AN ANALYTICAL SOLUTION FOR THE DIFFERENTIAL EQUATION OF THE EXCITER OF SYNCHRONOUS GENERATORS

Hălălae I., *„Eftimie Murgu" University of Reșița, România*
Răduca E., *„Eftimie Murgu" University of Reșița, România*
Stroia M. D., *„Eftimie Murgu" University of Reșița, România*

ABSTRACT: *The synchronous generator connected to a network of infinite power through a long transmission line is represented by a mathematical model formed from a set of 12 differential equations. This system can be divided in three parts: the Park's equations (9 equations, describing exclusively the synchronous generator), the connection of the generator at the energy network (2 equations) and the exciter equation. Generally, due to the technological interdependence restraints, this system is generally modelled using finite element methods. In this paper, we approach the exciter's equation from an analytical angle, and we find a general solution including constants depending of the concrete case (specific for each generator). In order to make our approach clearer and easy to replicate, we insert two case studies. Since the practitioners are usually approximating the exciter functioning as being linear, we added, for comparison, linear functions submitted to the same initial conditions.*
Our work here is part of a larger project, aiming the realisation of a software package for modelling the equations of a synchronous generator using simpler methods, preferably based on analytical solutions and not using approximating methods based on finite element.

KEYWORDS: *synchronous generator, exciter differential equation, analytical solution, case studies, Scilab®.*

1.INTRODUCTION

The synchronous generator connected to a network of infinite power through a long transmission line is represented by a mathematical model formed from a set of 12 differential equations grouped as such: first 9, known as Park's equations are exclusively describing the synchronous generator; next two are regarding the connection of the generator at the energy network, and the last one is the exciter equation.
In the present paper we study the exciter equation. We first determine an analytical solution for the equation and then we apply it in two case studies. Since, in practice, the exciter is approximate using a linear function, we computed for both cases a linear equivalent function and compared results with the analytical solution of the equation.
Generally, the equations of the synchronous generator are solved using approximation methods. We determine the analytical solution for the exciter functioning and, using the Scilab® simulation package; we display the evolution of the solutions.

2. MATHEMATICAL MODEL OF A SYNCHRONOUS GENERATOR CONNECTED TO A NETWORK OF INFINITE POWER THROUGH A LONG TRANSMISSION LINE

The synchronous generator connected to a network of infinite power through a long transmission line is represented, as a nonlinear mathematical model, by the following set of 12 differential equations grouped as such: first 9, known as Park's equations are exclusively describing the synchronous generator; next two are the regarding the connection of the generator at the energy network, and the last one is the exciter equation.

a. The equations of the synchronous generator (Park's equations):

$$\frac{\partial}{\partial t}(\delta) = \omega_0 \cdot s \qquad (1)$$

$$M \frac{\partial}{\partial t}(s) = -k_d \cdot s + T_m - T_e \qquad (2)$$

$$T_{d0}^{'} \frac{\partial}{\partial t}\left(e_q^{'}\right) = -V_f - \left(x_d - x_d^{'}\right) \cdot i_d^{'} - e_q^{'} \qquad (3)$$

$$T_{d0}^{'} \frac{\partial}{\partial t}\left(e_q^{'}\right) = e_q^{'} - \left(x_d^{'} - x_d^{''}\right) \cdot i_d - e_q^{'} \qquad (4)$$

$$T_{q0}^{''} \frac{\partial}{\partial t}\left(e_q^{''}\right) = \left(x_q - x_q^{''}\right) \cdot i_d - e_d^{''} \qquad (5)$$

$$e_d^{''} = v_d + \gamma_a \cdot i_d - x_q^{''} \cdot i_q \qquad (6)$$

$$e_q^{''} = v_q + \gamma_a \cdot i_q - x_d^{''} \cdot i_d \qquad (7)$$

$$T_e = e_d^{''} \cdot i_d + e_q^{''} \cdot i_q - \left(x_d^{''} - x_q^{''}\right) \cdot i_d \cdot i_q \qquad (8)$$

$$v_t^2 = v_d^2 + v_q^2 \qquad (9)$$

b. The equations for the connection of the generator at the energy network:

$$v_d = v_b \cdot \sin\delta + \gamma_e \cdot i_d - x_e \cdot i_q \qquad (10)$$

$$v_q = v_b \cdot \cos\delta + \gamma_e \cdot i_d + x_e \cdot i_q \qquad (11)$$

c. The exciter equation:

$$T_{ex} \frac{\partial}{\partial t}\left(v_j\right) = u - v_j \qquad (12)$$

where:

$T_{d0}^{'}$, $T_{d0}^{''}$, $T_{q0}^{''}$ - time constants;
T_{ex} - time constant of the exciter;
δ - the rotor angle;
ω_0 - the synchronicity speed;
T_m , T_e - mechanical and electrical torque;
i_d , i_q - the current projections on the axis *d* and *q*;
k_d - damping coefficient;
M - moment of inertia;
$e_q^{'}$ - transient electromotor voltage on the axe *q*;
$e_d^{''}$, $e_q^{''}$ – over-transient electromotor voltages on the axis *d* and *q*;
v_d , v_q - voltage circuit projections on the axis *d* and *q*;
v_f - excitation drive;
v_t - circuit voltage;
v_b - network voltage;
u - cue voltage for the exciter;
x_d , x_q - reactants on the axes *d* and *q*;
$x_d^{'}$, $x_q^{'}$ - transient reactants;
$x_d^{''}$, $x_q^{''}$ - over-reacting reactants;
x_e - transmission line reactance;
γ_a - internal resistance;
γ_e - transmission line resistance;
s - sliding.

The topics of the equations of a synchronous generator and modelling the set of differential equations by using computer methods are rather classical. We started our work using [2] and, then, we extracted from [1] the presentation above.

3. DETERMINATION OF THE ANALYTICAL SOLUTION FOR THE EQUATION OF THE EXCITER

In the system above, the equations (1)-(9) and (10)-(11) are grouped and interdepending. The equation (12) of the exciter is independent. We will compute an analytical solution of this equation.
First, we have the (12) form:

$$T_{ex}\frac{\partial}{\partial t}(v_j)=u-v_j$$

where:
T_{ex} - time constant of the exciter;
v_j - circuit voltage;
u - cue voltage for the exciter.

By rewring the equation (12) as follows:

$$T_{ex}\frac{\partial}{\partial t}(v_j)+v_j=u \qquad (13)$$

we notice that we have an nonhomogeneous linear differential equation of the first degree, with constant coefficients.
By applying the general methodology [4], we obtain a general solution:

$$v_j(t)=C_1\cdot e^{-\frac{1}{T_{ex}}\cdot t}+C_2\cdot u \qquad (14)$$

where C_1 and C_2 are constants specific for each generator, determined from the initial technologic specifications.

4. TWO CASE STUDIES

We present further two applicative examples for formula (14). In order to determine the C_1 and C_2 constants for the formula (14), we use the technological constraints:
u - cue voltage for the exciter (*V*);
v_{j0} - initial value of the voltage (*V*);
t_{ex} - cue current value of the exciter (*Amp*);
T_{ex} - time constant of the exciter.

We obtain a system as follows:

$$\begin{cases} v_j(0)=v_{j0} \\ v_j(T_{ex})=u \end{cases} \qquad (15)$$

We have to add an observation: in practice, the functioning of exciter is, usually, considered as being linear and approximated by a linear function of first degree. So, we also computed a linear function submitted to the same initial conditions. We start from a general linear equation of the first degree:

$$v_{jlin}(t)=a\cdot t+b \qquad (16)$$

and obtain the system:

$$\begin{cases} v_{jlin}(0)=v_{j0} \\ v_{jlin}(T_{ex})=u \end{cases} \qquad (17)$$

(we introduced the notation v_{jlin} for linear approximation of v_j)
We displayed both results. We used experimental data from [3]. According to the request of the beneficiary, we are not allowed to give nominal detail on the power stations (we will name them Case study 1 and Case study 2).

4.1. Case study 1

For the hydroelectric power station Case study 1, the technologic constraints are:
$u=24V$
$v_{j0}=1.5V$
$t_{ex}=40Amp$
$T_{ex}=7.5s$
By replacing those values in (15) we obtain the system:

$$\begin{cases} v_j(0)=1.5 \\ v_j(7.5)=40 \end{cases} \qquad (18)$$

with the solutions $C_1=-225.15$ and $C_2=9.443$.
So, the particular form of the exciter

equation for the hydroelectric power.
Case study 1 is:

$$v_j(t) = -225.15 \cdot e^{-\frac{t}{40}} + 226.663 \qquad (19)$$

The graphical representation of function (16) is in figure 1:

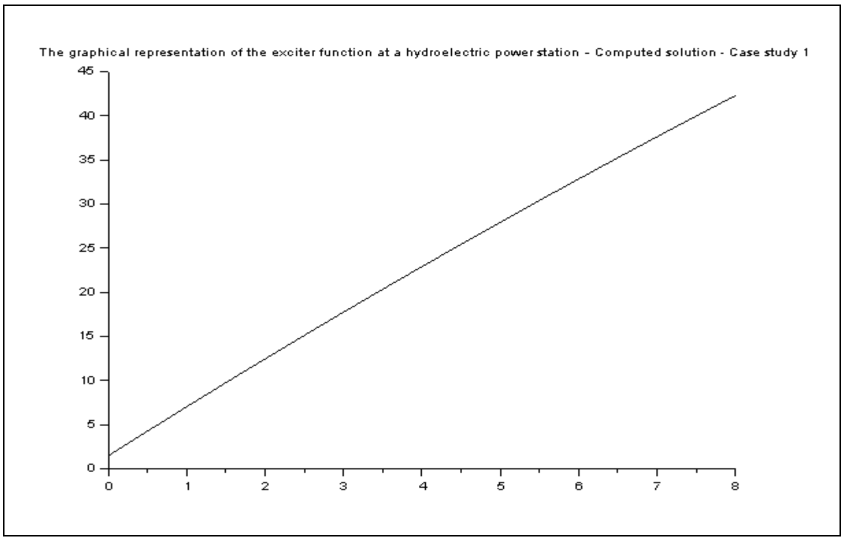

Figure 1. The graphical representation of the exciter function for Case study 1 hydroelectric power station – computed solution

By imposing the same values in (17), we obtain the empirical linear approximation of the function (19):

$$v_{jlin}(t) = 5.13 \cdot t + 1.5 \qquad (20)$$

The graphical representation of the function (20) is in figure 2:

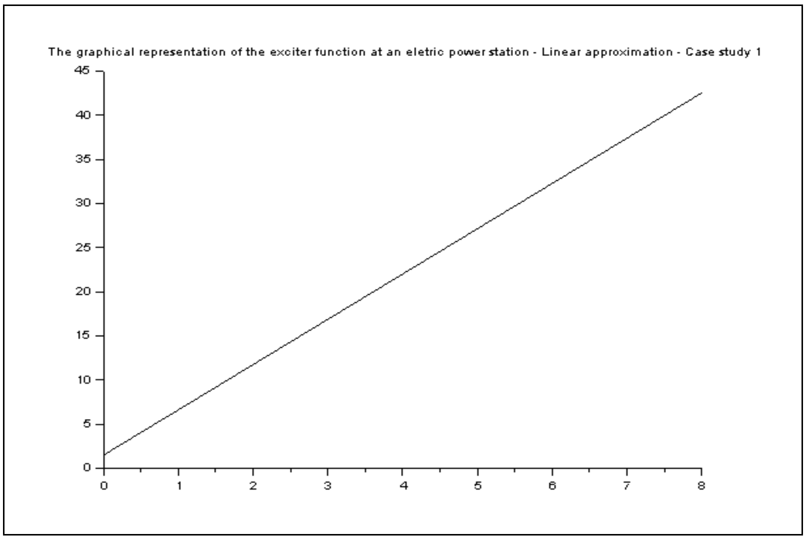

Figure 2. The graphical representation of the exciter function for Case study 1 hydroelectric power station – linear approximation

4.2. Case study 2

For the hydroelectric power station Case study 2, the technologic constraints are:

$$u = 24V$$

$v_{j0} = 2.5V$

$t_{ex} = 58.5Amp$

$T_{ex} = 9s$

By replacing these values in (15) we obtain the system:

$$\begin{cases} v_j(0) = 2.5 \\ v_j(7.5) = 58.5 \end{cases} \quad (21)$$

with the solutions $C_1 = -410$ and $C_2 = 17.18$. So, the particular form of the exciter equation for the hydroelectric power Case study 2 is:

$$v_j(t) = -400 \cdot e^{-\frac{t}{58.5}} + 412.32 \quad (22)$$

The graphical representation of the function (19) is in figure 3:

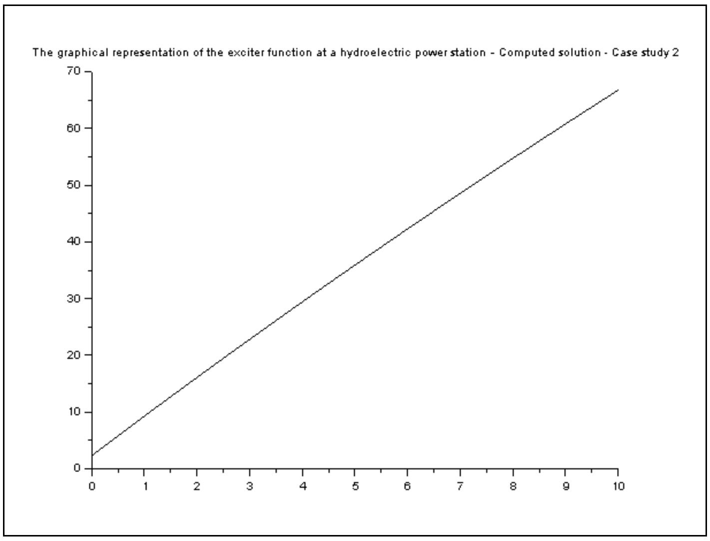

Figure 3. The graphical representation of the exciter function for Case study 2 hydroelectric power station – computed solution

By imposing the same values in (17), we obtain the empirical linear approximation of the function (22):

$$v_{jlin}(t) = 6.22 \cdot t + 2.5 \quad (23)$$

The graphical representation of the function (23) is in figure 4:

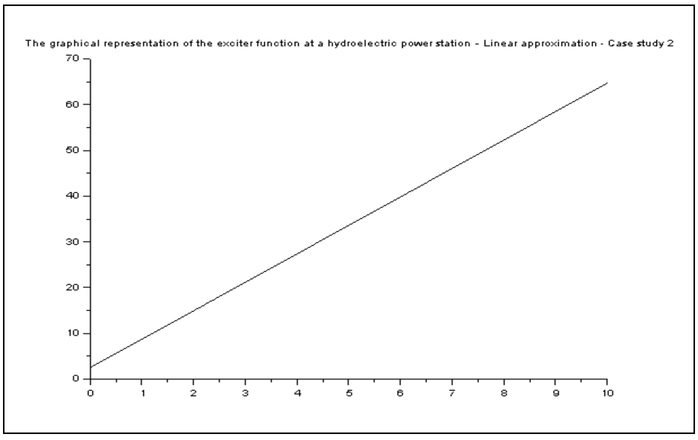

Figure 4. The graphical representation of the exciter function for Case study 2 hydroelectric power station – linear approximation

We used, for the representations, the Scilab® software package [5].

CONCLUSIONS

In this paper we presented analytical solution for the differential equation of the exciter for the synchronous generators.
After establishing a general solution, we realised two case studies, in order to illustrate how our model can be replicated.
Since, in practice, the functioning of exciter is usually considered as being linear and approximated by a linear function of first degree, for the case studied, we computed, for both cases, a linear function with the same technological constraints. We illustrated the evolution of each function in separate figures. The graphical similarities are representing a rather strange approximation of an exponential function by a linear one, and are confirming the empirical approximation.
Our work presented here is a part of a larger project, aiming to realise a complete set of simulation of the equations that model the synchronous generator. The choice of beginning with the equation of the exciter is based on the fact that, in the sets of 12 equations, this is independent, so it allows a separate study.
The main development idea for this work is to find, if possible, analytical solutions for the complete system.

Acknowledgements

We wish to express our special thanks and gratitude to professor Gheorghe Liuba from „Eftimie Murgu" University of Reșița, for his constant patience, support and encouragement and, also, for the access to experimental data from his research projects.

REFERENCES

[1] Filip I., Structuri Şi Algoritmi De Conducere Adaptivă, Editura Orizonturi Universitare, Timişoara, 2002

[2] Liuba Gh., Introducere în teoria maşinilor şi acţionărilor electrice, Editura Politehnica, Timişoara , 2005

[3] Liuba Gh., Oprimizarea funcţionării hidroagregatelor de la C.H.E. Ruieni prin folosirea datelor şi a sistemului de monitorizare, Contract Nr. 25/11.06.2004 încheiat între C.C.H.A.P.T./U.E.M.R. şi C.H.E. Ruieni

[4] Rogai E., Exerciţii şi probleme de ecuaţii diferenţiale şi integrale, Bucureşti, Editura tehnică, 1965

[5] http://www.scilab.org/