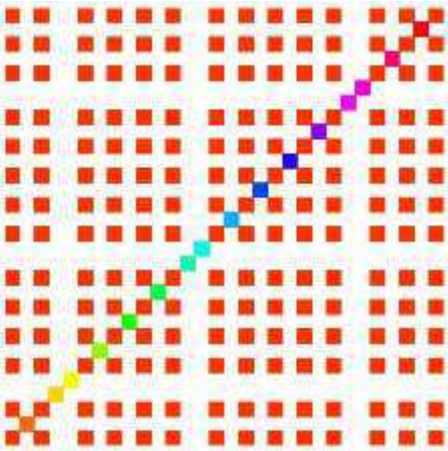
```
> gdred := groupoid_reduction(gd,
S):
```

We use the procedure orbits [3] to displays the graph of the equivalence relation (principal groupoid) associated with G($\mathbb{R}$,ψ)|X, respectively G($\mathbb{R}$,ψ)|S

```
>orbits(gd)
```

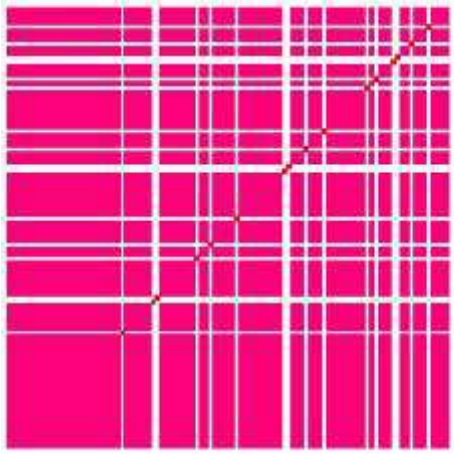


```
>orbits(gdred)
```

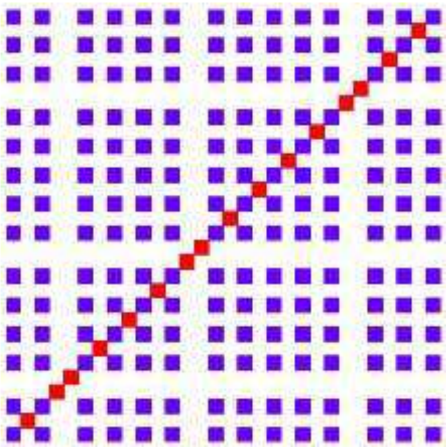


Let us use the procedure visualization [3] to represent each (r,d)-fibre G_v^u as the rectangle with top left corner (i-1,j) and bottom right corner (i,j-1) filled with a color uniquely determined by $k_X(u)$-$k_X(v)$ and $k_X(f(u))$.

```
>visualization(gd)
```



>visualization(gdred)



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