THEORETICAL SOLUTIONS TO REDUCE THE VIBRATION LEVEL OF AGGREGATES WITH TWO ROTORS COUPLED TO AN ELASTIC SHAFT

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ABSTRACT. *In this paper we presented the theoretical model for the study of the vibrations of a rotating assembly consisting of a turbine rotor coupled by an elastic shaft to the rotor of an electric generator, to which vibrations come from the deformation of the shaft during operation and the static imbalance. By using Lagrange's equations of second degree, we established the movement equations for rotors of the generator and of the turbine. From the analize of the approximate solutions found we draw some conclusions regarding the conditions for obtaining a stable functioning of the machine.*

KEY WORDS: *mechanical vibrations, rotating machines, rotors, static and dynamic balancing.*

1. INTRODUCTION

When designing aggregates comprising two rotors coupled by an elastic shaft, it must be taken into account that, following operations at high revolutions, the coupling shaft can deform. Due to this deformation the center of gravity moves towards the axis of the shaft with a certain distance ***e***. Thus, disturbing forces arise which cause transverse vibrations of the machine. To avoid their negative manifestations, they must be studied. This avoids both excessive deformation of the shaft and the resonance phenomenon. In addition, information can be obtained on the correlation between kinematic parameters and geometric elements.

For aggregates already in operation and which exhibits a significant level of vibrations, it is important to carry out prior expertise, and then to conclude on the causes of these vibrations. If vibrations come from imbalances this must be taken into account and to take action accordingly.

2. DETERMINING THE DYNAMIC MODEL AND OBTAINING THE DIFFERENTIAL EQUATIONS OF THE MOVEMENT

In order to determine the dynamic model we consider one of the rotors, for example the turbine rotor, mounted together with the second rotor, that of the electric generator, on an elastic shaft of diameter ***d;*** the distance between the two rotors is ***l*** (figure 1.a). We also know: the turbine rotor mass ***m,*** the turbine rotor eccentricity ***e*** (figure

1.b), the moment of inertia in relation to an axis passing through the center of gravity **C** parallel to the bearing line **J,** the elastic bending constant of the shaft right next to the turbine ***k***, the transverse elastic modulus of the shaft **G,** the polar inertia moment of the transverse section of the shaft **Ip**, the moment of inertia of the generator rotor relative to the shaft axis J_1, the moment with which the steam acts on the turbine M_1, and the moment with which the generator stator acts on the rotor M_2.

It is to notice that the generator rotor movement has a parallel-plane motion and, for the study, we choose a reference system ***Oxy,*** displayed in the plan of transverse symmetry of the rotor (figure 2.b). Point ***C*** is the center of gravity, point **A** is the rotor attachment point, and point **O** is the intersection point of the plane with the line of bearings.

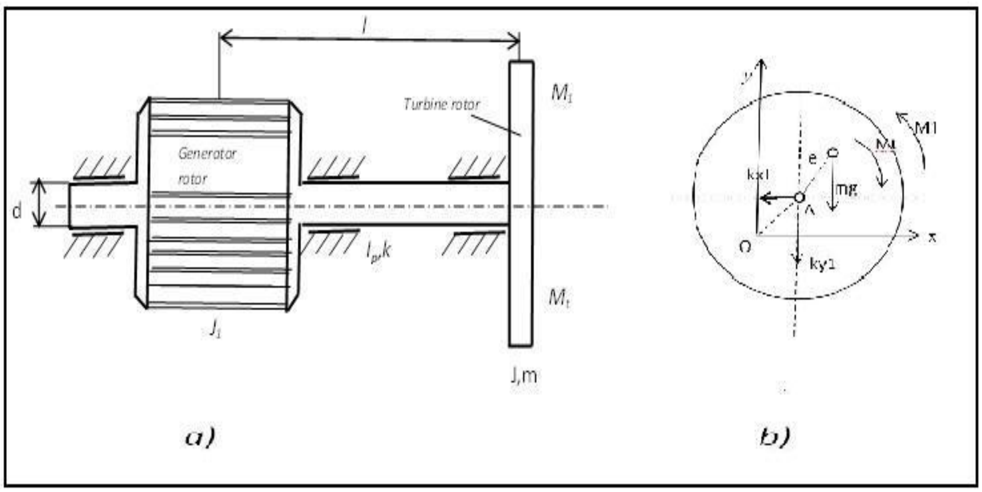

Fig. 1. The dynamic model of the rotating assembly: a) the generator rotor - turbine rotor assembly; b) the distribution of forces and moments

We obtain the differential equations of movement with the help of the Lagrange equations of second degree [5]:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_i}\right) - \frac{\partial E_c}{\partial q_i} = Q_i \qquad (1)$$

where:

E_c - kinematic energy;

q_i - generalized coordinate;

$\dot{q}_i$ - generalized speed;

Q_i – generalized force.

We choose as generalized coordinates the parameters:

- x_1, y_1 - the coordinates of point A;
- φ - the angle of rotation of the rotor (figure 2a).

The differential equations of movement, obtained from Lagrange's equation, are:

$$m[\ddot{x}_1 - e(\ddot{\varphi}\sin + \dot{\varphi}^2\cos\varphi)] = -kx_1 \qquad (2)$$

$$m[\ddot{y}_1 - e(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi)] = -ky_1 - mg \qquad (3)$$

$$(J+me^2)\,\ddot{\varphi} - em(\ddot{x}_1^2\sin\varphi - \ddot{y}_1\cos\varphi) = -mge\cos\varphi + M_1 - \frac{\varphi-\theta}{l}GI_p, \qquad (4)$$

with the notations:

- $q_1=x_1$;
- $q_2=y_1$;
- $q_3=\varphi$.

The differential equation of movement of the rotor is established using the kinetic momentum theorem:

$$J_1\ddot{\theta} = -\frac{\theta-\varphi}{l}GI_p - M_2 \qquad (5)$$

3. THE DETERMINATION OF APPROXIMATE SOLUTIONS OF MOTION EQUATIONS

The equations (2), (3), (4) and (5) form a system of unknowns x_1, y_1, θ and φ. Since this system cannot be integrated, approximate solutions are sought.

We assume that the angular velocity of rotation of the turbine $\dot{\varphi}$ is approximately equal to the angular velocity of the regime ω_0 (constant). Thus it results that the angular acceleration is null, so we can replace $\ddot{\varphi} = 0$ and the differential equations (2) and (3) become:

$$m(\ddot{x}_1 - e\,\dot{\omega}_o^2\cos\omega_o t) = -kx_1 \qquad (6)$$

$$m[\ddot{y}_1 - e\,\omega_o^2\sin\omega_o t] = -ky_1 - mg \qquad (7)$$

Equations (6) and (7) have the general solutions:

$$x_1 = \frac{e\omega_o^2 m}{k-\omega_o^2 m}\cdot\cos\omega_0 t + A\cos\omega_n t + B\sin\omega_n t \qquad (8)$$

$$y_1 = -\frac{mg}{k} + \frac{e\omega_o^2 m}{k-\omega_o^2 m}\cdot\sin\omega_0 t + C\cos\omega_n t + D\sin\omega_n t \qquad (9)$$

where:

- we noted $\omega_n = \sqrt{\frac{k}{m}}$;
- A, B, C, D are integration constants determined by the initial conditions.

We return to equation 4. First, we replace the solutions (8) and (9) and, using the notations:

- $F(t) = M_1 - mge\cos\omega_0 t + ek[\cos\omega_n t\,(C\cos\omega_0 t - A\sin\omega_0 t) + \sin\omega_n t\,(D\cos\omega_o t - B\sin\omega_o t)]$,
- $J_0 = J + me^2$ (Steiner's theorem)

we get:

$$J_o J_1(\ddot{\varphi} - \ddot{\theta}) + \frac{GIp}{l}(J_o + J_1)(\varphi - \theta) = M_1 J_1 + M_2 J_o + J_1 F(t) \qquad (10)$$

This is the differential equation of the torsional deformations of the shaft, where the term $F(t)$ has the role of disturbance, and has a solution in the form of a natural oscillation:

$$(\varphi - \theta) = A\cos\alpha t + B\sin\alpha t. \qquad (11)$$

4. THE INTERPRETATION OF APPROXIMATE SOLUTIONS OF MOTION EQUATIONS

By analyzing the solutions (8) and (9) we can conclude that the bending vibrations of the rotor are periodic, and their amplitude can be reduced if the eccentricity ***e*** decreases. Also, the beating or resonance phenomena can be prevented if the pulse ω_n is different from ω_n, ie the angular rotation speed of the turbine does not have to coincide with the natural pulsation of the bending oscillations of the shaft.

By analyzing the solution (11) it is to note that, in case of shaft torsion, it is a harmonic vibration, and the own pulsation of the relative oscillations is:

$$\alpha = \sqrt{\frac{GI_p\ (J_0+J_1)}{lJ_0 J_1}} \tag{12}$$

So, since the disturbance *F(t)* has a term in $\cos \omega_0 t$, a condition to avoid resonance would be that the pulse of the swing oscillations α is different from the pulse of the disturbance ω_0. Also, *F(t)* has components in sinus products and differences of sinus and cosine functions with the arguments $(\omega_0 - \omega_n)t$ and $(\omega_0 + \omega_n)t$, so to avoid the phenomenon of resonance, $\alpha \neq \omega_0 \pm \omega_n$.

The mathematical resonance avoidance conditions will be:

$$\omega_o \neq \omega_n = \sqrt{\frac{k}{m}} \tag{13}$$

$$\omega_o \neq \sqrt{\frac{GI_p\ (J_0+J_1)}{lJ_0 J_1}} \tag{14}$$

$$\omega_o \pm \omega_n = \sqrt{\frac{GI_p\ (J_0+J_1)}{lJ_0 J_1}} \tag{15}$$

If considered the case when $F(t) = 0$ (the perturbation is null) and the turbine rotor rotates evenly, from the equation (10) we obtain:

$$\varphi - \theta = \frac{M_1 l}{GI_p} \tag{16}$$

So, the relative deformation between the turbine and generator rotors would be constant.

CONCLUSIONS

When running two-rotor aggregates on the same shaft, there are different types of vibrations with the main causes: imbalance, beating phenomena, resonance phenomenon. The study shows that there are a number of theoretical solutions for the reduction of the vibration level, summarized by relationships (6). These theoretical solutions must be taken into account when designing. If vibrations occur after a certain amount of operating time, it has to be analyzed if there were any imbalances and a new dynamic rebalancing is needed.

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