A SIMPLE MATHEMATICAL MODEL OF A SYNCHRONOUS GENERATOR FOR DEMONSTRATING ITS SHORT CIRCUIT BEHAVIOR

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ABSTRACT: *The goal is the analytical calculation of the short circuit current in one phase in a synchronous generator. For this purpose, the geometric arrangement is abstracted e. g. by circles for the rotor and the stator. For this geometry then the important inductances of the rotor and the stator can be determined. Subsequently, with the impedances, the characteristic current curves are calculated in a simple manner.*

KEYWORDS: *Rotor inductance, stator inductance, mutual inductance, voltage equations, single-phase earth short-circuit current.*

1.INTRODUCTION

In the following, a simplified analytical calculation of the current propagation in the stator and in the exciter coil of the rotor of a turbo generator is shown, when one phase of the stator becomes a short circuit. To keep the calculation clear, some assumptions are made that reflect the technical conditions well. Four of them are:

1) the permeability of the iron is assumed as unlimited.
2) the inner contour of the stator as well as the outer contour of the rotor are assumed as a drilled hole respectively as cylinder
3) the scattering in the slots of the stator and the rotor is not considered. So the real currents will be smaller.
4) in real machines the ohmic resistance in the coils are much smaller as the reactive resistance. So the ohmic resistance is neglected.

2.THE MAGNETIC FIELD IN THE AIR GAP

With the unlimited permeability the magnetic field strength in the iron disappears. So it is only necessary to calculate the magnetic field in the air gap.

The geometry is rotationally symmetric. So the calculation should be made in polar coordinates as a planar problem. Figure 1 shows the high-permeability iron of the stator and the rotor.

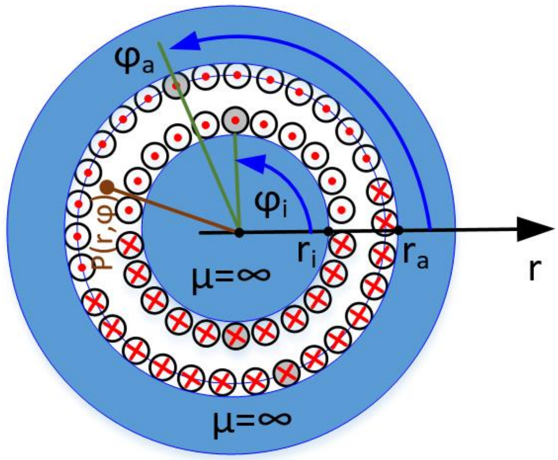

Figure 1 Cross section through the synchronous generator

The coils of the stator and the rotor have the same numbers w of turns. They are represented by 1D current densities $J_a(\varphi)$ and $J_i(\varphi)$ at the ambient and the inner circle as

$$J_a(\varphi) = C_{Sa}\cos(\varphi - \varphi_a) \quad (1a)$$

$$J_i(\varphi) = C_{Si}\cos(\varphi - \varphi_i) \quad (1b)$$

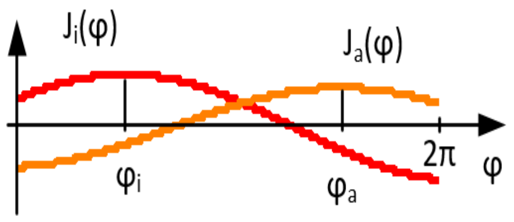

Figure 2 1D current densities of the coils at the ambient and inner circle

The constants C_{Sa} and C_{Si} can be calculated by integration of the current density, because the currents in the coils are well known as I_a and I_i.

$$w\, I_a = \int_{\varphi_a-\frac{\pi}{2}}^{\varphi_a+\frac{\pi}{2}} J_a(\varphi)\, r_a\, d\varphi = 2r_a\, C_{Sa}$$

In analogous way the constant for the inner current density can be calculated.

$$w\, I_i = 2r_i\, C_{Si}$$

In airspace the Laplace equation ΔA = 0 applies. Thus, in the chosen coordinate system with the z-directed currents, only the z-component of the vector potential exists

$\vec{A} = \vec{e_z} A(r,\varphi) = \vec{e_z} \sum R_n \Phi_n$ with

$$R_n = a_n r^n + b_n r^{-n},$$
$$\Phi_n = c_n \sin(n\varphi) + d_n \cos(n\varphi);\ n \in N$$

The φ-dependence of the vector potential can only be formed according to the current density, which means $n = 1$ and $c_n = 0$. For the vector potential, the following approach remains with the four unknown constants C_{Aa1}, C_{Aa2}, $C_{Ai1}$and C_{Ai2}.

$$A(r,\varphi) = \left(C_{Aa1}\frac{r}{r_a} + C_{Aa2}\frac{r_a}{r}\right)\cos(\varphi - \varphi_a) + \left(C_{Ai1}\frac{r}{r_i} + C_{Ai2}\frac{r_i}{r}\right)\cos(\varphi - \varphi_i) \qquad (2)$$

With equation (2) the tangential component of the magnetic field $H_\varphi(r,\varphi) = -\frac{1}{\mu_0}\frac{\delta A}{\delta r}$ could be calculated.

$$H_\varphi(r,\varphi) = -\frac{1}{r_a\,\mu_0}\left(C_{Aa1} - C_{Aa2}\frac{r_a^{\ 2}}{r^2}\right)\cdot\cos(\varphi - \varphi_a) - \frac{1}{r_i\,\mu_0}\left(C_{Ai1} - C_{Ai2}\frac{r_i^{\ 2}}{r^2}\right)\cos(\varphi - \varphi_i) \qquad (3)$$

The tangential component of the magnetic field strength on the surface of the rotor must correspond to the current density $H_\varphi(r_i,\varphi) = J_i(\varphi)$ of the rotor and it must correspond to the negative current density $H_\varphi(r_a,\varphi) = -J_a(\varphi)$ at the inner side of the stator. Now the expressions with equal φ-dependencies could be compared.

$$C_{Aa1} - C_{Aa2} = \frac{1}{2}\mu_0 w I_a\,; \qquad C_{Aa1} = C_{Aa2}\frac{r_a^2}{r_i^2}$$

$$C_{Ai1} - C_{Ai2} = -\frac{1}{2}\mu_0 w I_i\,; \qquad C_{Ai1} = C_{Ai1}\frac{r_i^2}{r_a^2}$$

With this the magnetic field in the air gap is completely described.

$$A(r,\varphi) = \frac{1}{2}\mu_0 w \frac{r_a r_i}{r_a^2 - r_i^2}\left[\left(\frac{r}{r_i} + \frac{r_i}{r}\right) I_a \cos(\varphi - \varphi_a) + \left(\frac{r}{r_a} + \frac{r_a}{r}\right) I_i \cos(\varphi - \varphi_i)\right] \qquad (4)$$

3.THE INDUCTANCES OF THE SYSTEM

With the vector potential (4) the magnetic energy in the air gap can be calculated $W_m = 1/2\int_{V-Leiter} \vec{J}\vec{A}\, dV$. The integration has to be execute over the conductor volume. The energy and thus the inductances are each related to the length.

$$W_m = \frac{1}{2}\int_0^{2\pi} J_a(\varphi) A(r_a,\varphi) r_a\, d\varphi + \frac{1}{2}\int_0^{2\pi} J_i(\varphi) A(r_i,\varphi) r_i\, d\varphi$$
$$= \frac{1}{2}\left(L_a I_a^2 + 2M I_a I_i + L_i I_i^2\right)$$

This leads directly to the self and mutual inductances.

$$L_a = L_i = L = \frac{\pi}{4}\mu_0 w^2 \frac{r_a^2 + r_i^2}{r_a^2 - r_i^2} \qquad (5a)$$

$$M = M_0 \cos(\varphi_a - \varphi_i) \qquad (5b)$$

$$M_0 = \frac{\pi}{4}\mu_0 w^2 \frac{2 r_a r_i}{r_a^2 - r_i^2}$$

The self-inductances of the two coils thus have the same constant value L, while the mutual inductance M between the two coils depends on the cosine of the differential angle $(\varphi_a - \varphi_i)$.

4.THE VOLTAGE EQUATIONS FOR THE COILS

The winding of the stator is initially open and may have the position $\varphi_a = 0$.

The windings of the rotor have the resistance R_i. They are connected to a DC voltage U. So the following DC current flows.

$$I = U/R_i \tag{6}$$

The rotor turns with the constant angular speed ω and the starting position is φ_0. So the actual position is

$$\varphi_i = \omega t + \varphi_0. \tag{7}$$

Now the stator winding with the ohmic resistance R_a is short-circuited. It flows in it a time dependent current i_a. In the rotor winding is flowing the DC current Eq. (6) plus a time dependent part i_v. The result is $i_i = I + i_v$.

With the inductances Eq. (5) it is possible to create the voltage equations for the coils.

$$0 = R_a i_a + \frac{d}{dt}\left(L i_a + M(I + i_v)\right) \tag{8a}$$

$$U = R_i(I + i_v) + \frac{d}{dt}\left(L(I + i_v) + M i_a\right) \tag{8b}$$

5.THE CURRENTS IN THE COILS

With Eq. (6) in Eq. (8b) the voltage can be reduced by the term $R_i I$. Since the inductance L is constant over time, the term LI is omitted, but not the term MI, since the mutual inductance is a function of time. The equations (8) are thus simplified.

$$0 = R_a i_a + \frac{d}{dt}\left(L i_a + M_0(I + i_v)\cos(\varphi_i)\right) \tag{9a}$$

$$0 = R_i i_v + \frac{d}{dt}\left(L i_v + M_0 i_a \cos(\varphi_i)\right) \tag{9b}$$

Now the simplification 4) from the introduction is used, according to which synchronous generators $R \ll \omega L$ can be used with good practical approximation. It is assumed that $R_a = R_i = 0$. Thus, the differential equations (9) can be integrated simple. Eq. (10) shows the general solution with the integration constants C_{Ia}, C_{Ii}.

$$C_{Ia} = L i_a + M_0(I + i_v)\cos(\varphi_i) \tag{10a}$$

$$C_{Ii} = L i_v + M_0 i_a \cos(\varphi_i) \tag{10b}$$

The two integration constants are determined from the initial conditions for the initially non-existent currents i_a, i_v.

$$i_a(t = 0) = 0 \tag{11a}$$

$$i_v(t = 0) = 0 \tag{11b}$$

With $t = 0$ in Eq. (7) leeds to the angle $\varphi_0 = \varphi_i$. At this angle the currents are zero. With this the constants are solved.

$$C_{Ia} = M_0 I \cos(\varphi_0) \tag{12a}$$

$$C_{Ii} = 0 \tag{12b}$$

With the definition for the constant $\upsilon = \frac{2 r_a r_i}{r_a^2 + r_i^2}$ the currents in the stator (13a) and in the rotor (13b) could be specified.

$$\frac{i_a}{I} = \frac{\upsilon(\cos(\varphi_0) - \cos(\varphi_i))}{1 - \upsilon^2 \cos^2(\varphi_i)} \tag{13a}$$

$$\frac{i_i}{I} = \frac{1 - \upsilon^2 \cos(\varphi_0)\cos(\varphi_i)}{1 - \upsilon^2 \cos^2(\varphi_i)} \tag{13b}$$

The calculation of the currents are carried out without the ohmic resistances in the two windings, whereby it does not decay. Thus, the evaluation of only the first period is performed. It can be seen that with a diameter ratio $r_a/r_i = 0.8$, a considerable peak value of the current $i_{a/i}/I = 40$ arises both in the rotor and in the stator.

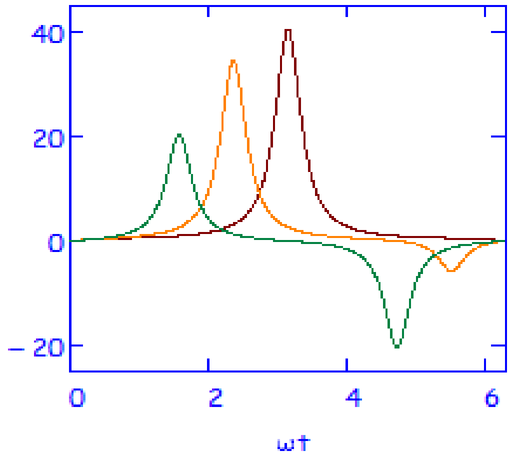

Figure 3 Current propagation in stator depending from the entry of the short circuit with $\varphi_0 = 0$ (brown), $\varphi_0 = \pi/4$ (orange), $\varphi_0 = \pi/2$ (green)

In the stator, the current is positive and negative, if the angle $\varphi_0 > 0$, figure 3. In the rotor, however, he always remains positive corresponding figure 4.

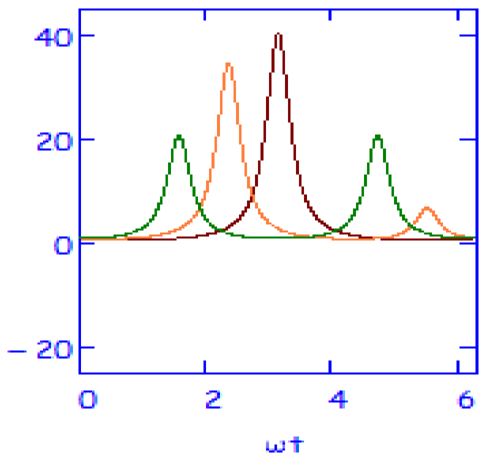

Figure 4 Current propagation in rotor depending from the entry of the short circuit with $\varphi_0 = 0$ (brown), $\varphi_0 = \pi/4$ (orange), $\varphi_0 = \pi/2$ (green)

It can be noted also, that at the same angles φ_0 the maxima in the figures 3 and 4 are to be found at the same times.

The minimum and maximum values of the current $\hat{i_a}$ are obtained by vanishing its derivative $di_a/dt = 0$ in Eq. (13a). That leads to the result $\sin(\varphi_i) = 0$ and for the cosine to the point $\cos(\varphi_i) = \begin{Bmatrix} -1 \\ +1 \end{Bmatrix}$.

$$\frac{\hat{i_a}}{I} = \frac{v\left(\cos(\varphi_0) - \begin{Bmatrix} -1 \\ +1 \end{Bmatrix}\right)}{1-v^2} \qquad (14)$$

It can be seen that the maximum current increases at a diameter ratio $r_a/r_i = 0.8$ and an initial angle $\varphi_0 = 0$ to 40 times of the direct current in the rotor. If the initial angle is $\varphi_0 = 90°$, the minimum is 20 times.

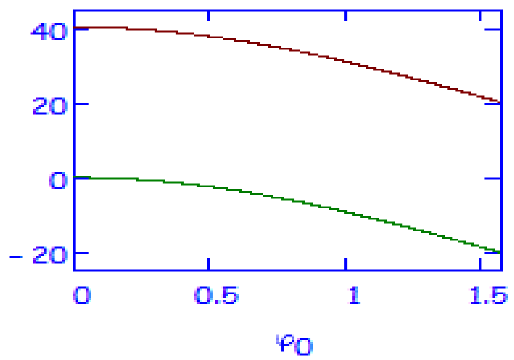

Figure 5 Maximum and minimum values of the stator current for initial values of the angle $\varphi_0 = 0 \,..\, \pi/2$, max (brown), min (green)

After the occurrence of the short circuit, a DC component $\overline{i_a}$ flows in the stator winding. It could to be found by integration over one period of the current.

$$\frac{\overline{i_a}}{I} = \frac{1}{2\pi}\int_0^{2\pi} \frac{i_a}{I}\, d(\omega t) = \frac{2 r_a r_i}{r_a^2 - r_i^2} \cos(\varphi_0) \qquad (15)$$

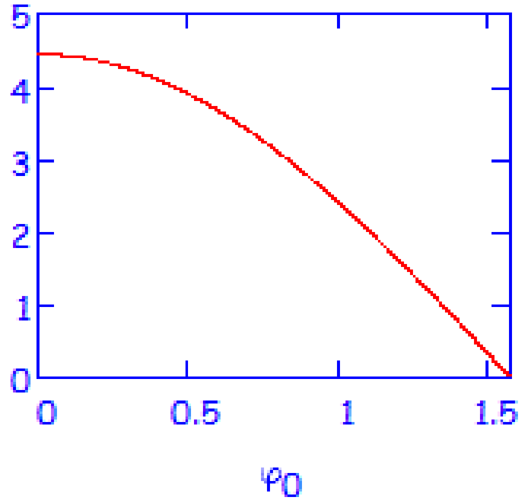

Figure 6 DC component when the short circuit appears

The specified average value is therefore depended on the position of the rotor winding φ_0, when the short circuit appears at $t = 0$.

If the angle is $\varphi_0 = 0$, the direct component is maximum, whereas at $\varphi_0 = \pi/2$ it disappears.

CONCLUSION

In a simple way, the currents in the stator and in the rotor are calculated in the event of a short circuit. Furthermore, maximum and minimum values and the direct current component are specified.

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