DETERMINATION OF THE FUNDAMENTAL CHARACTERISTICS FOR THE CASE OF LONG LINES RESULTING FROM DIMENSIONAL ANALYSIS

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ABSTRACT: *Is presented a comparative dimensional analysis between the propagation of mechanical energy and electromagnetic energy through long lines.*

KEYWORDS: *mechanical energy, electromagnetic energy, long lines, Theory of Sonics.*

INTRODUCTION

We write off long line equations in sonic case:

$$\begin{cases} \dfrac{d\underline{H}}{dx} = ja\underline{L}\underline{I} \\ \dfrac{d\underline{I}}{dx} = ja\underline{C}\underline{H} \end{cases} \qquad (1)$$

in which,

$$\underline{L} = L - j\frac{R}{a}, \quad \underline{C} = C - j\frac{G}{a}; \qquad (2)$$

For unit length of pipeline, l = 1m, the relationship of sonic resistance becomes:

$$R = K\frac{\gamma}{g\Omega} \qquad (3)$$

Under the same condition, l = 1m, the sonic inductance becomes:

$$L = \frac{\gamma}{g\Omega} \qquad (4)$$

With the help of relations (3) and (4) we can write the first relation in the group (2) in the form:

$$\underline{L} = \frac{\gamma}{g\Omega}\left(1 - j\frac{K}{a}\right) \qquad (5)$$

Analogously we can say:

$$G = K_1C \qquad (6)$$

where in the condition, l = 1m, C takes the form:

$$C = \frac{\Omega}{E} \qquad (7)$$

Conform relaţiei (I.2.5.6) a doua relaţie din grupul (2) devine:

$$\underline{C} = \frac{\Omega}{E}\left(1 - j\frac{K_1}{a}\right) \qquad (8)$$

So in the case of the parameters distributed on the unit of length, we have obtained the relations of the form:

$$R = KL \quad ; \quad G = K_1C \qquad (9)$$

These relationships are dimensionally verified for both sonic and electrical:

a) For sonic case :

$$\begin{cases} [R] = \dfrac{Kgf \cdot s}{cm^5} = \dfrac{N \cdot s}{m^5} \\ [L] = \dfrac{Kgf \cdot s^2}{cm^5} = \dfrac{N \cdot s^2}{m^5} \\ [G] = \dfrac{cm^5}{Kgf \cdot s} = \dfrac{m^5}{N \cdot s} \\ [C] = \dfrac{cm^5}{Kgf} = \dfrac{m^5}{N} \end{cases}$$

or, using S.I .:

$$[R] = M \cdot L \cdot T^{-2} \cdot T \cdot L^{-5} = M \cdot L^{-4} \cdot T^{-1}$$

$$\left[\frac{R}{l}\right] = M \cdot L^{-5} \cdot T^{-1}$$

$$[L] = M \cdot L \cdot T^{-2} \cdot T^2 \cdot L^{-5} = M \cdot L^{-4}$$

$$\left[\frac{L}{l}\right] = M \cdot L^{-5}$$

From which it follows clearly:

$$\mathbf{\frac{R}{l} = K \frac{L}{l}}, \ \textbf{în care [K] = s}^{\mathbf{-1}}$$

$$[G] = M^{-1} \cdot L^5 \cdot L^{-1} \cdot T^2 \cdot T^{-1} = M^{-1} \cdot L^4 \cdot T$$

$$\left[\frac{G}{l}\right] = M^{-1} \cdot L^3 \cdot T$$

$$[C] = M^{-1} \cdot L^5 \cdot L^{-1} \cdot T^2 = M^{-1} \cdot L^4 \cdot T^2$$

$$\left[\frac{C}{l}\right] = M^{-1} \cdot L^3 \cdot T^2$$

From which it follows clearly:

$$\mathbf{\frac{G}{l} = K_1 \cdot \frac{L}{l}}, \quad \textbf{în care: } [K_1] = s^{-1}$$

b) for the electric case:

$$\left[\frac{L}{l}\right] = \left[L^1 \cdot M^1 \cdot T^{-2} \cdot I^{-2}\right] = [\mu]$$

(magnetic permeability)

$$\left[\frac{R}{l}\right] = \left[L^1 \cdot M^1 \cdot T^{-3} \cdot I^{-2}\right]$$

Of which it is evident

$$\mathbf{\frac{R}{l} = K \frac{L}{l}}, \quad \textbf{în care: } \mathbf{[K] = s^{-1}}$$

$$\left[\frac{C}{l}\right] = \left[L^{-3} \cdot M^{-1} \cdot T^4 \cdot I^2\right] = [\varepsilon]$$

(electrical permeability)

$$\left[\frac{G}{l}\right] = \left[L^{-3} \cdot M^{-1} \cdot T^3 \cdot I^2\right]$$

From which it follows clearly:

$$\mathbf{\frac{G}{l} = K_1 \cdot \frac{C}{l}}, \quad \textbf{in wich:}$$

$$\mathbf{[K_1] = s^{-1}}$$

Analyzing the dimensional analogies presented in points a) and b) the following conclusions are highlighted:

By specialization, in the case of the propagation of electromagnetic energy through long lines (double-line or coaxial line type), the formulas for inductance L 'and C' distributed per unit length are:

1) **Ideal case (R = 0, G = 0)**

$$L' = 4\mu \ln\left[\frac{d+\sqrt{d^2-4r^2}}{2r}\right]$$

$$C' = \frac{\varepsilon}{4\ln\left[\frac{d+\sqrt{d^2-4r^2}}{2r}\right]}$$

2) **Double line:**

$$L' \cong \frac{\mu}{\pi}\ln\frac{d}{r_0}; \qquad C' \cong \frac{\pi\varepsilon}{\ln\frac{d}{r}}$$

3) **Coaxial cable:**

$$L' \cong \frac{\mu}{2\pi}\ln\frac{r_e}{r_i}; \qquad C' \cong \frac{2\pi\varepsilon}{\ln\frac{r_e}{r_i}}$$

Each conductor length unit contains the four distributed parameters: L, R, C, G.

CONCLUSIONS

1) The units of magnitude of inductance and distributed capacity per unit of length are identical to those of magnetic permeability μ and electrical permitivity ε.

2) In all the cases presented, the value of the inductance distributed on the unit of length L can be written as L = A μ, where A is a non-dimensional coefficient, which depends on the geometric shape of the line, and the value of the capacity distributed per unit of length , C can be written as C = Bε, where B is a non-dimensional coefficient, depending on the geometric shape of the line.

3) Coefficients A and B depend on the practical way of constructing and positioning the long line, so its geometry.

4) Relationships (9) are checked dimensionally in both sonic and electromagnetic cases.
In both cases, the proportionality ratios K and K1 have the same unit of measure: [s-1].

5) From the physical significance of the distributed quantities and the correlations between them, without referring to the form of energy, mechanical or electromagnetic, obviously results:

Energy is propagated in an alternative form: kinetic form, potential form, ...; In case of stationary waves the two forms of energy are decoupled: the kinetic form is manifested in the wave nodes and the potential form in the ventre.

Relationships (6) and (9) allow us to write:

$$W_2 = \frac{GH^2}{2} = K_1\frac{CH^2}{2} \qquad W_1 = \frac{RI^2}{2} = K\frac{LI^2}{2} \quad (10)$$

The following are required:

• **The coefficient K depends on the viscosity of the fluid.**

It plays the role of a coefficient of proportionality between the amount of energy accumulated in the kinetic form (defined by the parameter L) and the amount of dissipated energy in this form (defined by parameter R).

• **The coefficient K1 depends on the plasticity (hysteresis) of the fluid.**

It plays the role of a coefficient of proportionality between the amount of energy accumulated in the potential form (defined by parameter C) and the amount of dissipated energy in this form (defined by parameter G).

Coefficients K and K1 have numerical values given for a given type of material, representing the amount of energy dissipated in the propagation process in the line length unit.

Thus, in the case of mechanical energy propagation, K and K1 together with mass and coefficient of elasticity completely define the characteristics of a certain type of material.

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